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CROCKER'S

ELEMENTS

OF

LAND SURVEYING.

NEW EDITION,
CORRECTED THROUGHOUT, AND CONSIDERABLY IMPROVED
AND MODERNIZED,

BY T. G. BUNT,
LAND SURVEYOR, BRISTOL.

TO WHICH ARE ADDED,
TABLES OF SIX-FIGURE LOGARITHMS, &c.
SUPERINTENDED BY
RICHARD FARLEY,
OF THE NAUTICAL ALMANAC ESTABLISHMENT.

LONDON:
LONGMAN, ORME, BROWN, GREEN, & LONGMANS,
PATERNOSTER-ROW.
1841.

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EDITOR'S PREFACE

TO THE

FIFTH EDITION.

THE "Elements of Land Surveying," by the late Mr. Abraham Crocker, is a book well known to the public, having already gone through four editions. A few months ago an application was made to me (through the suggestions of a friend, whose name I feel it to be an honour to myself here to introduce*) by Messrs. Longman and Co., the present proprietors of the work, together with proposals, which were eventually accepted, to undertake the task of thoroughly revising and fitting it for an entirely new edition.

The very extensive circulation which Mr. Crocker's treatise has unquestionably obtained, sufficiently shows that it has long been regarded by the public as possessing, at least, the merit of being superior to any other work on the same subject; although the practical part of the science of Land Surveying, as is well known to persons connected with the profession, is very imperfectly explained by the author. For the purpose of supplying this great deficiency, a very considerable proportion of entirely new matter has been introduced into the present edition, and much of that which is retained has been remodelled, as will be seen in the following enumeration of particulars.

---

* John Phillips, Esq., F.R.S.

Several alterations and corrections have been made in the introductory parts of the work (from p. 1. to p. 140.), which treat on the elements of geometry and trigonometry. All the examples throughout the book, whose solutions had been given by the author, have been recalculated, and the answers corrected where necessary.

Many pages of the chapter on Land Measuring by Normals, and by the Plane Table, and many others relating to obsolete Customary Measures, have been entirely omitted; and the remainder much abridged and altered.

Parts VIII., IX., and X., extending from p. 191. to p. 244., are entirely new, and contain very full and minute directions relating to the modern practice of surveying, both with and without the aid of angular instruments. The methods of plotting estates, and casting or computing their areas, are described; and an engraved specimen is given of the field-book, containing the survey of the estate represented in the frontispiece, from which, together with the full explanation accompanying it, a complete map of the estate may be drawn by the pupil. Some of the instruments and methods which are here described are believed to be new; and the chief improvements which have, it is hoped, been made in the present edition, will be found in these pages.

The chapter on levelling (p. 270. to p. 292.) is also new, the subject having been entirely omitted in the original work.

Another very important and valuable addition to the present volume will be found in the Appendix; viz. a complete table of the logarithms of numbers, from 1 to 10·000, and of sines and tangents to every minute of the quadrant to six decimal places; which is given instead of the imperfect and comparatively useless table contained in the preceding editions of this work.

THOS. G. BUNT,
LAND SURVEYOR.

Small-Street-Court, Bristol,
November, 1839.

MR. CROCKER'S PREFACE.

In all ages and countries, where learning has prevailed, the Mathematical Science has been looked upon as the chief and most considerable branch of education.

It must be confessed, that no science in the world improves the mind of man so much as this does. — It gives a habit of close and demonstrative reasoning, by rendering it exact and capable of solving the greatest difficulties; and by regulating the imagination, and giving the mind the greatest extension and capacity that human nature is capable of attaining.

The usefulness of this science is almost infinite, there being scarcely any art or knowledge in the universe but may be assisted and advanced by it. It was a just observation of *Pope Ganganelli*, "that it was easy, on the perusal of a book, even a moral one, to determine whether or not its author was a mathematician. Such a man," says he, "at the first look, is sure to analyse and unravel a subject or proposition with *justness:* but a man who does not understand this science sees only in a vague, and almost in an imperfect manner. Even philosophy without geometry is like medicine without chemistry. The greater number of modern philosophers reason inconclusively only because they are unacquainted with geometry." *

Geometry (aided by its sister sciences, Arithmetic and Trigonometry) performs all the necessary subjects of mensuration, whether superficial or solid. By these we are enabled to state, in determinate measures, the superficial quantities of such board, stone, glass, lead, land, and water, as we have occasion to inquire about; and also the solidity or capacity of timber, stone, walls, casks, and all other vessels which we use in our domestic concerns, or employ in our commercial affairs. Among the subjects to which these sciences may be applied, and which is far from being the least important, is *Land-measuring.*

---

* See his Letters.

The surface of this globe being divided and subdivided among the sons of men, it becomes necessary that their respective portions should be known and ascertained; and, from thence, their intrinsic or determinate values demonstrated. — Such demonstration cannot be obtained without a previous accurate *mensuration* of the various portions whose respective values are required. The necessity of accuracy, *mathematical accuracy*, in measuring land, is obvious; for without this, the subsequent transaction of the Land-surveyor, in *valuing* it, must be erroneous: the ground-work being fallacious, the superstructure will inevitably be doubtful, if not absolutely false.

Not every one who knows a square from a circle, or a triangle from an ellipse, nor even he who has skill enough to lay a chain in a straight line, is properly qualified for a Land-measurer. He must be well-grounded in the knowledge of numbers; he should understand the arithmetic of Algebra and Logarithms; he must become acquainted with all the leading principles of Geometry; he must be conversant with the writings of mathematicians: or he will never be well qualified to ascertain the quantity of land contained within an irregular boundary, or to determine the *meum* and *tuum* between man and man.

With such qualifications, and with due application and perseverance, he has a fair prospect of success; but without these requisites, the mere pretender to the art can only blunder on, not knowing whether he be right or wrong, until his errors become so numerous, as to put him out of countenance.

So much preparatory knowledge being requisite to the practice of an art thus useful, it becomes a matter of surprise that, in this age of literature, we have no treatise on Land-measuring, which is professedly written for, and adapted to, the use of schools — the place where the groundwork ought to be laid.

We have, it is true, various books under the titles of Land-measuring, Land-surveying, &c.; and we have scraps and fragments relative to the subject in various books of Mensuration; some of which, I am ready to admit, possess many useful hints and instructions relative to a partial knowledge of the subject*; but none systematically formed for that purpose, or which may be considered as a safe and unerring guide to the art.

These considerations, together with a full assurance of the importance of the subject, have induced the Author of the following pages to draw up such a work as he conceived wanting on the subject—a work not merely speculative, but such a one as is fairly the result of more than forty years' experience in the theory and practice of the profession; and having herein acted with the most upright intentions, he has the greater confidence in submitting his labours to the candour of the public.

---

* What Dr. Hutton published forty years ago, on the areas of superficies in general, and of land in particular, in his valuable "Treatise on Mensuration," far exceeded all that his predecessors had done; and his writings seem to have been the ground-work of all which has been since written on the subject; but even all this was never intended by him as a perfect treatise on Land-measuring.

This work is not intended to take the pupil by the hand at the portal of the temple of numbers; but the Author expects that his disciple has been previously practised in all the rules of common and decimal arithmetic, and in the extraction of the first roots, either by the usual school exercises, or by long and close application to introductory books; then will he be prepared to enter on the subject of this Treatise, and easily to comprehend and follow the Author throughout the whole.

This work comprises Geometry, Trigonometry, Distances, Areas, reducing and equalising Figures, Land-measuring, and dividing of Land. *Algebraic Notation* and *Logarithmic Arithmetic* are comprised in an introductory chapter.

PRACTICAL GEOMETRY is employed on the mathematical principles of Lines, Angles, Triangles, Parallelograms, Polygons, the Circle, the Ellipse, and on inscribing and circumscribing Figures.

TRIGONOMETRY is herein exemplified, in all the cases which relate to Land-measuring; and is also applied to a variety of cases of inaccessible distances.

Then follows the AREA, or superficial content of Plane Figures; comprising that of the square, of the parallelogram, rhombus, rhomboid, triangle, trapezium, trapezoid, polygon, circle, with its sector and segment, ellipse, and circular rings.

The reducing and equalising the Figures comprehend that of the parallelogram to a square of equal area, the rhombus and rhomboid to a square, the trapezium to a triangle and to a square, and a heptagon to a triangle.

The *division of land* is not confined to the *quantity* which each claimant may be entitled to; but regard is also had to the *quality* thereof.

From the foregoing is deduced the whole art of Land-measuring, so far as can be taught or practised in schools, comprising single fields of all the geometrical forms before spoken of, both single and compounded; two or more fields connected, forming an estate; and several estates united, forming a Manor or Lordship: the whole illustrated by suitable diagrams, figures, and plans; with a variety of examples in each part, wrought out; and others unwrought, for the exercise of the pupil's abilities.

Of the necessary instruments for Land-measuring, little more is laid down in this work than a brief description of each, as it occurs; their practical uses being best learnt from a professional man in the field; and that more satisfactorily, and in infinitely less time, than can be obtained from the study of many volumes written professedly on the subject.

To this new edition I have added a brief yet comprehensive Supplement, containing practical instructions for the use of the chain, plane-table, and theodolite in the field, according to the various modes of modern practice.

An APPENDIX is subjoined, containing brief, distinct tables of Logarithmic Numbers, of Sines, Tangents, and Natural Sines, &c., under a new mode of arrangement; sufficiently extensive for all purposes of Land-measuring.

Thus it may be perceived, the following Treatise contains all the necessary *preparatory* knowledge to the *practice* of Land-measuring; and the principles here laid down (when well understood by the pupil) will render the field-practice easy, pleasant, and certain; and thereby enable him to prosecute a profession in a way creditable to himself and satisfactory to his employers.

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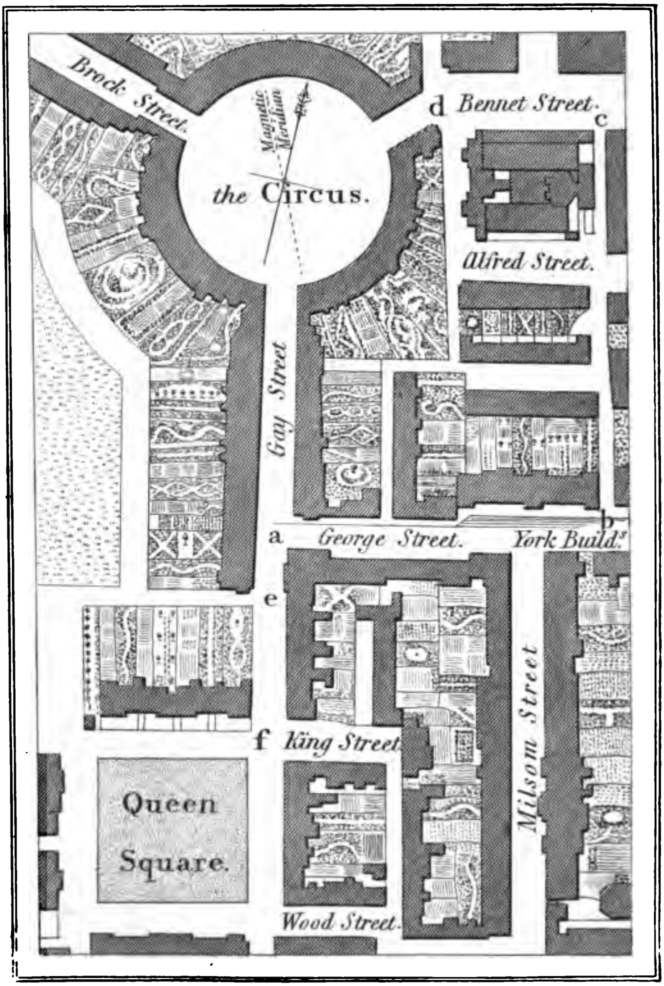
PART OF THE CITY OF BATH.



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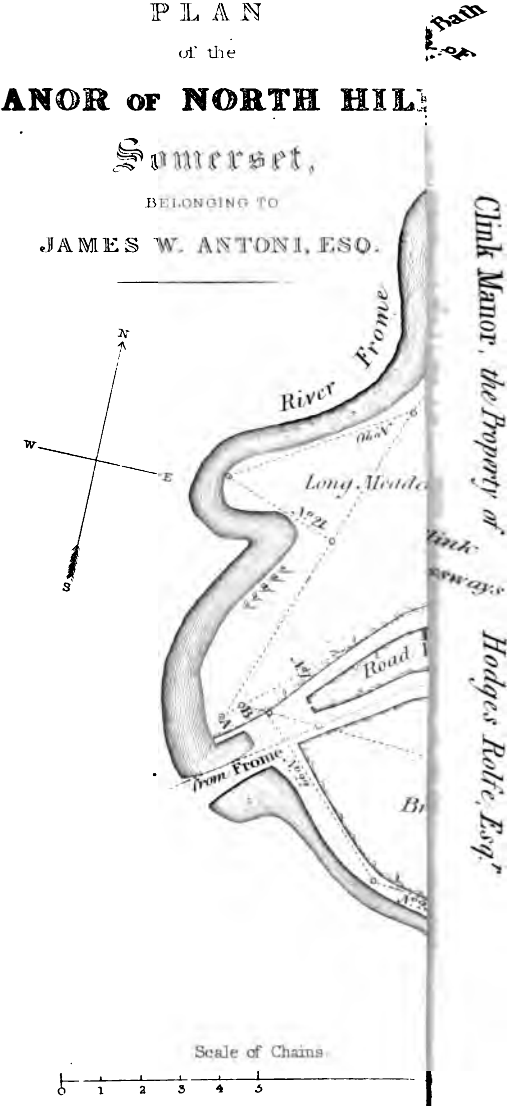
PLAN
of the
MANOR OF NORTH HIL
Somerset,
BELONGING TO
JAMES W. ANTONI, ESQ.



To face the Title

ELEMENTS

OF

LAND SURVEYING.

INTRODUCTORY CHAPTER.

OF ALGEBRAIC NOTATION, &c.

WRITERS on mathematical subjects have long been aware of the convenience of Algebraic symbols, even in works which are not wholly confined to that species of arithmetic. The brevity of the notation, the perspicuity of the arrangements, and the simplicity of the characters, enable the Algebraist to form a true picture of a theorem or problem; and thereby to comprehend, in the clearest manner, the whole of any mathematical proposition.

For the better understanding of the following pages and of other mathematical works, it is deemed useful to explain Algebraic Notation, and a few of the first principles of that abstract species of arithmetic.

The characters employed in Algebra, in lieu of figures, are the letters of the alphabet; the former of which (as a, b, c, &c.) are arbitrarily used to express any quantities or numbers which are known or given; and the latter (as x, y, z,) usually denote such quantities as are not known, or as are sought for by mathematical operation. When geometrical *lines* are the subject, capital letters are used to denote such lines.

Various other characters, called *signs*, are employed, to denote sundry arithmetical steps in an algebraic process: thus,

The sign or character + (called *plus*, or more) denotes addition: as $a + b$ (read a plus b), means that the respective *quantities* which happen to be denoted by the letters a and b, are to be added together. In the case of geometrical lines, generally represented by capital letters, $AB + CD$, means that a line marked with the letters A and B, is to be added to a line marked with the letters C and D.

The sign — (called *minus* or less) denotes subtraction; as $a - b$ (read a minus b), means that the quantity understood by the operator to be represented by b, is to be subtracted from the number or quantity understood by him, to be represented by a. Sometimes, when it is not immediately known which of the two quantities is the greater, the character ∽ is employed; $x \backsim y$, alludes to the difference of these quantities, whichever of them may afterwards be found to be the greater: also, in geometrical lines, $GH - IK$, means that the line IK is to be subtracted from the line GH.

The sign × (*into* or multiplied by) denotes multiplication; as $a \times b$, means that the quantities of a and b are to be multiplied one into the other. Sometimes in multiplication, the symbols are used alone, without this sign; as $a\,b\,c$ means that the quantities represented by a, b, and c, are to be multiplied continually together; as also $BC \times IL$, means that the lines represented by BC and IL, are to be multiplied together.

The sign $\div$ (called *by*, or divided by) signifies division; as $a \div b$, means that the quantity a is to be divided by b. The division of two numbers is more frequently represented as a vulgar fraction; thus, $\frac{a}{b}$ signifies that a is to be divided by b; or $\frac{AB}{3}$ means that a line AB is to be divided by 3.

The signs $: \, :: \, :$ (called *proportionals*) denote proportionality; as $a : b :: c : x$, signifying that a bears the same proportion to b, as c does to x; or in other words, as a is to b, so is c to x.

The sign $=$ (called *equality*) means that the respective quantities, before and after this sign, are equal; as $a=b$; — or $b+c=d$, means that the sum of b and c are equal to d.

The sign $\overline{\qquad\quad}$ (called *vinculum*) a band, to tie or connect several quantities together; as $\overline{a+b-c} \times d$; signifying that the sum of a and b, minus c, are united as one quantity; which, in the expression above, is to be multiplied by d.

The sign ^{2}, placed above the head of any quantity, as a^2, represents the square of that quantity. The same power is otherwise represented by $a \times a$; or aa.

The sign $\sqrt{}$ (square-root), placed before any letter, means the square-root of the quantity represented by that letter; as $\sqrt{a}$, means the square-root of the quantity represented by a: also, $\sqrt{AB}$ signifies the square-root of a line AB. The square-root of a compound quantity may be expressed in like manner; as $\sqrt{ab+cd}$ signifies the square-root of the sum of ab and cd.

Signs are *like* and *unlike*.

Like signs are all affirmative ($+$), or all negative ($-$); as $a+4b$; or $-b-7c-8d$.

Unlike signs are some affirmative and some negative, as $a-b+c-d$.

A figure standing before a letter representing a quantity, is called the *co-efficient* of that quantity; as $5a$, signifying that the quantity denoted by a is to be taken five times. When no co-efficient stands before a symbol, 1 is understood to be its co-efficient.

Quantities may be *like*, *unlike*, *simple*, *compound*, *affirmative*, or *negative*.

Like quantities are those which consist of the same letters and powers; as a and $3a$; or $2a^2bc$, and $9a^2bc$.

Unlike quantities are such as consist of different letters or powers; as $2a$ and a^2.

Simple quantities are such as consist of one term only; as $3a$, or $6ab$.

Compound quantities are such as consist of two or more terms, united by some sign; as $a+b$; or $3b+c-4d$. When a compound quantity consists of two terms, as $a+b$, it is called a *binomial;* when of three, as $7a+b-5c$, it is called a *trinomial*, &c.

Affirmative or *positive quantities*, are such as are to be added, as $a+b$. In the first term of affirmative quantities, the sign $+$ is understood, although not written.

Negative quantities are such as are to be subtracted; as $-a$, or $-7ab$.

Although it is usual to range the letters, expressing any simple quantity, in their alphabetical order, yet they may be placed in any other order at pleasure: for the product of a and b, which is usually expressed by ab, may as well be represented by ba; and that of abc

may be expressed by *acb*, or *bac*, or *bca*, or *cab*, or *cba*, as the final product will be the same in either case.

Persons who are disposed, at their leisure, to pursue the study of this truly elegant science, may consult the ingenious writings of Messrs. Hutton, Saunderson, Bonnycastle, Emerson, and Simson.

OF LOGARITHMS.

By means of these artificial numbers, tedious operations in arithmetic are performed with a tenth part of the labour which would be required to arrive at the same results without them. Multiplication is performed by addition, Division by subtraction, and the tedious processes of involution and evolution, by single acts of multiplication and division.

Logarithms are a series of numbers, in an arithmetical progression, corresponding to as many others in geometrical proportion, in such way that 0 in the arithmetical series, corresponds to, or is the exponent of 1, in the geometrical series: thus,

0,	1,	2,	3,	&c. are indices or logarithms.
1,	10,	100,	1000,	&c. are geometric progressions, increasing by a multiplier of 10.

It may be observed, that the same indices will serve for any geometric series; however, let it be understood here, that the logarithms most convenient for practice are such as are adapted to a geometric series, increasing in a *tenfold* proportion, as in the above form.

When a natural number consists of one figure, or of units, the index of the logarithm of such number is 0; but when it hath two figures or tens, the index is 1; when it hath three figures or hundreds, the index is 2; when it hath four or thousands, the index is 3; and so on uniformly, as in the foregoing series, the index being one less than the number of figures in the integral part of the natural number.

The decimal part of a logarithm is the same, whether the natural number to which it belongs be a whole number, decimal, or partly a whole number, and partly a decimal. The index only changes, for which reason it is better to omit it in the logarithmic table.

Whilst logarithms are not wanted for natural numbers less than a single figure of whole numbers, the indices will be affirmative; but when indices are to be written for natural numbers less than unity, those indices will be negative. The affirmative sign (+) may be omitted; but the negative sign (−) must be always expressed. This sign applies only to the index, the decimal parts of the logarithm being always positive.

Thus, the logarithm of 2651 being 3.4234097, the logarithms of $\frac{1}{10}$, $\frac{1}{100}$, &c. parts thereof will be as follows: —

NATURAL NUMBERS.	LOGARITHMS.
	INDEX. DECIMAL.
2651	3.4234097
265.1	2.4234097
26.51	1.4234097
2.651	0.4234097
.2651	−1.4234097
.02651	−2.4234097

Different writers have arranged their logarithmic tables differently, both in manner and extent, in such way as best suited the purposes for which their tables were intended. Those which are best adapted to general and scientific purposes are Dr. Hutton's; but of those which have been published in a more compendious form, it is hoped that none are superior to the one which is appended to this work.

The first table consists of logarithms to six decimal places, with their differences, corresponding to the natural numbers, from 1 to 10,000. The second table contains the logarithmic sines and tangents, to six places of decimals, corresponding to every degree and minute of the quadrant.

A new arrangement has been introduced into the present table of the logarithms of natural numbers, for the purpose of preventing an error in taking out a logarithm from the central columns, when the second figure of the logarithm (which is printed only in the first logarithmic column) changes by an increase of 1. Wherever this change occurs, the remainder of the line is left blank, and the last four figures of the logarithm are contained in the line below; the natural number is *repeated* in the outer column, and the first two figures of the logarithm, which have increased by unity, appear with their proper value.

To find the logarithm of any natural number consisting of 4 places of figures, seek the first three figures of the number in the left-hand column, and the fourth at the head of one of the vertical columns. The last four places of the logarithm will be found where the vertical and horizontal rows of figures meet; the first two are seen only in the first of the columns of logarithms. To these six figures prefix the proper index and decimal point. (For the index, see the preceding page.)

Thus the logarithm of	47	is	1·672098
of	375	is	2·574031
of	234·4	is	2·369958
of	234·5	is	2·370143

To find the logarithm of any number consisting of five places. Take out the logarithm of the first four places, as before; also the difference between that and the next greater logarithm. Multiply this difference by the fifth or last figure in the said number; add the product, rejecting the right-hand figure, to the logarithm first taken, and prefix the true index and decimal point.

Thus to find the logarithm of 23·233

Logarithm of	2323	- -	366049
Difference	187 multiplied		
by the 5th figure	3		
gives, rejecting the right-hand figure	56|1	which added,	56

and the index 1· prefixed gives log. 23·233 = 1·366105

If the fifth figure be a cypher, the decimal of the logarithm will be the same as for the first four figures.

To find the logarithm of a vulgar fraction, or of a mixt number.

Reduce the vulgar fraction to a decimal, and find its logarithm as before.

The decimal of $\frac{3}{16}$ = ·1875; and the logarithm of this decimal is = − 1·273001.

To find the natural number of any given logarithm.

It is to be observed that the index of the logarithm will show how many integral places are in the required number, when it is affirmative; viz. one more than the index number represents.

If the logarithm be exactly found in the table, the natural number is found in the first column, by inspection: thus,

The natural number of the log. - 1·146128 = 14·

- - - - - - 2·603144 = 401

But if the logarithm cannot be exactly found, nor nearly so, in the table (and accuracy be required), take out of the table the logarithm next less than the one given, together with its corresponding natural number consisting of four places; also take, from the right-hand column, the tabular difference.

Subtract from the given logarithm the logarithm which has been taken out of the table; divide the remainder by the tabular difference, and *annex* the quotient to the first four figures of the natural number.

EXAMPLE.

What is the natural number corresponding to the logarithm 1·470023?

Log. given	·470023	
Log. next less	·469969	natural number 29·51

Remainder 54 divided by the tabular difference 147 gives ·37 nearly, which being annexed to 29·51, gives 29·5137 for the natural number required.

OF LOGARITHMIC ARITHMETIC.

MULTIPLICATION by means of logarithms is performed by adding together the logarithms of the factors. The sum is a logarithm, the corresponding natural number of which will be the product required.*

Multiply 14 by 4
The logarithm of 14 = 1·146128
- - - of 4 = 0·602060

1·748188, which answers in the table to 56.

DIVISION, by means of logarithms, is performed by subtracting the logarithm of the divisor from the logarithm of the dividend, and the remainder will be the logarithm of the quotient.

Divide 56 by 4.
Logarithm of 56 = 1·748188
- - - of 4 = 0·602060

1·146128, answering to the natural number 14.

PROPORTION, or RULE OF THREE, is performed by the following rule:

Add the logarithms of the second and third terms together, and from the sum subtract the logarithm of the first; and the remainder will be the logarithm of the fourth term.

---

* Multiplication by logarithms will readily apply to the areas of right-lined figures; as will be shown hereafter.

EXAMPLE.

If 4·1 be to 5, as 65 is to a fourth number, what is that number?

To the logarithm of 5 = 0·698970
Add the logarithm of 65 = 1·812913

2·511883
Subtract the logarithm of 4·1 = 0·612784

answering to 79·26, nearly 1·899099

Instead of subtracting one logarithm from another, we may add its *arithmetical complement*, and the result will be the same.

Note. The arithmetical complement of a logarithm, is what that logarithm wants of 10·000000. The readiest way of finding which complement is, to begin at the left hand, and subtract each figure from 9, except the last on the right hand, which must be subtracted from 10. And for every complement which is added, subtract 10 from the last sum of the indices.

The arithmetical complement is commonly used in trigonometrical calculations, when radius is not the first term in the analogy.

It is further to be noted, that the arithmetical complement of any sine is the same as the co-secant of the same number of degrees.

From - - - 10·000000
Subtract the logarithm of 4·1 = 0·612784

The arithmetical complement of 4·1 = 9·387216

If, as before, 4·1 be to 5, as 65 is to a fourth number, what is that number?

To the logarithm of - 5 = 0·698970
Add the logarithm of - 65 = 1·812913
And the arithmetical complement of 4·1 = 9·387216

answering nearly to 79·26, as before. 1·899099

Involution, or Raising a Power, is performed by this rule:

Multiply the logarithm of the root by the index of the power: thus, to square any number, multiply its logarithm by 2; and the product will be the logarithm of the required power.

EXAMPLE.

Required the square of 15.

The logarithm of - - 15 = 1·176091
Multiplied by the index - - 2

2·352182

answering to the logarithm of 225, the square required.

Evolution, or Extracting Roots, is the reverse of the foregoing; for to extract the root of any number, divide the logarithm of the power by its index, and the quotient will be the logarithm of the root.

EXAMPLE.

Extract the square root of 225.

Logarithm of - - 225 = 2·352182
½

1·176091

answering to the logarithm of 15.

OF SINES AND TANGENTS.

The second table in the Appendix contains the logarithmic Sines and Tangents, to every minute of a degree; by means of which all Trigonometrical Operations may be performed.

It must be understood, that the circumference of every circle in Geometry is divided, or supposed to be divided, into 360 parts, called degrees; and every degree into 60 minutes. Degrees are expressed by °, on the right side of the number, and minutes by ′; so that 39 degrees 15 minutes, are expressed thus: 39° 15′.

Observe, that from 0 to 45, the degrees are placed at the top of each page in the table, and the minutes descend from the top to the bottom on the left side; and that from 45 to 90, the degrees are found at the bottom of the page, and the minutes ascend from the bottom to the top, on the right side. The difference of each logarithm and its next greater, increased in the proportion of 1 minute (or 60 seconds) to 100 seconds, is also given in a separate column.

Further, it is to be observed, that the sine, tangent, &c. of any arc is expressed by the same number as the sine, tangent, &c. of its supplement: so that when an arc is greater than 90 degrees, take its supplement, viz. subtract it from 180, and take the sine, &c. of the remainder for that of the arc given. The sine, &c. of 110 degrees is the same as that of 70; that of 130 degrees 15 minutes, is the same as that of 49 degrees 45 minutes; and so of others.

Observe also, that the radius, or logarithmic sine of 90 degrees, is 10. The radius of the natural sines is 1. The tangent of 90°, whether natural or logarithmic, is infinite.

To find the logarithm of any sine, or tangent, below 45°, look in the table for the degrees on the top of the page, and the minutes in the left-hand column, descending, and the logarithm is found by inspection; if above 45°, seek the degrees at the bottom of the page, and the minutes in the right-hand column, ascending. Thus,

The logarithmic sine of 7° 4′ is = 9·089990
81° 41′ = 9·995409
The logarithmic tangent of 110° 50′ = 10·419611
71° 4′ = 10·464672

When the logarithm is required to seconds of a degree, multiply the tabular difference between the logarithm of the given degrees and minutes, and that next greater by the number of seconds, reject the two right-hand places in the product, and add the remaining places to the logarithm of the degrees and minutes given: thus,

To find the logarithmic sine of 82° 44′ 20″:

Log. sine of 82° 44′ = - - 9.996498
Tabular difference for 100″ more } =27
Multiplied by number of seconds=20

Rejecting two places to the right-hand gives - - - - - - 5

Which added is the log. sine of 82° 44′ 20″ = 9·996503

To find the degree and minute corresponding to any given logarithm, look in the table for the logarithm nearest to the one given, and the number of degrees and minutes will be found by inspection.

If fractions of a minute, or seconds, are required, subtract from the logarithm given the next less logarithm found in the table, and take out also the tabular difference for 100 seconds: then say,

As the tabular difference, is to 100″: so is the remainder left by subtracting, to the number of seconds, which are to be annexed to the degrees and minutes already found. Thus,

To find the number of degrees, minutes, and seconds, corresponding to the logarithmic tangent 9·122898

From logarithm given -	9·122898
Subtract the next less found in the table, answering to the tangent of 7° 33′	9·122348
Remainder - - - -	550
Tabular difference, for 100″ more than 7° 33′	1615

Then say,

Tab. diff.		Sec.		Rem.		Sec.
As 1615	:	100	: :	550	:	34

Which being annexed gives 7·° 33′ 34″, the arc corresponding to the given logarithmic tangent.

This introductory Chapter may be read at the Student's leisure, while he is perfecting himself in the subsequent Problems of Practical Geometry.

PART I.

PRACTICAL GEOMETRY.

Geometry, in the express sense of the word, signifies the art of measuring the earth: an art so useful and important, that it must, in some measure, have been known in the earliest ages of the world.

Geometry may be distinguished into *theoretical* and *practical;* the first demonstrates the properties and relations of right lines, figures, &c., in speculation; from which is deduced the second, or the practical part, applicable to various purposes of human society.

The theory of Geometry received great advances about 2000 years ago, from Euclid of Alexandria, who there taught the mathematics, with great applause, and wrote many ingenious books on that and other subjects; the most valuable of which to the land-measurer is his Elements of Geometry.*

The practical applications of Geometry have, in point of accuracy and dispatch, been carried by the moderns to a higher degree of perfection than any other branches of human knowledge.

The perfection of the art of Land-measuring, that branch of Practical Geometry which is the object of this treatise, depends solely on the accuracy employed in taking the length of lines with some known standard measure, such as feet and inches, or chains and links, &c.; and on carefully observing, and accurately measuring, the inclinations of those lines relatively to each other.

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* A book to which I shall occasionally refer, and of which the best edition is that by Robert Simson, in one volume 8vo, price 7s.

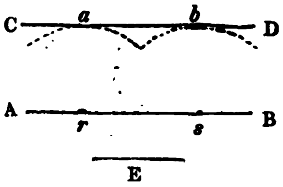
PROBLEM I.

PARALLEL LINES.

Parallel lines are such as may be produced to any distance whatever, without meeting or inclining to each other.

To draw a line parallel to a given line A B.*

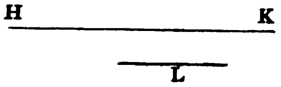
Case 1. When the parallel line is to be at a given distance E.



From any two points, *r* and *s*, in the original line, with the compasses opened to a radius equal to the given distance E, describe the arcs *a* and *b*.

Draw the line CD, to touch those arcs, without cutting them, and it will be the parallel line required.

For Practice.

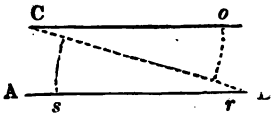


1. Draw a line parallel to HK of a distance = L.
2. Draw another line, parallel to the original, of twice the distance of the length L.
3. Another, distant half the length of L.

Case 2. When the parallel line is to pass through a given point, as C.

---

* To perform these problems, the student should be provided with a case of mathematical instruments, the price of which varies according to the size and number of the instruments. A case, sufficient for the ordinary purposes of the land-measurer, contains two pairs of compasses, three drawing pens, pencil-sweep, six-inch protractor and scale, and twelve-inch sector.



From the given point C, draw a line at pleasure, which shall cut the given line AB at *r*.

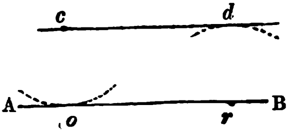
With any radius, from *r* describe an arc *s*; and with the same radius, from C describe another arc *o*.

Make the arc *o* equal to the arc *s*.

A line drawn through C and *o* will be the line required.*

Otherwise.

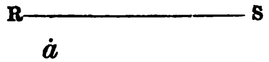
From the point *c*, sweep an arc which shall touch the original line at *o*.



With the same radius, and one foot in any other convenient part of the line AB, as at *r*, sweep the arc *d*.

From *c*, draw a line, touching the arc *d*, and it will be the parallel line required.†

For Practice.



1. By the last method, draw a line parallel to RS,

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* Euclid's Elements, Book I. Proposition 31.

† Parallel lines are drawn with much facility by an instrument called a parallel ruler (fig. 1.). These rulers are of various lengths,

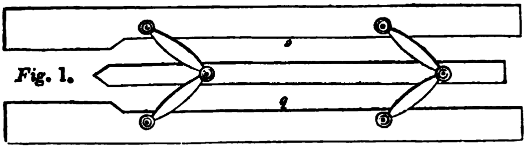


Fig. 1.

from one to three feet. The most convenient form of this ruler is when it is made to roll on two wheels fixed on one axis.

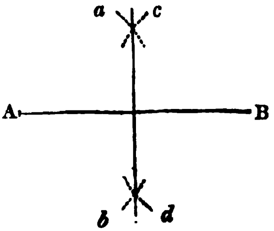
which shall pass through the point *a*, below the given line.

2. Draw other lines parallel to RS, of double the distance *a*, above the line; by both the foregoing methods.

PROBLEM II.

BISECTED LINES.

Any right line, as AB, may be bisected, i. e. divided into two equal parts, in the manner following:



Set one point of a pair of compasses at the end of the line, at A, and with any extent greater than half the length of the line, describe the arcs *a* and *b*, above and below the said line.

With the same extent, set one foot of the compasses in the end B; and describe other arcs, *c* and *d*, intersecting the former.

A line drawn from one intersection to the other will divide the original line into two equal parts. Euclid, I. 10.*

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* The same thing is readily performed by proportional compasses (fig. 2.). Those of seven or eight inches are of convenient length.

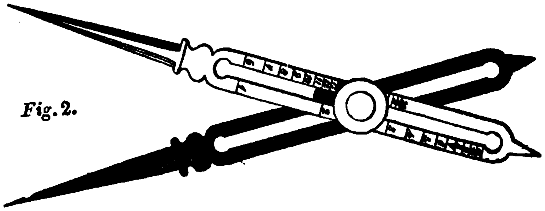


Fig. 2.

For Practice.

C——————————D

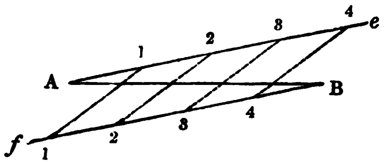
1. Divide the line CD into two equal parts.
2. Divide each of those parts into two equal parts.

PROBLEM III.

DIVIDED LINES.

Any line, as AB, *may be divided into three, or any other proposed number of equal parts.*

Case 1. To divide it into five equal parts.



From the end A of the given line, draw the line A*e*, at pleasure; and from the other end, B, draw the line B*f* parallel to A*e*.

From A towards *e*, set off one less than the required number of equal divisions, of any reasonable length.

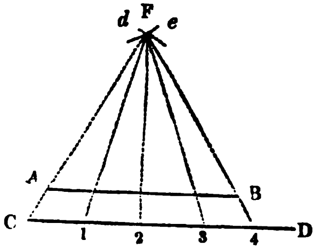
Do the same from B towards *f*, numbering the points of division, as in the figure.

Draw lines from the corresponding numbers, intersecting the original line; and they will divide it, as required.

Case 2. It is proposed to divide the line AB into four equal parts.

A——————————B

Draw the line CD, of any convenient length, greater than AB, and thereon, from C, set off as many equal parts as the given line is to be divided into; and number those parts 1, 2, 3. 4.



Take C4 in the compasses, and, with that extent, upon C draw the arc *d*. With the same extent, and one foot on 4, draw the arc *e*.

From the intersecting point F, draw FC and F4.

Set off the length of the given line from F towards C and 4, on which draw AB.

From F, to the given points, 1, 2, 3, 4, on the assumed line CD, draw the lines F1, F2, F3, F4. These lines will cut the line AB into the equal parts required.

For Practice.

A——————————B

1. Divide the line AB into three equal parts, by the first method.

2. Divide the same into four equal parts.

E——————————G

3. By the second method, divide EG into five equal parts.

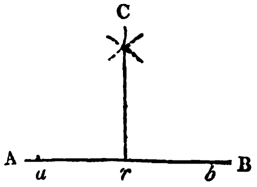
4. Divide the same into six equal parts.

PROBLEM IV.

PERPENDICULAR LINES.

One line may be drawn perpendicular to another.

A perpendicular, or normal, line is one that meets a given line AB, in such a manner that the angles on both sides of it may be equal to each other.



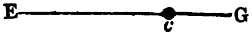
Let *r* be the point assigned for raising a perpendicular from.

From this point set off equal distances, *a* and *b*.

With the compasses, opened to the extent *ab*, set one foot in *a*, and describe an arc. With the same extent, and the foot in *b*, draw another arc intersecting the former at C.

From the intersection, draw the line C*r*, and it will be perpendicular to the given line. EUCLID, I. 11.

For Practice.

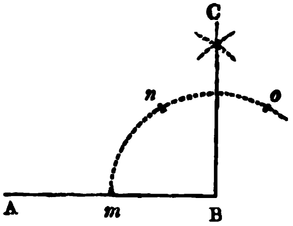


1. From the point *c*, in the line EG, erect a perpendicular.
2. At mid-way between E and *c* erect another.

CASE 2. When the perpendicular is required to be raised on the end of the line AB.

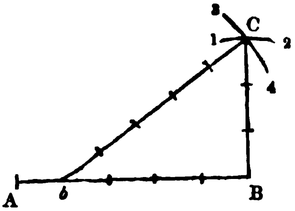
With the centre B, and any radius, as B*m*, describe the arc *mno*.

With the same radius, from *m* turn the compasses twice over on the arc, making the points *n* and *o*.



From these points describe arcs intersecting at C.

Draw BC, and it will be the perpendicular required.



Otherwise,

Divide the line AB into five equal parts.

Take three of these parts in your compasses, and with the sextant, and one foot in B, describe the arc 1 2.

With the extent of five parts, and one foot in the fourth division (at *b*), intersect the former arc at C.

Draw a line from C to B, and it will be perpendicular to the end B of the line AB.

For Practice.

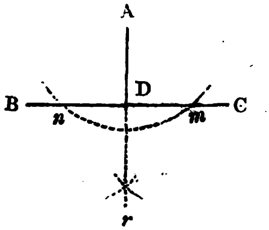
I————————K

1. On the end I, of the line IK, raise a perpendicular by the last method.

2. By the last method, erect a perpendicular at K.

CASE 3. When a perpendicular is to be demitted from a given point, A, over a line, BC, near to the middle thereof.

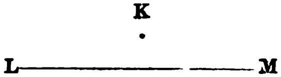
With the centre A, and any convenient radius, describe an arc, cutting the given line in *m* and *n*.



With the centres *m* and *n*, describe arcs intersecting at *r*.

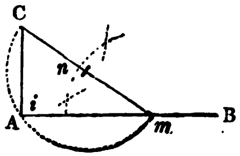
A line demitted towards *r* from A to D, will be the perpendicular required.

For Practice.



1. From the point K, let fall a perpendicular to the line LM.

CASE 4. When the point is nearly opposite to the end of a given line, as C.



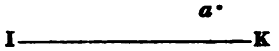
From any point, *m*, in the line AB, draw the line *m*C.

Bisect this line in the point *n*.

From *n*, with the radius *nm*, describe the arc C*im*, cutting AB in *i*.

From the point C, draw the line C*i*, and it will be the perpendicular required.

For Practice.



1. From the point *a* demit a perpendicular to the line IK.

Perpendiculars from below a line, may be as readily drawn as from above: this will be evident by inverting the diagrams.*

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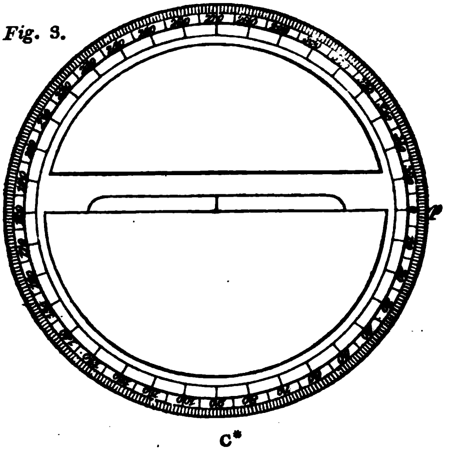
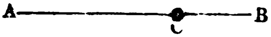
* Perpendiculars may be readily drawn from any part of a line, either above or below, by means of an instrument called a protractor (of which there are various constructions, as figs. 3.

Fig. 3.

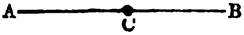


Peculiar Properties of Lines.

Suitable to our present purpose are sundry theorems, respecting various properties of right lines, collected from the Elements of that profound geometrician before mentioned; for the demonstration of which I refer to the Elements themselves.



If the right line AB be any how divided, in the point C for instance, the rectangle contained under AB and BC, together with that contained under AB and BC, is equal to the square of AB. — EUCLID, II. 2.



If the right line AB be any how cut, as in C, the

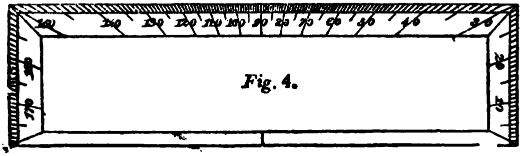


Fig. 4.

and 4.); or otherwise, on a drawing board, with a T square (fig. 5.).

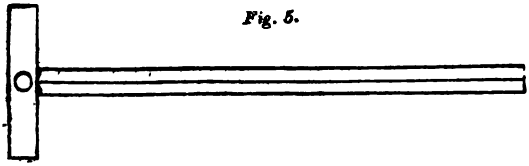
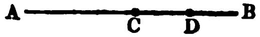


Fig. 5.

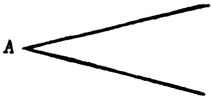
of AB is equal to the squares of AC, and CB, r with twice the rectangle contained under d CB.—Euclid, II. 4.



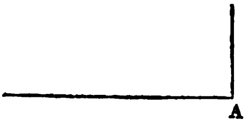
ny right line AB, be cut into two equal parts in d into two unequal parts in D; the rectangle ained under AD, and DB, together with the square D, is equal to the square of half the line, or AC. UCLID, II. 5.

ANGLES.

PROBLEM V.



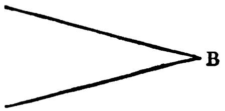
Two lines, which mutually incline to each other, do, at the vertex, or point of meeting, form an *angle* of some certain number of degrees *; as A,



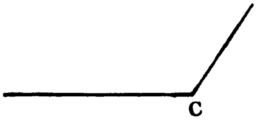
If one leg of an angle be perpendicular to the other, it is called a *right angle;* as A.

---

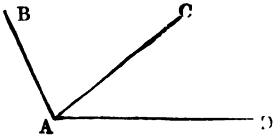
* That the circumference of every circle may be supposed divisible into degrees, &c., has been previously hinted at, in the chapter on logarithms; and will be more fully explained in a subsequent chapter.



If the inclination of the legs, one to the other, be less than a right angle, the angle is *acute;* as B.

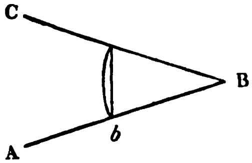


If the inclination be greater than a right angle, it is *obtuse;* as C.

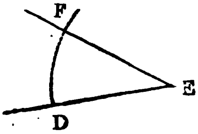


If the angle be simple, viz., no other lines meeting at the vertex than the two which form the angle, it is denoted by the letter placed at the angular point; and in the preceding diagrams, would be called the angle A, B, or C. But if other lines fall into the same angular point, as in the present example, the upper angle would be called the angle BAC; and the other (or lower) would be called the angle CAD: the middle letter in such case being always that which is placed at the angular point.

To make an angle equal to any given right-lined angle



Let ABC, in the foregoing figure, be the given angle, and DE, in the following figure, a line given, on which, at the end E, it is required to make an angle which shall be equal to the given angle ABC.



With any radius at discretion, on the vertex of the given angle B, describe the arc *ab*, cutting the two sides AB and BC, in the points *a* and *b*.

Draw a right line to join *a* and *b*.

With the same radius, on the point E in the last figure, draw the arc DF.

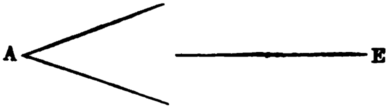
Take the measure of the arc *ab* in the compasses; and make DF equal to *ab*.

From E draw a line through F, and the angle DEF will be equal to the former angle ABC.* — EUCLID, I. 23.

---

* Angles of any number of degrees, are readily set off by a protractor, or by compasses and a scale of chords on a plain scale.

For Practice.

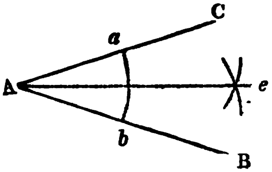


Draw an angle from E, which shall be equal to the angle A.

PROBLEM VI.

BISECTED ANGLE.

To bisect, or divide into two equal parts, any given angle.



Let the angle A be divided into two equal parts.

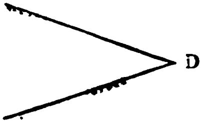
From the angular point A, with any convenient radius, describe the arc *ab*.

From *a* and *b*, with the same or any other convenient radius, describe arcs, crossing each other in *e*.

From the angular point A, to the intersection *e*, draw the line A*e*, and it will divide the angle into two equal parts.—EUCLID, I. 9.

The same may more readily be done with compasses and a scale of chords, by setting off from *b*, one-half the number of degrees contained between *a* and *b*.

For Practice.

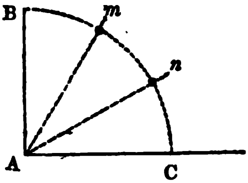


1. Bisect the angle D.

PROBLEM VII.

TRISECTED ANGLE.

To trisect, or divide into three equal parts, a right angle.



From the angular point A, with any radius, describe an arc BC,

From the point C, with the same radius, set off on the said arc, the distance C*m*, towards B.

From the point B, set off the like distance towards C, as *n*.

Lines drawn from A to *m* and *n*, will divide the original angle into three equal parts.*

---

* The exact trisection of angles engaged the attention of many of the ancient geometricians, particularly PAPPUS; as may be seen in his Mathematical Collections. It is one of those celebrated problems, the complete solution of which has not yet been found.

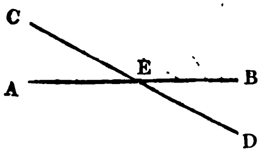
By a scale of chords on the plain scale, or on a sector, angles may readily be trisected, or otherwise divided.

For Practice.

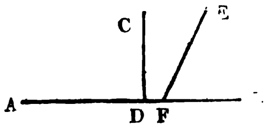
1. Trisect the right angle B.

Peculiar Properties of Angles.

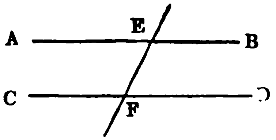
The following properties of angles are worthy of notice.



If two right lines, AB and CD, mutually cut each other in the point E, the opposite angles are equal; that is, the angle AEC is equal to the angle BED; and the angle AED is equal to the angle CEB. —EUCLID, I. 15.



If a right line, CD, or EF, stand upon, or meet with, a line AB, making ∠s therewith, they will be either two right angles, as ADC and CDB; or they will be equal to two right angles, as EFB and EFA.



If a right line EF, falling upon two right lines AB, CD, make the alternate angles AEF and EFD equal to each other, the lines AB and CD are parallel to each other; and each pair of adjacent angles will be equal to two right angles. — EUCLID, I. 27. and 29.

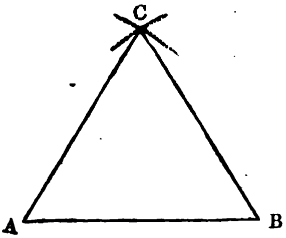
TRIANGLES.

PROBLEM VIII.

A figure bounded by three right lines, comprising three angles, is called a *triangle*, or *trigon*.*

An equilateral triangle is such as has all its sides of equal length.

To describe an equilateral triangle, whose base is AB.



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* The bottom side is called the *base*, and the other sides are called *legs*.

From the centres A and B, with the radius AB, describe arcs intersecting at C.

Draw AC and BC, and the triangle will be formed.

For Practice.

K——————L

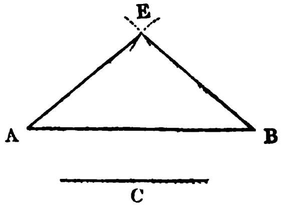
1. Draw an equilateral triangle on the base KL.

PROBLEM IX.

ISOSCELES TRIANGLES.

An *isosceles triangle* is such as has only two of its sides equal.

To describe an isosceles triangle on the base AB, *whose other sides shall be equal to* C.



On the centres A and B, with the radius C, describe arcs intersecting in E.

Draw the lines AE and BE, and the triangle will be perfected.

For Practice.

F——————G

H——————I

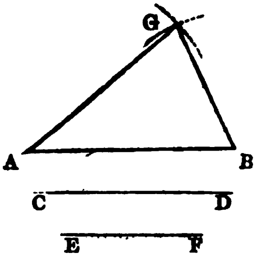
1. Draw an isosceles triangle on the base FG, whose other sides shall be equal to HI.

PROBLEM X.

SCALENOUS TRIANGLE.

A *scalenous triangle* is that which has all its sides unequal.

To draw a scalenous triangle, whose three sides are equal to AB, CD, *and* EF.

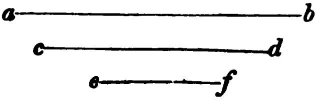


On the centre A, with the line CD, in the compasses, describe an arc, at G.

With the centre B, and line EF, in the compasses, intersect the arc, at G.

Draw lines from A and B to G, and the triangle will be formed.

For Practice.



Draw a scalenous triangle, with the lines *ab*, *cd*, and *ef*.

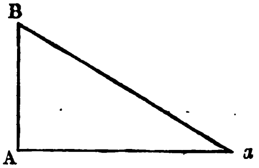
PROBLEM XI.

RIGHT ANGLED TRIANGLE.

A *right angled triangle* is such as has one right angle.

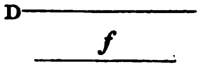
To describe a right angled triangle, on the line A*a.*

On the end A erect AB perpendicular. Draw B*a*, and the triangle is formed.



Note—All triangles differing from a right angled one, are called *oblique angled triangles.*

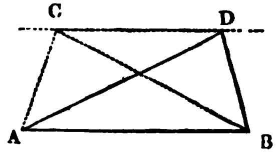
For Practice.



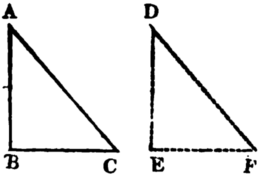
Describe a triangle, on the base D*d*, which shall be right angled at *d*, and whose perpendicular shall be *f.*

Peculiar Properties of Triangles.

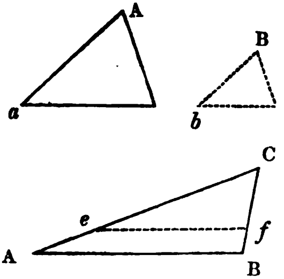
Triangles possess various useful properties; of which the following have been selected as the most useful to the measurer. They are deduced from the writings of Euclid; and rendered evident by geometrical construction.



Triangles constituted upon the same base, as AB, and under the same parallel of altitude as CD, are equal to each other; that is, the triangle ABC, is equal in area to the triangle ABD. — Euclid, I. 37.



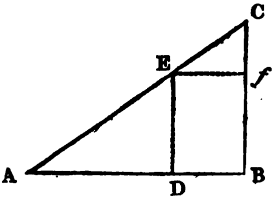
If two triangles, ABC, and DEF, have two sides of the one (AB and CA), equal to the two sides of the other (DE and DF), and the bases equal, the angles contained under the equal sides, will be equal.—Euclid, I. 8.



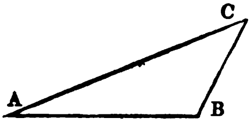
Similar triangles are such as have the angles of the one equal to the angles of the other: thus the triangle

B is *similar* to the triangle A: the angle *b*, in the one, being equal to the angle *a*, in the other; and so of the other angles. So, if a line *ef*, be drawn parallel to either side of a triangle (as AB), the small triangle C*ef*, is similar to the larger, ABC.

The sides of such triangles will be proportional to each other as follows:



AB : AD :: BC : DE.
Or, AB : BC :: AD : DE.
Or, AD : DE :: AB : BC.
Or, AD : DE :: E*f* : *f*C, &c.*



The longest side, as AC, of any triangle, is opposite to or subtends the greatest angle, as B: and the shortest side, as BC, is opposite to the least angle, A. —EUCLID, I. 18 and 19.

---

* Similar triangles are always in proportion as the squares of their like sides.

The sum of all the three angles, of any right-lined triangle, taken together, is equal to two right angles.

All the outer angles of any triangle make, together, four right angles.*—EUCLID, I. 32.

Various other curious properties of triangles may be found in EUCLID'S Elements of Geometry; and in Dr. HUTTON'S Mathematical and Philosophical Dictionary, under the article TRIANGLE.

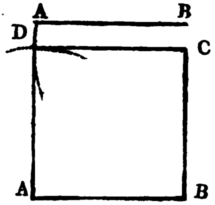
PARALLELOGRAMS.

PROBLEM XII.

SQUARE.

A *square*, or tetragon, is a figure consisting of four equal sides, and as many right angles.

To construct a square, each of whose sides shall be equal to the line AB.



Draw two perpendiculars, on A and B, equal to AB, and draw DC, and the square will be completed. —EUCLID, I. 46.

---

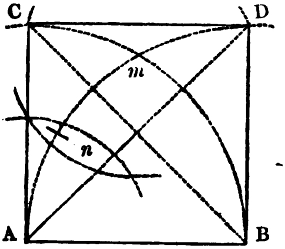
* This also holds good of any other right-lined figure, of any number of sides.

Or, draw BC perpendicular and equal to AB.

From A and C, with the radius AB, describe arcs intersecting in D.

Draw AD and CD, and the square will be completed.

Another way.



On the centres A and B, with the radius AB, describe arcs crossing at *m*.

Bisect A*m* in *n*; and with the centre *m*, and radius *mn*, cross the two arcs in C and D.

Draw AC, CD, and DB, and the square will be completed.*

For Practice.

A——————B

H——————I

---

* If two diagonals, viz. lines drawn from one angle to another be drawn in a square, they will intersect each other at right angles.

Construct a square, on the line AB, by the first method.

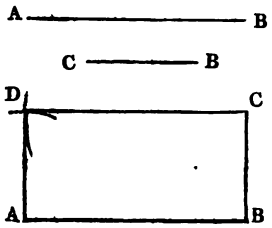
By the last method, construct a square on the line HI.

PROBLEM XIII.

RECTANGULAR PARALLELOGRAM.

A *rectangular parallelogram* is a four-sided figure, all whose angles are right angles, and whose opposite sides are parallel to each other.

To describe a rectangular parallelogram of the length AB, *and breadth* CB.



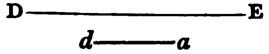
Erect the perpendicular BC of its due length, upon the end B of the base.

With the centre A, and radius BC, describe an arc.

With the centre C, and radius AB, describe another arc intersecting the former in D.

Draw AD and CD, and it is done.

For Practice.



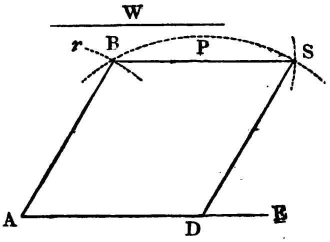
Construct a rectangular parallelogram of the length DE, and the breadth *da*.

PROBLEM XIV.

RHOMBUS.

A *rhombus* is a quadrilateral figure, whose sides are equal and parallel; but the four angles not all equal, two opposite ones being acute, and two obtuse; which, in the regular rhombus, are 60 and 120 degrees.

To make a regular rhombus equal to a given line, W.



Draw the line AE to any length greater than the given line W.

From A set off the length of the line W, to D.

With the same extent in the compasses, and one foot in D, draw the arc P.

With the same extent, and one foot in A, describe the arc *r*, intersecting the former at B.

With the same extent, and one foot in B, intersect the first arc in S.

Draw the lines AB, BS, and SD, and the rhombus will be completed.*

For Practice.

K——————L

Draw a regular rhombus on the base KL.

---

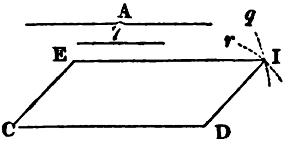
* If two diagonals be drawn in the rhombus, they will cross each other at right angles, as in the square.

PROBLEM XV.

RHOMBOIDES, OR RHOMBOID.

When two sides and angles in a parallelogram are unlike the other two sides and angles, the figure is called a *Rhomboid,* or *Rhomboides.*

To make a Rhomboid whose longest side shall be equal to A, and whose shortest shall be equal to Z.



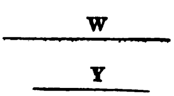
Draw the line CD equal to A.

Take the length Z in the compasses, and (if there be no limitation to the angle) set it from the point C to the point E.

Take the line A in the compasses, and from the point E make the arc *q*.

Take the line Z again in the compasses, and with one foot in D intersect the arc in I.

Draw the lines CE, EI, and DI, and CEID will be the rhomboides.



For Practice.

Construct a rhomboides, whose sides shall be as W and Y.

Note. Parallelograms constituted upon the same base (or on equal bases), and between the same parallels, are equal to one another.—EUCLID, I. 35. and 36.

In every quadrilateral (or four-sided) figure, the four angles, taken together, are equal to four right angles.

All the inward angles of any right-lined figure, taken together, make twice as many right angles, abating four, as the figure has sides. Or, what amounts to the same thing, multiply 180 by the number of sides, wanting 2.

So that the $\angle$s of a figure of

3 sides amount to	180°
4 - - -	360
5 - - -	540
6 - - -	720
7 - - -	900
8 - - -	1080
9 - - -	1260
10 - - -	1440
11 - - -	1620
12 - - -	1800, &c.

And all the external angles taken together, of any right-lined figure, whose sides are produced, make four right angles.

If a parallelogram and a triangle stand on the same base, and between the same parallels, the triangle will be exactly one half of the parallelogram.

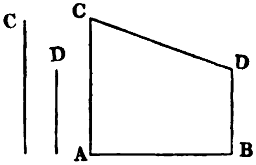
Parallelograms (as well as triangles), having equal bases, are to each other as their altitudes.

PROBLEM XVI.

TRAPEZOID.

A four-sided figure, having two sides parallel, standing on the same base, is called a *trapezoid.*

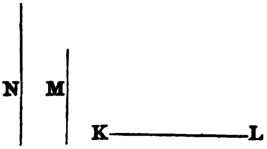
To construct a trapezoid on the base AB, *whose parallel sides shall be equal to* C *and* D.



If the sides are perpendicular to the base (as com monly is the case in land-measuring), on the end A erect a perpendicular of the length C.

On the end B erect another perpendicular of the length D; aud ABDC will be the required trapezoid.

For Practice.



Draw a trapezoid on KL, whose parallel sides shall be as M and N.

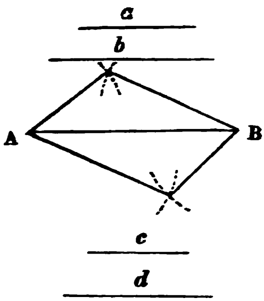
PROBLEM XVII.

TRAPEZIUM.

A *trapezium* is a four-sided figure, whose sides are neither equal, nor parallel to each other.

Construct a trapezium on the diagonal AB, *whose two upper sides shall be equal to* a *and* b, *and whose under sides shall be equal to* c *and* d.

This construction is no other than drawing two



scalenous triangles, where one side, AB, is common to them both; for which I refer back to Problem X.

POLYGONS.

Note. The figures of more than four sides, are in general called *polygons;* and receive other particular names, according to the number of their sides: thus,

a figure of five sides is called		a *pentagon;*
of six sides	-	a *hexagon;*
of seven	-	a *heptagon;*
of eight	-	an *octagon;*
of nine	-	a *nonagon;*
of ten	-	a *decagon;*
of eleven	-	an *undecagon;*
of twelve	-	a *dodecagon,* &c.

If the sides and angles of any such polygon be equal, it is called a *regular polygon;* otherwise, it is called an *irregular polygon.*

As their construction depends on their angular quantities, I shall forbear further notice of them until I have treated of Trigonometry.

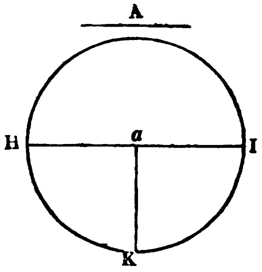
PROBLEM XVIII

CIRCLE.

A *circle* is a plain figure, comprehended under one curve line only, called the *circumference*; every part of which is equally distant from a point therein, called the centre.

A line drawn from any point of the circumference, through the centre, to another point of the circumference, is called the *diameter*; the distance from the centre to the circumference is called the *radius*.

With the radius A, *to describe a circle from the centre a.*



Take the line A in the compasses, and placing one foot in the centre *a*, sweep the curve line round until it returns into itself; and the circle will be completed.

Note. A semi-circle, as HKI, is one half the circle: and a quadrant, as HK, is a quarter of the circle.

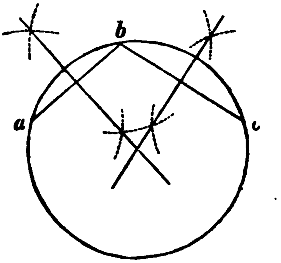
For Practice.

n————*o*

On the centre *o*, and radius *on*, describe a circle.

PROBLEM XIX.

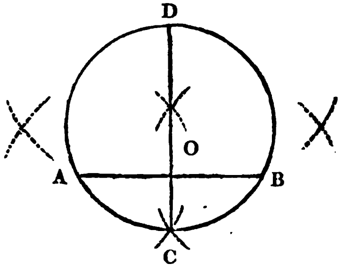
Having any three points given, as a, b, and c, not lying in a right line, to describe a circle which shall pass through all those points.



From *a* to *b*, and from *b* to *c*, draw right lines.

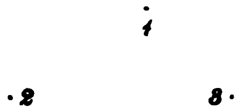
Bisect those lines by Problem II.

The lines of bisection, continued, will cross each other in the centre of the circle, which will pass through the three points given.*



* The centre of a circle may be found another way; thus, draw any chord, AB; bisect it perpendicularly with CD. — Bisect CD, and it will give the centre O.

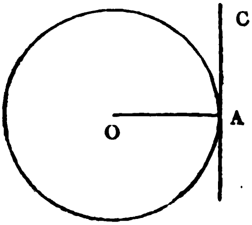
For practice.



Through the three points, 1, 2, 3, describe a circle.

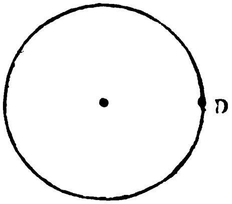
PROBLEM XX.

To draw a tangent to a given circle, through a given point A.



From A, draw AO to the centre; perpendicular to which draw BAC; and it will be the tangent required.

For practice.

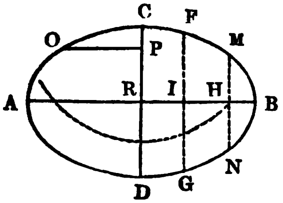


Draw a tangent to the circle, from the point D.

PROBLEM XXI.

ELLIPSIS.

An ellipsis, or oval, is a plane figure, bounded by a regular curved line, falling into itself; which, not being circular, is described from two central points.



Any line drawn through the centre (R) of an ellipse to the periphery, is called a diameter; as AB, or CD.

The longest diameter which can be drawn, as AB, is called the *transverse;* and the shortest, DC, is called the *conjugate.*

Any line drawn from a point in the curve parallel to the transverse or conjugate diameter, is called an *ordinate,* as FI, or IG, or OP.

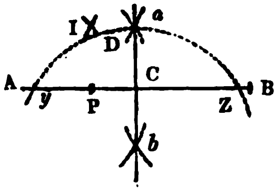
If either the transverse or the conjugate diameter be cut by an ordinate in any point, the two segments made by such section are called *abscissas;* as AI and IB; CP and PD.

Foci or *focuses,* are two points in the transverse diameter, from which, as from two centres, the periphery of the ellipsis is described.

Latus rectum, or *right parameter,* is that double ordinate which passes through a focus; as MN.

The transverse and conjugate diameters of an ellipse being given, to find the foci of that ellipse.

Draw the transverse diameter of the given length.



Bisect it in C, and set off the semi-conjugate from thence, above and below, to *a* and to *b*.

With the radius AC, and centre *a* or *b*, describe an arc D, cutting the transverse diameter in *y* and Z, which are the foci required.

Having the two diameters and foci given, to construct the ellipse.

Draw the two diameters, and mark off the foci, as above.

Take any point in the transverse, as P; then with the radii AP and PB and centres *y* and Z, describe two arcs, intersecting in I; so will the point I be in the circumference of the ellipse.

Assume as many points, P, in the transverse as may be necessary, and make similar intersections; through all which, let the curve line be carefully drawn by hand.

Otherwise.

Having the foci found, as before, take a thread of the length of the transverse; fasten its ends with pins in the foci; then stretch the thread with a pencil, and moving the pencil half round within the thread, keeping it always stretched, half the curve will be formed; do the like for the other half, and the ellipse will be completed. *Or*, with two pins fixed in the foci, and another in one end of the conjugate, tie the thread, and that extent reached with a pencil will, by moving the pencil round, describe the curve.

Another way.

An ellipse may be generated by the shadow of a circle, or ring, projected on a plane, not parallel to the ring, the luminous point not being in the plane of the circle; and from thence may be drawn, by tracing the extreme shadow.

Example for practice.

Having the transverse given = A, and conjugate = B; it is required to construct the ellipse by the second method.

A————————————

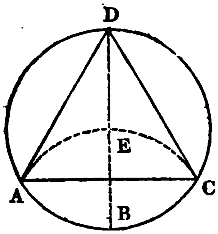
B——————————

PART II.

OF INSCRIBING AND CIRCUMSCRIBING FIGURES.

PROBLEM I.

To inscribe an equilateral triangle in a given circle.



In the circle ABCD, draw the diameter DB.

With the radius in the compasses, and centre B, draw AEC. The points A and C comprise one side of the triangle.

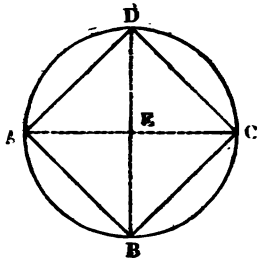
Draw DA and DC, and they will be the other sides.

PROBLEM II.

To inscribe a square in a circle.

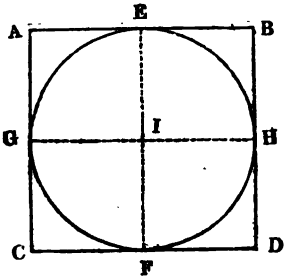
Draw two diameters, AC and DB, crossing each other at right angles, in the centre E.

Join the four extremes of the diagonals with right lines, and they will form the inscribed square.



PROBLEM III.

To inscribe a circle in a given square.



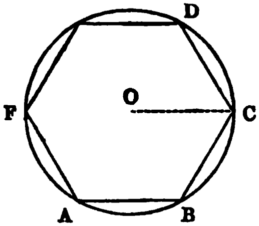
Bisect the sides AB and CD, in the points E and F, and draw EF parallel to the sides AC and BD.

In like manner, draw GH parallel to AB and CD, intersecting EF in the centre I.

From I, with the radius IE, or IF, describe the circle, touching the square in EFG and H.

PROBLEM IV.

To inscribe a regular hexagon in a circle.



The radius of a circle being equal to a side of its inscribed hexagon, take the radius in the compasses, and turn it over on the circumference, from A to B; from thence to C, &c. which points will be the angular points of the hexagon.

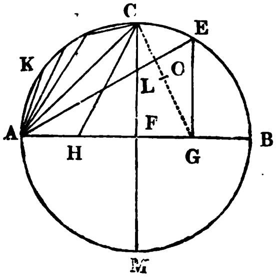
PROBLEM V

All other regular polygons may be inscribed in a circle, for each of which a particular rule may be given; but I choose rather to give one general scheme for approximately inscribing all the usual regular polygons in a circle.

In the circle AB CM, draw two diameters, crossing each other at right angles; then,

For the equilateral triangle.

Take FC in the compasses, and set it from A to D, and from D to E; draw AE, and it will be a side of the inscribed equilateral triangle.



For the square.

Draw AC, and it will be a side of the square inscribed.

For the pentagon.

Bisect FB in G, draw CG; make GH = GC, draw CH for a side of the pentagon.

For the hexagon.

The radius FC set off from A to D, will be a side of the hexagon.

For the heptagon.

The line EG is a side of the heptagon.

For an octagon.

Bisect the arc AC in I, and draw AI for a side of the octagon.

For the nonagon.

Divide the arc ADE into three equal parts; at K draw AK, and it will be a side of the nonagon.

For the decagon.

The part of the diameter intercepted between H and F, will be a side of the decagon.

For the hendecagon.

Bisect the line CG in O for the side of the hendecagon.

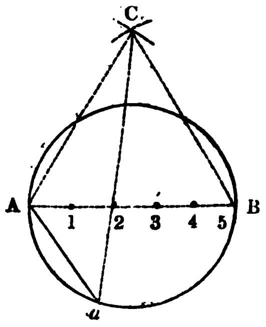
For the dodecagon.

The line from C to D will be a side of the dodecagon.

Another approximate method is as follows:

Draw the diameter, AB, of the circle; and therewith construct an equilateral triangle, ABC.

Divide the diameter into as many equal parts as the proposed polygon has sides.



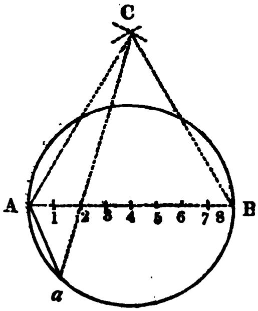
From C, the apex of the equilateral triangle, draw a line through the *second division* in the diameter, until it touches the circle below, as at *a*; from whence, if a line be drawn to A (where the numbering began), it will give the length of a side of the required polygon.

1. EXAMPLE.

Required the side of a *pentagon*, by the above figure.

Divide the diameter into five parts. Draw lines from C, through 2 to *a*, and from thence to A; so will A*a* be the side of the pentagon

2. Required the side of an *octagon*.

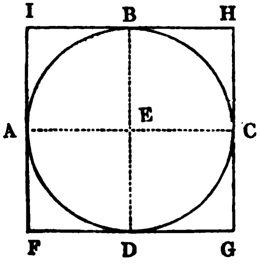


The diameter being divided into eight parts, draw the line C2, as before directed, to meet the circumference at *a*. Draw A*a*, and it will be the length of a side of the octagon required.

PROBLEM VI

To circumscribe a square about a circle.

Draw two diameters, AC and BD, intersecting each other at right angles in the centre of the circle E.

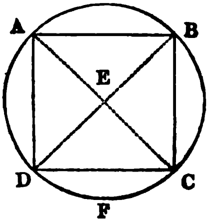


Through the extremities thereof draw FG and IH parallel to AC; and IF and GH, parallel to BD;

and the square, IFGH, will circumscribe the circle, ABCD.

PROBLEM VII.

To circumscribe a circle about a square.

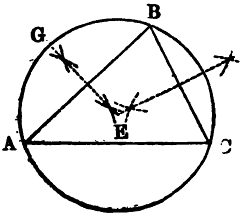


Draw the diagonals AC and BD, intersecting each other in the centre E.

With the radius EA, describe the circle ABCFD.

PROBLEM VIII.

To circumscribe a circle about a triangle.



Bisect any two sides, as AB and BC, with the perpendiculars, G and F, and they will cross each other at E, the centre of the circle which circumscribes the triangle.

With EB, or EA, or EC, as a radius, describe the circle ABC.

PART III.

TRIGONOMETRY *

Is that branch of mathematical science which is employed in calculating the sides and angles of a *Triangle;* and its doctrines are founded on the mutual proportions which subsist between the sides and angles of that figure. These proportions are discovered by first finding the relations between the *radius of a circle*, and certain other lines drawn in and about the circle, called *chords*, *sines*, *tangents*, and *secants;* which lines are either expressed in numbers, and arranged in tables, called the tables of sines, tangents, &c.; or are actually laid down in lines upon scales, sectors, &c. The parts, then, of a supposed triangle being ready calculated in the tables of Sines and Tangents, the parts of other *similar angled* triangles are easily found by the rule of Proportion; for the lines in all triangles having equal angles, are in direct proportion to each other.

The circumference of every circle is divided (or supposed to be divided) into 360 degrees; every degree into 60 minutes, &c. And there are various right lines so related to these degrees, or parts of a circle, as to be called their sines, tangents, &c. as in *fig.* 6. The radius is supposed to be 1·00000, and

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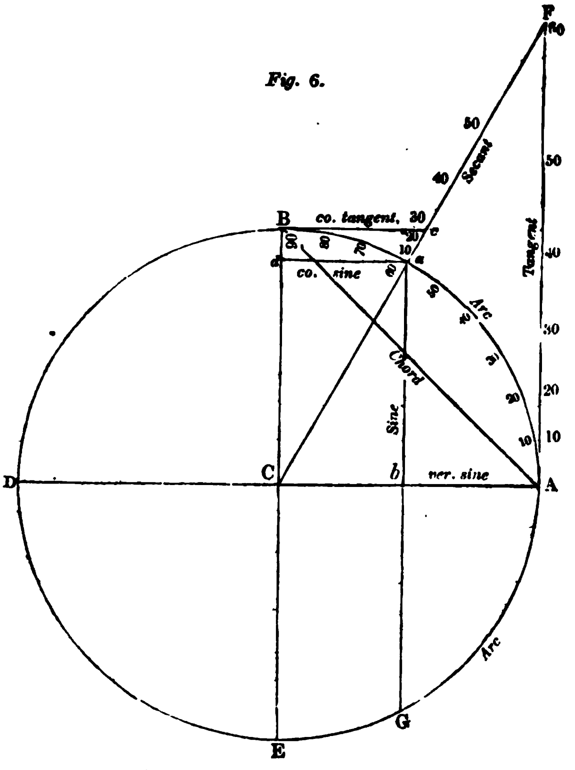
* Trigonometry not being essential to the mere practical Surveyor, may be passed over by the student whose time for study is limited.

Fig. 6.



all the other lines are exactly calculated to every degree and minute, after this standard, in the tables.

By inspecting the figure, it will be perceived that a chord of an arc is a right line which divides a circle into two unequal parts; and is common to both those parts: as AB is a chord, as well to the part AEDB, as it is to that part of the circle included between A and B.

A *right sine* of an arc is a line drawn from one end of that arc, perpendicularly to the radius; as *ab* is the sine of the arc A*a*. A *versed sine* is that part of the radius which is contained between the right sine and the circumference, as *b*A. The cosine is *da*.

A *tangent* of an arc is a right line drawn touching one extremity of the arc, and terminating at the secant of that arc; as AF is the tangent of the arc A*a*. B*c* is the *co-tangent* of the same arc.

A *secant* of an arc is a right line drawn from the centre through the extremity of the arc, and produced until it meets the tangent; as CF. The *co-secant* is C*c*.*

These lines, with a line of equal parts, are commonly laid down parallel to one another, on wood or brass rules; and constitute what is commonly called (from its inventor) *Gunter's* [*] plain scale (fig. 7.); and is used in constructing triangles and other plane figures.

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* The *complement* of an arc is what it wants of a quadrant, or 90°; the *supplement* of an arc is what it wants of a semicircle, or 180°.

The sine, tangent, and secant, of the complement of an arc, are called the co-sine, co-tangent, and co-secant of that arc.

Some peculiar properties of the lines in and about a circle are as follows: —

1. The square of the diameter is equal to the sum of the squares of the chord of an arc, and of the chord of its supplement to a semi-circle.

2. The square of the radius is equal to the sum of the squares of the sine and co-sine.

3. The sum of the co-sine and versed-sine is equal to the radius.

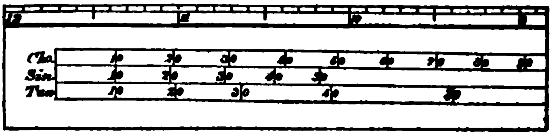
4. The sine is a mean proportional between the versed sine of double the arc, and the sum of the co-sine and radius.

5. Radius is to the sine as twice the co-sine is to the sine of twice the arc: or, as the secant is to the tangent.

6. As the co-sine is to the sine, so is radius to the tangent.

7. Radius is a mean proportional between the tangent and the co-tangent. And also between the secant and co-sine.

Fig. 7.



On the same scales are other lines of sines, tangents, numbers, &c., constructed from the logarithms of numbers; which lines are used in finding (by proportion) the angles and sides of triangles.[†]

Plane triangles are composed of six parts; three angles and three sides; and in all solutions, three of these parts (one of which must be a side) are always given, to find any or either of the others.

There are three methods of resolving triangles; viz. *geometrical construction, instrumental operation,* and *arithmetical computation.* Each of which I shall explain; but the latter particularly recommend.

In the first method, the triangle is constructed by drawing and laying down the given sides, from a scale of equal parts; and the given angles, from the line of chords or protractor: then the unknown parts are measured by the same scales, and so become known.

In the second method, by the logarithmic lines on *Gunter's* scale, extend the compasses from the first term to the second or third, whichever happens to be of the same kind as the first; then, that extent laid off from the other term (whether second or third) will reach to the fourth, or answer. In these logarithmic lines, the 10 in the line of numbers, 90 on the sines, and 45 on the tangents, are set perpendicular to each other.

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[*] Mr. Gunter was a celebrated Professor of Astronomy at Gresham College.

[†] For further particulars of the construction of these and other lines on plain scales, the reader is referred to BION on Mathematical Instruments, Book I.; or more especially to HUTTON'S *Mathematical and Philosophical Dictionary,* under the article *Scale.*

In the third method, the terms of the proportion must be stated according to rule; which terms consist partly of the numbers which express the given length of sides, and partly of the logarithmic sines or tangents of the given angles, taken from the tables; in which case the logarithms of the second and third terms are to be added together; and, from their sum, the first must be subtracted, as has been heretofore directed in logarithmical arithmetic: or else, when radius is not concerned in the analogy, by taking the arithmetical complement of the first term, and adding it to the logarithms of the second and third terms, the natural number of which aggregate logarithm is the fourth term of the proportion.

There are other methods of finding the proportions which the sides and angles of triangles bear to each other; but as they consist of long and tedious multiplications and divisions, I shall not waste the learner's time in explaining them.

TRIGONOMETRY has three cases; viz.

Case I.

When two of the given parts are a side and its opposite angle.

Case II.

When two sides and their included angle are given.

Case III.

When the three sides are given.

Case I.

Rule for the First Case.

As one given side
Is to the sine of its opposite angle,
So is either of the other given sides
To the sine of its opposite angle.

Also,

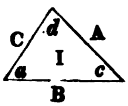
As the sine of any given angle
Is to its opposite side,
So is the sine of any other given angle
To its opposite side.

So that, to find an angle, the proportion must begin with a side which is opposite to a given angle: and to find a side, the proportion must begin with an angle opposite to a given side.*

If the given angle be acute, and opposite to a lesser side, the triangle may be ambiguous; as the sine, thus found. answers to two angles, which are supplements to each other; and the construction would form two triangles from the same given parts. But if the given

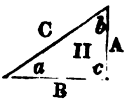
---

* If in any triangle (as I.) A B and C stand for the respective sides, *a d* and *c* for the respective opposite angles, and *s* for the sine of any ∠, then $\frac{A}{s.\,a} = \frac{B}{s.\,d} = \frac{C}{s.\,c}$



Note also. — In these and in the subsequent computations, *s.* stands for the sine of an angle; *co-s.* for the *co-sine*; *t.* for *tangent*; *co-t.* for *co-tangent*; and *r.* for *radius*.

When the trigon (II.) is right angled, then $B = \sqrt{C^2 - A^2} = \sqrt{\overline{C + A} \times \overline{C - A}}$.

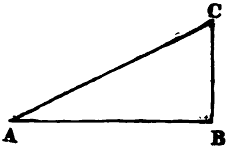


This is deducible from EUCLID, Book I Prop. 47.; where $C^2 = B^2 + A^2$; and the sides themselves become known, by extracting the square-roots of these squares.

angle be right or obtuse, there can be but one triangle constructed with the same dimensions. This ambiguity must be determined before the operation.

EXAMPLE I.

In the triangle ABC, let there be given
The side AB = 735·9 links
——— BC = 320
And angle A = 23° 30′.



Required, 1st the angle C (assuming it as acute),
——— 2d the angle B,
——— 3d the side AC.

First method, *by Geometrical Construction.*

Draw AB = 735·9, from some scale of equal parts of proper dimensions.

From some scale of chords, with the radius 60°, sweep the arc *ab;* then set off the chord of 23° 30′, from *b* to *a.*

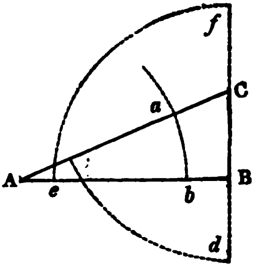
Draw AC, of an indefinite length, passing through the point *a.*

On the same scale of equal parts, take 320 in the compasses; and with one foot in B, intersect the line AC in C.

Draw BC, and the triangle will be completed.

To find the angle C.

With the radius of 60° on the chords (a radius which for setting off angles must be always used),



from C sweep the arc *cd*, crossing the line CB, continued in *d*.

Take the extent *cd* in the compasses; and with one foot in 0 of the scale of chords, extend the other over the chord scale, and it will reach to 66° 30′ for the angle C.

To find the angle B.

In like manner, from B sweep the arc *ef*; and the extent from *e* to *f*, applied to the scale of chords, will reach to 90°.*

To find the side A C.

Take the extent of the line, and apply it to the same scale of equal parts with which the triangle was constructed, and it will reach from 0 to 802·5.

Second method, *by Gunter's Logarithmic Scale.*

To find the angle C.

Analogy.

As CB : *s* ∠† A :: AB : *s* ∠C.

---

* All the three angles of a triangle are equal to 180°; therefore if the sum of any two be subtracted from thence, the remainder will be the quantity of the third angle. The sum of the angles A and C (23° 30′ + 66° 30′ = 90), subtracted from 180° leaves 90° for the angle B.

† ∠ means the *angle* denominated by its accompanying letter.

Practical Rule.

Extend the compasses from the first to the third term, on the line of numbers, viz. from 320 to 735·9.

With this extent, place one foot in 23° 30′, on the line of sines; and the other will reach to 66° 30′ for the required angle.

To find the angle B.

Subtract the sum of these angles from 180°, and the remainder (90°) will be the angle B.

To find the side A C.

Analogy.

As *s* ∠ A : BC :: *s* ∠ B : AC

Extend the compasses from 23° 30′ to 90° on the sines: that extent set from 320 will reach to 802·5, for the side AC.*

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* The same may be performed with the *sector* (fig. 8.), whereon the lines of natural sines, natural tangents, and equal parts, or

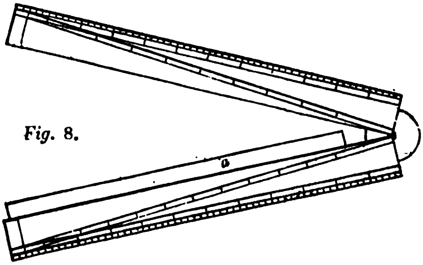


Fig. 8.

lines, are laid on the two legs, and marked SS, TT, LL; thus, from the central point extend the compasses to 735·9 (= AB) on one of the lines marked L. Open the legs until this distance reaches from 66° 30′ to 66° 30′ (= ∠ C) on the two lines of sines marked SS. Take the distance from 90° to 90° (= ∠ B) on the same lines SS, and it will extend from the centre of the brass joint to 802·5 on the line of lines, for the side AC.

Third method, *by Arithmetical Computation with Logarithms.*

To find the angle C

As the side BC = 320 links, arith. comp.	7·494850
Is to the sine of its opp. angle A, 23° 30′	9·600700
So is the side AB = 735·9 - - -	2·866819
To *s.* of opp. ∠ C = 66° 30′ - -	9·962369

To find the angle B.

Subtract the sum of the foregoing angles (90°) from 180°, and the remainder will be ∠ B, 90°.

To find the side AC.

As *s.* ∠ A, 23° 30′ - - -	9·600700
: BC, 320 links - - -	2·505150
:: *s.* ∠ B, 90° - - - -	10·000000
	12·505150
: AC=802·5 - - -	2·904450

In the *right-angled* triangle, where the base, with the lesser angle A, are given, the *perpendicular* may be thus found:

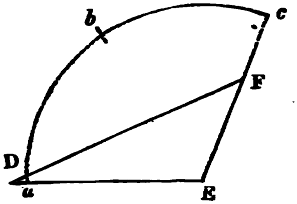
As radius : the given leg, AB, (see the last fig.)
:: the *tangent* of the ∠ A
: the perpendicular, BC.

And the *hypothenuse*, thus:

As radius : the given leg, AB.
:: the *secant* of ∠ A
: the hypothenuse, AC.

EXAMPLE II.

In the triangle DEF, given,
The base DE = 610 links,
∠E 115°
∠F 42° 30′
Required DF, ∠D and EF.



First, *by Construction*.

Draw DE, by the scale of equal parts = 610.

At E set off, by the line of chords. the ∠E = 115°.*

Draw E*c*.

The sum of the ∠s E and F (157° 30′), being subtracted from 180°, leaves 22° 30′ for ∠D.

Set off ∠D, and draw DF cutting E*c* in F.

Measure EF on the scale of equal parts, and it will be found = 345·53. And on the same scale, DF will be found = 818·32.

Second method, *by Gunter's Logarithmic Scale*.

To find the side DF.

Analogy.

As *s.* ∠F : DE :: *s.* ∠E : DF

---

* When the angle is greater than 90° take half the angle and set it off at twice; as from *a* to *b*, and from *b* to *c*.

Extend the compasses, on the line of sines, from 42° 30′ (the ∠F) to the supplement of 115° (∠E) = 65°.

That extent, from 610 (DE) will, on the line of numbers, reach to 818·32 = DF.

To find the ∠D.

Subtract the sum of the other angles (E and F = 157° 30′) from 180°, and the remainder will be 22° 30′, the required angle.

To find the side EF.

Analogy.

As *s.* ∠F : DE :: *s.* ∠D : EF.

Extend the compasses on the line of sines from 42° 30′ (∠F) backwards, to 22° 30′ (∠D), that extent from 610, backwards, will reach to 345·53 = EF.

Third method, *by Logarithms.*

To find the side DF.

As *s.* ∠F, 42° 30′, arith. com. or co-sec.*	0·170317
: DE 610 - - - - - -	2·785330
:: *s.* suppl. ∠E 65° - - -	9·957276
: DF 818·32 - - - -	2·912923

The ∠D is found by subtraction = 22° 30′.

---

* It has been before noted, that when the arithmetical complement of any sine is wanted, the co-secant of that number (abating 10 in the characteristic) is the arithmetical complement sought.

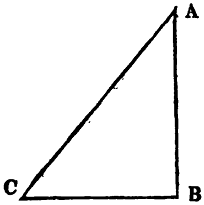
To find the side EF.

As *s.* ∠F 42° 30′ arith. comp. - - 0·170317
: DE 610 - - - - 2·785330
:: *s.* ∠D 22° 30′ - - - - 9·582840

: EF = 345·53 - - - - 2·538487

Examples for Practice.

1. *In the right-angled triangle* CBA,
Given CB = 560 links.
∠ A = 48° 30′.
∠ B = 90°.



Required, first, by construction, the side AC,
—— 2d, by logarithms, the side AB,
—— 3d, by subtraction, the ∠C.

Ansr. AC = 747·7, AB = 495·45, and ∠C = 41° 30′.

2. *In the same triangle.*
Given ∠C = 41° 30′.
BA = 495.45 links.

Required, first, by Gunter's scale, BC,
—— 2d by logarithms, CA,

Ansr. AC 747·7, BC 560.

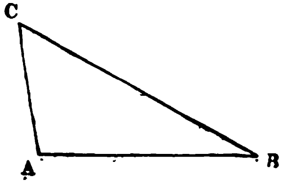
3. Suppose a right-angled triangle has its base 740 links, its ∠C = 50° 5′.

Required, by logarithms, the sides AB and AC.

Ansr. AB = 884·5, AC = 1153·2.

The construction of the triangle is also required.

4. *In the obtuse triangle* ABC,
Given AB = 765 links,
AC = 290,
∠C = 58° 5′



Required, first, by logarithms, ∠B.
—— 2d, by subtraction, ∠A.
—— 3d, by the scale CB.

Ansr. ∠B=18° 46′, ∠A=103° 9′, and CB=877·6.

5. In an acute-angled triangle.
Suppose the side AB 848 links,
∠C 88° 30′,
∠B 44° 15′.
Required the ∠A, by subtraction,
the side CB, by logarithms,
and AC, by the scale.

Ansr. ∠A = 47° 15′. CB = 622·9. AC = 591·9.

Case II

Rule for the Second Case.

As the sum of the two given sides is to the difference of those sides, so is the tangent of half the sum of their opposite angles, to the tangent of half their difference.

This half-difference, added to the half sum, will give the greater angle *: and taken from the half sum, will give the less angle.†

---

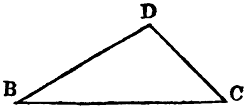
* The greatest is always opposite the longest side.

† The same holds good of two quantities of any kind; for if the sum of two numbers be 60, and their difference 20; half the

All the angles being thus known, and two sides given, the other side is found by Case I.

In the triangle BCD.

Given BD = 1636 links,
DC = 1272,
∠D = 97° 30′.
Required the ∠s B and C, and the side BC.



First, *By Construction.*

Draw the line BD = 1636.

At the point D make the ∠ = 97° 30′.

Draw the line DC = 1272; and join these legs by BC.

By a scale of equal parts, measure the base, and it will be found = 2200.

On the scale of chords, the ∠ B = 35°.

- - - - the ∠ C = 47° 30′ nearly.

Second, *by Gunter's Scale.*

Extend the compasses, on the line of numbers, from the sum of the sides (= 2908) to the difference (= 364); that extent will reach from the tangent of the half sum of the opposite ∠s (= 41° 15′) to the tangent of half-difference (= 6° 16′).

This half-difference, added to the half sum, gives 47° 30′ for the greater ∠ C; and subtracted, leaves 35° for the lesser ∠ B, nearly.

---

difference (10), added to the half sum (30), will give (40) for the greater number; and subtracted therefrom, will leave (20) the less.

Then, by Case I., the side BC is found = 2200

Third method, *by Logarithms.*

As sum of sides BD + DC = 2908,
co-arith. - - - = 6·536406
: their difference - = 364 = 2·561101
:: the *t.* of half opp. ∠s. 41° 15′ = 9·942988

: *t.* of half difference - 6° 16′ = 9·040495

Their sum - 47 31 = ∠C.

Difference - 34 59 = ∠B.

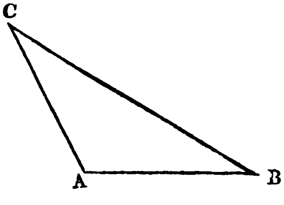
Then, *by the First Case.*

As *s.* ∠B = 34° 59′ co-arith. or co-sec. = 0·241589
: side DC = 1272 - - - = 3·104487
:: *s.* ∠D = 97° 30′ supplement - = 9·996269

: BC = 2199·6 - - - = 3·342345

Examples for Practice.

1. In the triangle ABC,



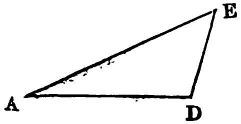
Given, AB = 530 links.
BC = 923·6
∠B = 29° 25′.
Required ∠s A and C, by logarithms.
And the side AC, by Gunter's scale.

Ansr. ∠A 121° 9′, ∠C 29° 26′, and AC 530.

2. Suppose the ∠C, in the foregoing triangle, be 36°, the side AC 560 links, and BC as before; required the other parts by logarithms.

Ansr. ∠A 109° 1′, ∠B 34° 59′, AB 574.

3. In the triangle ADE.



Given, AE = 630
AD = 520
∠A = 22° 37′

To find, by Gunter's Scale, the ∠*s* E *and* D; *and logarithmically, by Case the* 1*st, the side* DE.

Ansr. ∠E 53° 8′, ∠D 104° 15′, and DE 250.

Case III.

Rule for the Third Case.

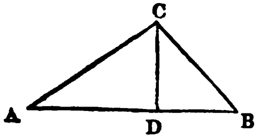
From the greatest angle, let fall a perpendicular to the base or opposite side, dividing it into two segments, and the whole triangle into two right-angled triangles.

Then, by the common rule of proportion,

As the whole base
Is to the sum of the other two sides,
So is the difference of those sides
To the difference of the segments of the base.

Then, half this difference of segments added to half the base, will give the greater segment; and subtracted from the half base, will leave the less segment *: and the whole triangle will be divided into two right-angled triangles; with two sides and an angle given in each. The remaining sides and angle may be found by the rule to the first Case.

In the triangle ABC.



Given, AB = 2200 links.
—— AC = 1636
—— BC = 1272
Required the angles.

First, *by Construction.*

This triangle is drawn as Problem X. in Geometry; and the perpendicular demitted as Problem IV. The parts are then to be measured as hereinbefore directed.

Second, *by Gunter's Scale.*

To find the difference of the segments.

Analogy.

As AB : AC + CB :: AC – CB : diff. of segs.

Extend the compasses, on the line of numbers, from 2200 (the base), to 2908 (the sum of the sides); that extent will reach from 364 (the difference of sides), to 481 (the difference of segments), or fourth number.

Hence the segments are found to be about 1340 the greater, and 860 the less.

---

* The greater segment will adjoin the longer side; and the less segment the shorter side.

Secondly, to find ∠ A.

Analogy.

As AC : *rad.* :: AD : *co-s.* ∠ A.

Extend from 1636 (AC), on the line of numbers, to 1340, the greater segment; that extent, from radius on the sines will reach to 55° = the ∠ ACD, the complement of which is 35° for ∠ A.

Thirdly, to find ∠ B.

Extend from 1272 (BD), to 860, the less segment; that extent, from radius, will reach to 42° 32′ = ∠ BCD; the complement of which is 47° 28′ for ∠ B.

The supplement of the sum of these two angles, 97° 32′ = the ∠ ACB.

Third method, *by Logarithms*, &c.

As 2200 : 2908 :: 364 : 481 diff. of segs.

Half diff. of segs. -	240·5
+ and − to and from ½ base	1100
	1340·5 great seg.
	859·5 less seg.

For the ∠ A.

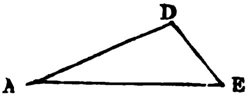
As AC = 1636 - - - -	3·213783
: radius - - - - -	10·000000
:: AD = 1340 - - - -	3·127105
	13·127105
: *Co-s.* of ∠ A = 35° - - -	9·913322

For the ∠B

As BC = 1272 - - - -	3·104487
: radius - - - -	10·000000
:: DB = 860 - - - -	2·934498
	12·934498
: *Co-s* of ∠B = 47° 27′ - -	9·830011

The supplement of the sum of the ∠A + ∠B = 97° 33′, will be the ∠ACB.

Examples for Practice.



Given, AE = 630
AD = 520
DE = 250

Required the three angles.

Ansr. ∠A = 22° 37′, ∠E 53° 8′, and ∠D 104° 15′.

2. Suppose AE = 400
AD = 385
DE = 190

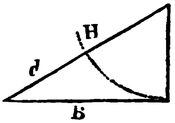
Required the angles.

Ansr. ∠A = 27° 56′, ∠E 71° 38′, and ∠D 80° 26′.

There are various properties of right-angled triangles, which are deduced from Algebra, or analytical investigation; a few of which follow:

Given, the base, B, and the difference between the hypothenuse and perpendicular, = *d*, to find the perpendicular P.

Theorem, $\frac{B^2-d^2}{2d}=P$.

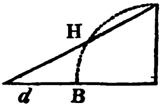


Given, the hypothenuse, H, and the sum of the legs, B, P (which call s) thence to find the legs respectively.

1st. *For the base,* $\frac{s+\sqrt{2H^2-ss}}{2}=B$.

2d. *For the perpendicular,* $\frac{s-\sqrt{2H^2-ss}}{2}=P$.

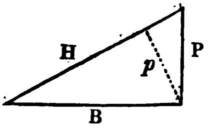
Given, the hypothenuse, H, and the difference of the other sides, $=d$, to find the sides B and P.



1st. *For the base,* $\frac{d+\sqrt{2H^2-d^2}}{2}=B$.

2d. *For the perpendicular,* $\frac{\sqrt{2H^2-d^2}-d}{2}=P$.

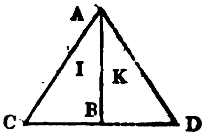
Given, the difference d, of the legs B, P, and the perpendicular p, demitted from the right angle to the hypothenuse; to find the hypothenuse.



$p+\sqrt{d^2+p^2}=H$.

If, in a right-angled triangle, one of the acute angles be 30°, the side opposite that angle will be equal to half the hypothenuse: for,

Let ABC be the triangle given, right angled at B, and the $\angle$ CAB$=30°$, then BC$=\frac{AC}{2}$



DEMONSTRATION.

Produce CB to D, so that BD may be equal to CB, and drawing AD, the two triangles I and K will be similar and equal; therefore, the $\angle$ CAD will be $=$ 60°, and the triangle CAD will be equilateral; therefore the line CB, which is half of CD, will also be half of AC.

PART IV.

THE APPLICATION OF TRIGONOMETRY TO DISTANCES, &c.

By the mensuration of one or more lines, and by taking the angle which one object bears towards another, we find out, trigonometrically, the relative distances between such objects as may be inaccessible by impediments of water, wood, or other obstruction.

The instrument which a land measurer ought to choose, for taking the angles of bearing of one object to another, is the theodolite.* And for such lengths as he may have occasion to measure, he will, doubtlessly, employ a *Gunter's* chain†; as being best calculated for accuracy and dispatch.

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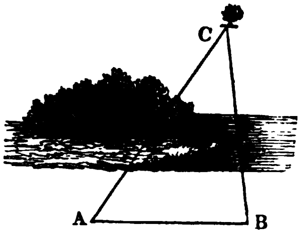
* For a description of this instrument see page 191.

† The measuring chain is composed of 100 links of strong iron wire, each link 7·92 inches; consequently, the whole chain is 22 yards or 4 poles, in length. At every 10 links is fastened a piece of brass, so cut into points as to denote the number of tens. The better to carry on the account of the number of chains, there are ten (or rather eleven) small iron arrows, 16 inches in length, used by the assistant who has the foremost end of the chain, one of which he sticks into the ground at the end of each chain's length; which are collected by the hindermost chainman, or master-measurer.

EXAMPLE I.

In the triangle ABC, given the ∠A = 60°, the ∠B = 80°, and the base AB = 500 links.

The distance from A to C (inaccessible on account of wood) is required.



By subtracting the sum of the angles A and B from 180°, the ∠C is found = 40°.

Then, by Case I. in Trigonometry,

As *s.* ∠C = 40° - - - - 9·808067

: AB = 500 links - - 2·698970

:: *s.* ∠B = 80° - - - 9·993351

12·692321

: AC = 766 links - - 2·884254

Examples for Practice.

1. In the foregoing triangle, given as before, required, by a scale of 6 chains to an inch, and also by logarithms, the side BC, inaccessible on account of water.

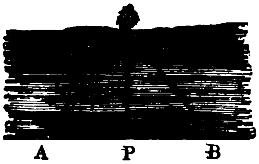
Ansr. 673·7 links.

2. Supposing, also, ∠ B was 85° 30′ required the distance BC.

Ansr. 764·5 links.

EXAMPLE II.

On the side of a river, whose breadth is required, suppose two stations taken at A and B, the distance from each other being 450 links; and the ∠BAC = 53°, and the ∠CBA = 60°, the perpendicular CP is required.



By subtraction of ∠A + ∠B from 180, the ∠ACB is found = 67°.

Then, by Case I. in Trigonometry,

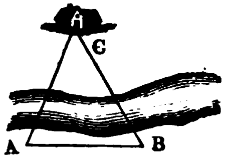
First, As *s.* ∠ACB = 67°	-	9·964026
Is to AB = 450	-	2·653212
So is *s.* ∠B = 60°	-	9·937531
		12·590743
To side AC = 423·36	-	2·626717

Second, The ∠P being a right angle,

As Rad. = 90°	- -	10·000000
: AC = 423·36	-	2·626717
:: *s.* ∠A = 53°	- -	9·902349
		2·529066

: CP = 338·11, the perpendicular required.

Examples for Practice.

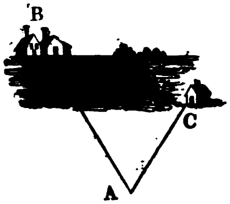


Let A and B be two stations, by a river's side, 400 links apart; let C be a house, on the other side, the distance of which from A, and from B, is required; and let the ∠ A be 70°, and ∠ B 54° 30′; required, by Gunter's scale, the distance AC, and, by logarithms, the distance BC.

Ansr. AC 395·1, and BC 456·1 links.

EXAMPLE III.

Let the inaccessible distance between two houses, B and C, be required, from the following data, viz. The distance from A to B = 500, from A to C = 350; and the ∠ A = 65°.



By Case II. of Trigonometry.

As AB + AC = 850 co-arith. - - 7·070581
: AB — AC = 150 - - - 2·176091
:: *t.* of $\frac{\angle B + \angle C}{2}$ = 57° 30′ - - 10·195813

: *t.* of half their diff. = 15° 29′ - - 9·442485

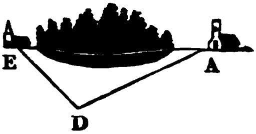
This, added to the half-sum = ∠ C, 72° 59′.

Then, by Case I.

As s. ∠C = 72° 59′, co-sec. - - 0·019442
: AB = 500 - - - - 2·698970
:: s.∠A = 65° - - - - 9·957276

: BC, the required dist. = 473·9 - 2·675688

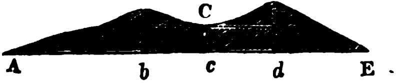
Examples for Practice.



1. Suppose the distance of two churches, E and A, separated by a wood, was required; and that, by mensuration, we find the distance from D to E = 2500, and from D to A 5200, and the ∠D = 104° 15′; what is the distance from the one church to the other?

Let this problem be worked by logarithms.

2. Suppose, of necessity, a person were to measure over the two points of hill, B and D, whose horizontal base, AE, was required; and that at A, the ∠ of acclivity to B was 12°; and length 760; the ∠ of declivity from B to C, 14°, and length 320; the acclivity from thence to D, 11°, and length 400; and that the ∠ of declivity from D to E was 16°, and length 520; what is the horizontal distance from A to E?



1st. *For the distance* A*b*.

As Rad. - - - - 10·0
: AB = 760 - - - - 2·880814
:: *co-s.* ∠ A = 12° - - - 9·990404
: A*b* = 743·4 - - - - 2·871218

2d. *For the distance a*C = *bc*.

As Rad. - - - - 10·
: BC = 320 - - - - 2·505150
:: *co-s.* ∠ of decliv. at B = 14° - - 9·986904
: *a*C = *bc* = 310·5 - - 2·492054

3d. *For the distance* C*e* = *cd*.

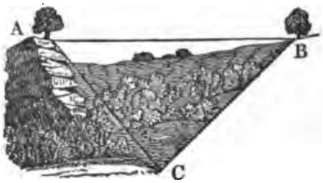
As Rad. - - - - 10·
: CD = 400 - - - - - 2·602060
:: *co-s.* ∠ of accliv. at C = 11° - 9·991947
: C*e* = *cd* = 392·7 - - - 2·594007

4th. *For the distance d*E.

As Rad. - - - - 10·
: DE = 520 - - - - 2·716003
:: *co-s.* of decliv. at D = 16° - - 9·982842
: *d*E = 499·9 - - - 2·698845

Then the sum of A*b* + *bc* + *cd* + *d*E = 1946·5 = AE.

Examples for Practice.



The horizontal breadth of a glen, from the tree at A to that at B, is required; having the descent from A to C given = 580, and the ascent from C to B = 650; and the ∠ of declivity at A = 40°, and of acclivity from C towards B = 35°.

Let this be worked by logarithms.

PART V.

THE AREAS OF PLANE FIGURES.

AREA, with Mathematicians, means the superficial content of any plane geometrical figure, estimated by the squares of some known lineal measure, as inches, links, chains, &c. which, in land-measuring, are converted into square perches, roods, acres, &c.

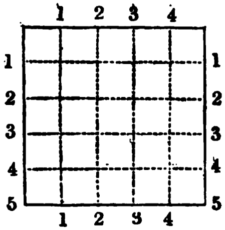
PROBLEM I.

To find the Area of a Square.

RULE.

Multiply the base AB, by the perpendicular AC; and the product will be the area.* — See next fig.

That is, AB × AC = the area.



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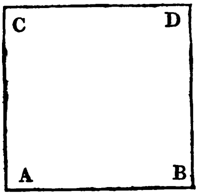
* This rule will appear manifest, by drawing the square, and dividing each side into the proposed number of the measuring unit, suppose *5* chains; and from the opposite points let lines be drawn, dividing the square into a number of small squares, each of which will contain one square chain. The number of these lesser squares will be equal to the base of the square multiplied by its perpendicular = *25* square chains.



The length of the square being equal to its height, the area will be found by squaring either of its sides; as $AB^2 =$ area.

The area of any rectangular quadrilateral may be found by logarithms, thus: — Add the logarithm of one side to the logarithm of the side at right angles thereto, and the sum will be the logarithm of the area; the natural number of this last logarithm is the area itself. In the square, twice the logarithm of a side will be the logarithm of the area. This arises from the logarithmical arithmetic, p. 10.

EXAMPLE.

1. Suppose the side of a square be 500 links, what is its area?

$$\begin{array}{r} 500 \\ 500 \\ \hline 250000 \\ \hline \end{array}$$

Examples for Practice.

2. What is the area, in links, of a square, the sides of which are each equal to 3 chains, 50 links?

Ansr. 122500 square links.

3. How many square chains are in the square, the side of which is 370 links?

Ansr. 13·69 sq. chains.

4. The area of a square is required, the side being 590. *Ansr.* 348100 sq. links.

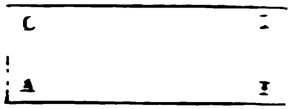
5. The area of another is required, whose sides are each 747. *Ansr.* 558009 sq. links.

To find the Area of a rectangular Parallelogram.

RULE.

Multiply the length by the breadth, and the product will be the area.

That is, AB × AC = area.



EXAMPLE.

1. Let AB be 600, and AC or BD 220; what is the area?

600 × 220 = 132000, for the answer.

Examples for Practice.

2. Find the area of the parallelogram, whose length is 575 links, and breadth 257. *Ansr.* 147775 sq. links.

3. Required the area of a parallelogram, the length of which is 1200, and breadth 740. *Ansr.* 888000.

4. What is the area of a parallelogram, whose length and breadth are 17·5 and 8·75 chains? *Ansr.* 153·125 chains.

5. Required, also, the area of a parallelogram, whose length is 950, and breadth one-half the length. *Ansr.* 451250.

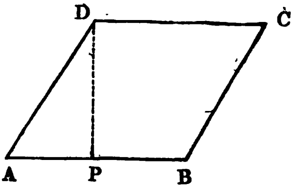
To find the Area of a Rhombus, or of a Rhomboides.

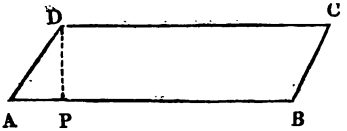
RULE I.

When the base and perpendicular height are given.

Multiply the base by the perpendicular height, and the product will be the area.

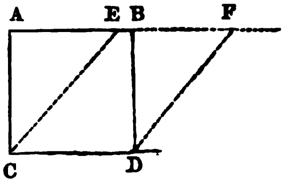
That is, AB × PD = the area.





The logarithms of these numbers may be added, to find the area; as in the last case.

Parallelograms standing on the same base, and between the same parallels, are equal. For, let ABDC in the following figure be a right-angled parallelogram, and CDFE an oblique one. Then, because of the equal and parallel sides AC and BD, CE and DF, the triangles ACE and BDF will be in every respect equal. Then, if to the common space, CEBD, the triangle AEC be added, it will complete the right-angled parallelogram ABDC, whose area is found by multiplying the base by the perpendicular. And, if to the common space, CEBD, the triangle DBF be added, it will make the rhombus CDFE equal to ABDC; whose area is also found by multiplying its base by its perpendicular height.



RULE II.

When two sides and the included ∠ are given.

By logarithms.

As radius (viz. *s.* of 90°)
Is to the sine of any ∠ of a rhombus or rhomboides*,
So is the product of the sides including the ∠
To the area.

Or by the following table of natural sines.

The product of the sides × nat. sine of the included ∠ = area.

---

* It has been noted before, that the angles of a regular rhombus are 60° and 120°; but those of a rhomboid may be greater or less.

TABLE OF NATURAL SINES AND COSINES.

0 Degree.

M.	Nat. Sines.	Nat. Co-s.	
0	0	·10000000	60
5	·0014544	·9999989	55
10	·0029089	·9999958	50
15	·0043633	·9999905	45
20	·0058177	·9999831	40
25	·0072721	·9999736	35
30	·0087265	·9999619	30
35	·0101809	·9999482	25
40	·0116353	·9999323	20
45	·0130896	·9999143	15
50	·0145439	·9998942	10
55	·0159982	·9998720	5
60	·0174524	·9998477	0
	Nat. Co-s.	Nat. Sines.	M.

89 Degrees.

1 Degree.

M.	Nat. Sines.	Nat. Co-s.	
0	·0174524	·9998477	60
5	·0189066	·9998213	55
10	·0203608	·9997927	50
15	·0218149	·9997620	45
20	·0232690	·9997292	40
25	·0247230	·9996943	35
30	·0261769	·9996573	30
35	·0276309	·9996182	25
40	·0290847	·9995770	20
45	·0305385	·9995336	15
50	·0319922	·9994881	10
55	·0334459	·9994405	5
60	·0348995	·9993908	0
	Nat. Co-s.	Nat. Sines.	M.

88 Degrees.

2 Degree.

M.	Nat. Sines.	Nat. Co-s.	
0	·0348995	·9993908	60
5	·0363530	·9993390	55
10	·0378065	·9992851	50
15	·0392598	·9992290	45
20	·0407131	·9991709	40
25	·0421663	·9991106	35
30	·0436194	·9990482	30
35	·0450724	·9989837	25
40	·0465253	·9989171	20
45	·0479781	·9988484	15
50	·0494308	·9987775	10
55	·0508835	·9987046	5
60	·0523360	·9986295	0
	Nat. Co-s.	Nat. Sines.	M.

87 Degrees.

3 Degrees.

M	Nat. Sines.	Nat. Co-s.	
0	·0523360	·9986295	60
5	·0537883	·9985524	55
10	·0552406	·9984731	50
15	·0566928	·9983917	45
20	·0581448	·9983081	40
25	·0595967	·9982225	35
30	·0610485	·9981348	30
35	·0625002	·9980449	25
40	·0639517	·9979530	20
45	·0654031	·9978589	15
50	·0668544	·9977627	10
55	·0683055	·9976644	5
60	0697565	·9975641	0
	Nat. Co-s.	Nat. Sines.	M.

86 Degrees.

NATURAL SINES, &c.

4 Degrees.

M.	Nat. Sines.	Nat. Co-s.	
0	·0697565	·9975640	60
5	·0712073	·9974615	55
10	·0726580	·9973569	50
15	.0741085	·9972502	45
20	·0755589	·9971413	40
25	·0770091	·9970303	35
30	·0784591	·9969173	30
35	·0799090	·9968022	25
40	·0813587	·9966849	20
45	·0828082	·9965655	15
50	·0842576	·9964440	10
55	·0857067	·9963204	5
60	·0871557	·9961947	0
	Nat. Co-s.	Nat. Sines.	M.

85 Degrees.

5 Degrees.

M.	Nat. Sines.	Nat. Co-s.	
0	·0871557	·9961947	60
5	·0886046	·9960669	55
10	·0900532	·9959369	50
15	·0915016	·9958049	45
20	·0929499	·9956708	40
25	·0943979	·9955345	35
30	·0958458	·9953962	30
35	·0972934	·9952557	25
40	·0987408	·9951132	20
45	·1001881	·9949685	15
50	·1016350	·9948217	10
55	·1030819	·9946728	5
60	·1045285	·9945219	0
	Nat. Co-s.	Nat. Sines.	M.

84 Degrees.

6 Degrees.

M.	Nat. Sines.	Nat. Co-s.	
0	·1045285	·9945219	60
5	·1059748	·9943688	55
10	·1074210	·9942136	50
15	·1088669	·9940563	45
20	·1103126	·9938969	40
25	·1117580	·9937355	35
30	·1132032	·9935718	30
35	·1146482	·9934062	25
40	·1160929	·9932383	20
45	·1175374	·9930685	15
50	·1189816	·9928964	10
55	·1204256	·9927224	5
60	·1218693	·9925462	0
	Nat. Co-s.	Nat. Sines.	M.

83 Degrees.

7 Degrees.

M.	Nat. Sines.	Nat. Co-s.	
0	·1218693	·9925462	60
5	·1233128	·9923678	55
10	·1247560	·9921874	50
15	·1261990	·9920049	45
20	·1276416	·9918204	40
25	·1290841	·9916337	35
30	·1305262	·9914449	30
35	·1319681	·9912539	25
40	·1334096	·9910609	20
45	·1348509	·9908659	15
50	·1362919	·9906687	10
55	·1377327	·9904694	5
60	·1391731	·9902680	0
	Nat. Co-s.	Nat. Sines.	M.

82 Degrees.

NATURAL SINES, &c.

8 Degrees.

M.	Nat. Sines.	Nat. Co-s.	
	·1391731	·9902680	60
5	·1406132	·9900646	55
10	·1420531	·9898590	50
15	·1434926	·9896514	45
20	·1449319	·9894416	40
25	·1463708	·9892298	35
30	·1478094	·9890158	30
35	·1492477	·9887998	25
40	·1506857	·9885817	20
45	·1521234	·9883615	15
50	·1535607	·9881392	10
55	·1549978	·9879148	5
60	·1564345	·9876883	0
	Nat. Co-s.	Nat. Sines.	M.

81 Degrees.

10 Degrees.

M.	Nat. Sines.	Nat. Co-s.	
0	·1736482	·9848077	60
5	·1750803	·9845542	55
10	·1765121	·9842985	50
15	·1779435	·9840407	45
20	·1793746	·9837808	40
25	·1808052	·9835189	35
30	·1822355	·9832549	30
35	·1836654	·9829888	25
40	·1850949	·9827206	20
45	·1865240	·9824504	15
50	·1879528	·9821781	10
55	·1893811	·9819036	5
60	·1908090	·9816271	0
	Nat. Co-s.	Nat. Sines.	M.

79 Degrees.

9 Degrees.

M.	Nat. Sines.	Nat. Co-s.	
0	·1564345	·9876883	60
5	·1578708	·9874598	55
10	·1593069	·9872291	50
15	·1607426	·9869964	45
20	·1621779	·9867615	40
25	·1636129	·9865246	35
30	·1650476	·9862856	30
35	·1664819	·9860445	25
40	·1679159	·9858013	20
45	·1693495	·9855561	15
50	·1707828	·9853087	10
55	·1722156	·9850592	5
60	·1736482	·9848077	0
	Nat. Co-s.	Nat. Sines.	M.

80 Degrees.

11 Degrees.

M.	Nat. Sines.	Nat. Co-s.	
0	·1908090	·9816271	60
5	·1922365	·9813486	55
10	·1936636	·9810680	50
15	·1950903	·9807853	45
20	·1965166	·9805005	40
25	·1979425	·9802136	35
30	·1993679	·9799247	30
35	·2007930	·9796337	25
40	·2022176	·9793406	20
45	·2036417	·9790455	15
50	·2050655	·9787483	10
55	·2064888	·9784490	5
60	·2079117	·9781476	0
	Nat. Co-s.	Nat. Sines.	M.

78 Degrees.

NATURAL SINES, &c.

12 Degrees.

M.	Nat. Sines.	Nat. Co-s.	
0	·2079117	·9781476	60
5	·2093341	·9778441	55
10	·2107561	·9775387	50
15	·2121777	·9772311	45
20	·2135988	·9769215	40
25	·2150194	·9766098	35
30	·2164396	·9762960	30
35	·2178593	·9759802	25
40	·2192786	·9756623	20
45	·2206974	·9753423	15
50	·2221158	·9750203	10
55	·2235337	·9746962	5
60	·2249511	·9743701	0
	Nat. Co-s.	Nat. Sines.	M.

77 Degrees.

13 Degrees.

M.	Nat. Sines.	Nat. Co-s.	
0	·2249511	·9743701	60
5	·2263680	·9740419	55
10	·2277844	·9737116	50
15	·2292004	·9733792	45
20	·2306159	.9730449	40
25	·2320309	.9727084	35
30	·2334454	.9723699	30
35	·2348594	·9720293	25
40	·2362729	·9716867	20
45	·2376859	·9713421	15
50	·2390984	·9709953	10
55	·2405104	·9706465	5
60	·2419219	·9702957	0
	Nat. Co-s.	Nat. Sines.	M.

76 Degrees.

14 Degrees.

M.	Nat. Sines.	Nat. Co-s.	
0	·2419219	·9702957	60
5	·2433329	·9699428	55
10	·2447433	·9695879	50
15	·2461533	·9692309	45
20	·2475627	·9688718	40
25	·2489716	·9685108	35
30	·2503800	·9681476	30
35	·2517879	·9677825	25
40	·2531952	·9674152	20
45	·2546019	·9670459	15
50	·2560082	·9666746	10
55	·2574139	·9663012	5
60	·2588190	·9659258	0
	Nat. Co-s.	Nat. Sines.	M.

75 Degrees.

15 Degrees.

M.	Nat. Sines.	Nat. Co-s.	
0	·2588190	·9659258	60
5	·2602237	·9655484	55
10	·2616277	·9651688	50
15	·2630312	·9647873	45
20	·2644342	·9644037	40
25	·2658365	·9640181	35
30	·2672384	·9636305	30
35	·2686396	·9632408	25
40	·2700403	·9628490	20
45	·2714404	·9624553	15
50	·2728400	·9620594	10
55	·2742390	·9616616	5
60	·2756374	·9612617	0
	Nat. Co-s.	Nat. Sines.	M.

74 Degrees.

NATURAL SINES, &c.

24 Degrees.

M.	Nat. Sines.	Nat. Co-s.	
0	·4067366	·9135454	60
5	·4080649	·9129529	55
10	·4093923	·9123584	50
15	·4107189	·9117620	45
20	·4120446	·9111637	40
25	·4133693	·9105635	35
30	·4146932	·9099613	30
35	·4160163	·9093571	25
40	·4173385	·9087511	20
45	·4186597	·9081432	15
50	·4199801	·9075333	10
55	·4212996	·9069215	5
60	·4226183	·9063078	0
	Nat. Co-s.	Nat. Sines.	M.

65 Degrees.

25 Degrees.

M.	Nat. Sines.	Nat. Co-s.	
0	·4226183	·9063078	60
5	·4239360	·9056921	55
10	·4252528	·9050746	50
15	·4265687	·9044551	45
20	·4278838	·9038338	40
25	·4291979	·9032105	35
30	·4305111	·9025853	30
35	·4318234	·9019582	25
40	·4331348	·9013292	20
45	·4344453	·9006982	15
50	·4357548	·9000654	10
55	·4370634	·8994307	5
60	·4383712	·8987940	0
	Nat. Co-s.	Nat. Sines.	M.

64 Degrees.

26 Degrees.

M.	Nat. Sines.	Nat. Co-s.	
0	·4383712	·8987940	60
5	·4396779	·8981555	55
10	·4409838	·8975151	50
15	·4422887	·8968727	45
20	·4435927	·8962285	40
25	·4448957	·8955824	35
30	·4461978	·8949343	30
35	·4474990	·8942844	25
40	·4487992	·8936326	20
45	·4500985	·8929789	15
50	·4513967	·8923233	10
55	·4526941	·8916659	5
60	·4539905	·8910065	0
	Nat. Co-s.	Nat. Sines.	M.

63 Degrees.

27 Degrees.

M.	Nat. Sines.	Nat. Co-s.	
0	·4539905	·8910065	60
5	·4552859	·8903453	55
10	·4565804	·8896821	50
15	·4578739	·8890171	45
20	·4591664	·8883502	40
25	·4604580	·8876815	35
30	·4617486	·8870108	30
35	·4630382	·8863383	25
40	·4643269	·8856639	20
45	·4656145	·8849876	15
50	·4669012	·8843095	10
55	·4681869	·8836294	5
60	·4694716	·8829476	0
	Nat. Co-s.	Nat. Sines.	M.

62 Degrees.

NATURAL SINES, &c.

28 Degrees.

M.	Nat. Sines.	Nat. Co-s.	
0	·4694716	·8829476	60
5	·4707553	·8822638	55
10	·4720380	·8815782	50
15	·4733197	·8808907	45
20	·4746004	·8802014	40
25	·4758801	·8795102	35
30	·4771588	·8788171	30
35	·4784364	·8781222	25
40	·4797131	·8774254	20
45	·4809888	·8767267	15
50	·4822634	·8760263	10
55	·4835370	·8753239	5
60	·4848096	·8746197	0
	Nat. Co-s.	Nat. Sines.	M.

61 Degrees.

29 Degrees.

M.	Nat. Sines.	Nat. Co-s.	
0	·4848096	·8746197	60
5	·4860812	·8739136	55
10	·4873517	·8732058	50
15	·4886212	·8724960	45
20	·4898897	·8717844	40
25	·4911571	·8710710	35
30	·4924236	·8703557	30
35	·4936889	·8696385	25
40	·4949533	·8689196	20
45	·4962165	·8681988	15
50	·4974787	·8674762	10
55	·4987399	·8667517	5
60	·5000000	·8660254	0
	Nat. Co-s.	Nat. Sines.	M.

60 Degrees.

30 Degrees.

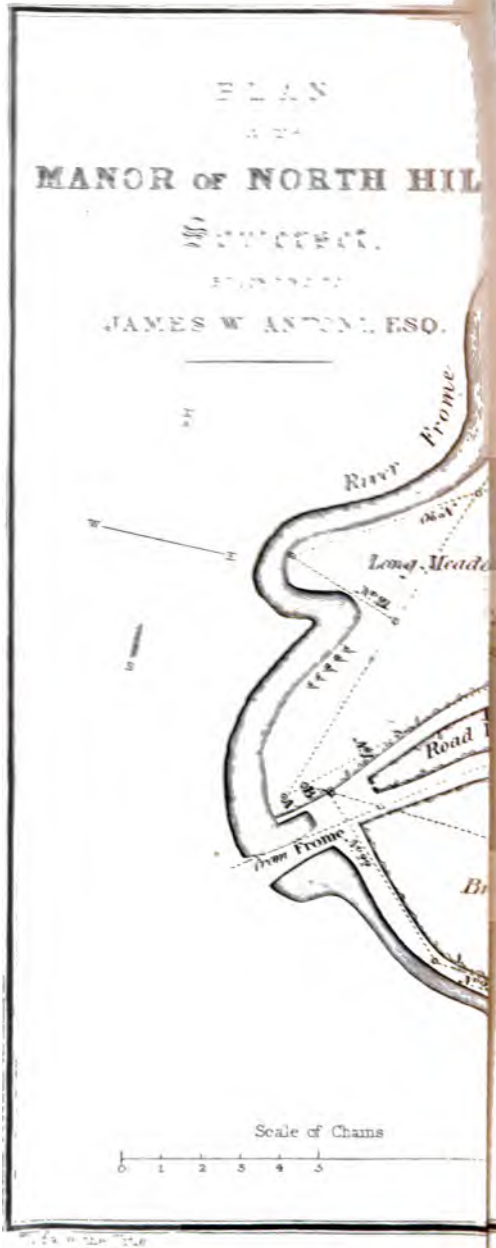
M.	Nat. Sines.	Nat. Co-s.	
0	·5000000	·8660254	60
5	·5012591	·8652972	55
10	·5025170	·8645673	50
15	·5037740	·8638355	45
20	·5050298	·8631019	40
25	·5062846	·8623664	35
30	·5075384	·8616292	30
35	·5087910	·8608900	25
40	·5100426	·8601491	20
45	·5112931	·8594064	15
50	·5125425	·8586619	10
55	·5137908	·8579155	5
60	·5150381	·8571673	0
	Nat. Co-s.	Nat. Sines.	M.

59 Degrees.

31 Degrees.

M.	Nat. Sines.	Nat. Co-s.	
0	·5150381	·8571673	60
5	·5162842	·8564173	55
10	·5175293	·8556655	50
15	·5187733	·8549118	45
20	·5200161	·8541564	40
25	·5212579	·8533992	35
30	·5224986	·8526402	30
35	·5237381	·8518793	25
40	·5249766	·8511167	20
45	·5262139	·8503522	15
50	·5274502	·8495860	10
55	·5286853	·8488179	5
60	·5299193	·8480481	0
	Nat. Co-s	Nat. Sines.	M.

58 Degrees.


PLAN
MANOR OF NORTH HIL
Somerset.
JAMES W. AN[illegible] ESQ.
River Frome
Long Mead
Road
from Frome
Br
Scale of Chains
0
1
2
3
4
5

RAL SINES, &c.

38 Degrees.

		M.	Nat. Sines.	Nat. Co-s.	
[illegible]	60	0	·6156615	·7880107	60
[illegible]	55	5	·6168069	·7871145	55
[illegible]	50	10	·6179511	·7862165	50
[illegible]	45	15	·6190940	·7853169	45
[illegible]	40	20	·6202355	·7844157	40
[illegible]	35	25	·6213757	·7835128	35
[illegible]	30	30	·6225146	·7826082	30
[illegible]	25	35	·6236522	·7817019	25
[illegible]	20	40	·6247885	·7807940	20
[illegible]	15	45	·6259235	·7798845	15
[illegible]	10	50	·6270571	·7789733	10
[illegible]	5	55	·6281894	·7780604	5
[illegible]	0	60	·6293204	·7771460	0
	M.		Nat. Co-s.	Nat. Sines.	M.

51 Degrees.

39 Degrees.

		M.	Nat. Sines.	Nat. Co-s.	
[illegible]	60	0	·6293204	·7771460	60
[illegible]	55	5	·6304500	·7762298	55
15	50	10	·6315784	·7753121	50
[illegible]	45	15	·6327053	·7743927	45
[illegible]	40	20	·6338309	·7734716	40
[illegible]	35	25	·6349553	·7725489	35
[illegible]	30	30	·6360782	·7716246	30
071	25	35	·6371998	·7706986	25
792	20	40	·6383201	·7697710	20
806	15	45	·6394390	·7688418	15
083	10	50	·6405566	·7679110	10
054	5	55	·6416727	·7669785	5
107	.0	60	·6427876	·7660444	0
Sines.	M.		Nat. Co-s.	Nat. Sines.	M.

50 Degrees.

NATURAL SINES, &c.

32 Degrees.

M.	Nat. Sines.	Nat. Co-s.	
0	·5299193	·8480481	60
5	·5311521	·8472764	55
10	·5323839	·8465030	50
15	·5336145	·8457278	45
20	·5348440	·8449508	40
25	·5360724	·8441720	35
30	·5372996	·8433914	30
35	·5385257	·8426091	25
40	·5397507	·8418249	20
45	·5409745	·8410390	15
50	·5421971	·8402513	10
55	·5434187	·8394618	5
60	·5446390	·8386706	0
	Nat. Co-s.	Nat. Sines.	M.

57 Degrees.

33 Degrees.

M.	Nat. Sines.	Nat. Co-s.	
0	·5446390	·8386706	60
5	·5458583	·8378775	55
10	·5470763	·8370827	50
15	·5482932	·8362861	45
20	·5495090	·8354878	40
25	·5507236	·8346877	35
30	·5519370	·8338858	30
35	·5531492	·8330821	25
40	·5543603	·8322768	20
45	·5555702	·8314696	15
50	·5567790	·8306607	10
55	·5579865	·8298500	5
60	·5591929	·8290376	0
	Nat. Co-s.	Nat. Sines.	M.

56 Degrees.

34 Degrees.

M.	Nat. Sines.	Nat. Co-s.	
0	·5591929	·8290376	60
5	·5603981	·8282234	55
10	·5616021	·8274074	50
15	·5628049	·8265897	45
20	·5640065	·8257703	40
25	·5652070	·8249491	35
30	·5664062	·8241262	30
35	·5676043	·8233015	25
40	·5688011	·8224751	20
45	·5699968	·8216469	15
50	·5711912	·8208170	10
55	·5723844	·8199854	5
60	·5735764	·8191521	0
	Nat. Co-s.	Nat. Sines.	M.

55 Degrees.

35 Degrees.

M.	Nat. Sines.	Nat. Co-s.	
0	·5735764	·8191521	60
5	·5747672	·8183169	55
10	·5759568	·8174801	50
15	·5771452	·8166415	45
20	·5783323	·8158013	40
25	·5795183	·8149593	35
30	·5807030	·8141155	30
35	·5818864	·8132701	25
40	·5830687	·8124229	20
45	·5842497	·8115740	15
50	·5854294	·8107234	10
55	·5866080	·8098710	5
60	·5877853	·8090170	0
	Nat. Co-s.	Nat. Sines.	M.

54 Degrees.

NATURAL SINES, &c.

36 Degrees.

M.	Nat. Sines.	Nat. Co-s.	
0	·5877853	·8090170	60
5	·5889613	·8081612	55
10	·5901361	·8073038	50
15	·5913096	·8064446	45
20	·5924819	·8055837	40
25	·5936530	·8047211	35
30	·5948228	·8038569	30
35	·5959913	·8029909	25
40	·5971586	·8021232	20
45	·5983246	·8012538	15
50	·5994893	·8003827	10
55	·6006528	·7995100	5
60	·6018150	·7986355	0
	Nat. Co-s.	Nat. Sines.	M.

53 Degrees.

38 Degrees.

M.	Nat. Sines.	Nat. Co-s.	
0	·6156615	·7880107	60
5	·6168069	·7871145	55
10	·6179511	·7862165	50
15	·6190940	·7853169	45
20	·6202355	·7844157	40
25	·6213757	·7835128	35
30	·6225146	·7826082	30
35	·6236522	·7817019	25
40	·6247885	·7807940	20
45	·6259235	·7798845	15
50	·6270571	·7789733	10
55	·6281894	·7780604	5
60	·6293204	·7771460	0
	Nat. Co-s.	Nat. Sines.	M.

51 Degrees.

37 Degrees.

M.	Nat. Sines.	Nat. Co-s.	
0	·6018150	·7986355	60
5	·6029760	·7977593	55
10	·6041356	·7968815	50
15	·6052940	·7960020	45
20	·6064511	·7951208	40
25	·6076069	·7942379	35
30	·6087614	·7933533	30
35	·6099147	·7924671	25
40	·6110666	·7915792	20
45	·6122173	·7906896	15
50	·6133666	·7897983	10
55	·6145147	·7889054	5
60	·6156615	·7880107	.0
	Nat. Co-s.	Nat. Sines.	M.

52 Degrees.

39 Degrees.

M.	Nat. Sines.	Nat. Co-s.	
0	·6293204	·7771460	60
5	·6304500	·7762298	55
10	·6315784	·7753121	50
15	·6327053	·7743927	45
20	·6338309	·7734716	40
25	·6349553	·7725489	35
30	·6360782	·7716246	30
35	·6371998	·7706986	25
40	·6383201	·7697710	20
45	·6394390	·7688418	15
50	·6405566	·7679110	10
55	·6416727	·7669785	5
60	·6427876	·7660444	0
	Nat. Co-s.	Nat. Sines.	M.

50 Degrees.

NATURAL SINES, &c.

40 Degrees.

M.	Nat. Sines.	Nat. Co-s.	
0	·6427876	·7660444	60
5	·6439011	·7651087	55
10	·6450132	·7641714	50
15	·6461240	·7632325	45
20	·6472334	·7622919	40
25	·6483414	·7613497	35
30	·6494480	·7604060	30
35	·6505533	·7594606	25
40	·6516572	·7585136	20
45	·6527598	·7575650	15
50	·6538609	·7566147	10
55	·6549606	·7556630	5
60	·6560590	·7547096	0
	Nat. Co-s.	Nat. Sines.	M.

49 Degrees.

41 Degrees.

M.	Nat. Sines.	Nat. Co-s.	
0	·6560590	·7547096	60
5	·6571560	·7537546	55
10	·6582516	·7527980	50
15	·6593458	·7518398	45
20	·6604386	·7508800	40
25	·6615300	·7499187	35
30	·6626201	·7489557	30
35	·6637087	·7479912	25
40	·6647959	·7470251	20
45	·6658817	·7460574	15
50	·6669661	·7450881	10
55	·6680491	·7441172	5
60	·6691306	·7431448	0
	Nat. Co-s.	Nat. Sines.	M.

48 Degrees.

42 Degrees.

M.	Nat. Sines.	Nat. Co-s.	
0	·6691306	·7431448	60
5	·6702107	·7421708	55
10	·6712895	·7411953	50
15	·6723668	·7402181	45
20	·6734427	·7392394	40
25	·6745172	·7382592	35
30	·6755902	·7372773	30
35	·6766618	·7362939	25
40	·6777320	·7353090	20
45	·6788007	·7343225	15
50	·6798681	·7333345	10
55	·6809339	·7323448	5
60	·6819984	·7313537	0
	Nat. Co-s.	Nat. Sines.	M.

47 Degrees.

43 Degrees.

M.	Nat. Sines.	Nat. Co-s.	
0	·6819984	·7313537	60
5	·6830613	·7303610	55
10	·6841229	·7293667	50
15	·6851830	·7283709	45
20	·6862416	·7273736	40
25	·6872988	·7263748	35
30	·6883545	·7253744	30
35	·6894089	·7243724	25
40	·6904617	·7233690	20
45	·6915131	·7223640	15
50	·6925630	·7213574	10
55	·6936114	·7203494	5
60	·6946584	·7193398	0
	Nat. Co-s.	Nat. Sines.	M.

46 Degrees.

NATURAL SINES, &c.

44 Degrees.			
M.	Nat. Sines.	Nat. Co-s.	
0	·6946584	·7193398	60
5	·6957039	·7183287	55
10	·6967479	·7173161	50
15	·6977905	·7163019	45
20	·6988315	·7152863	40
25	·6998711	·7142691	35
30	·7009093	·7132505	30
35	·7019459	·7122302	25
40	·7029810	·7112086	20
45	·7040147	·7101854	15
50	·7050469	·7091607	10
55	·7060776	·7081345	5
60	·7071068	·7071068	0
	Nat. Co-s.	Nat. Sines.	M.
45 Degrees.			

RULE III.

When the diagonals, and the angle made by their intersection, are given.

By logarithms.

As radius

Is to the *s.* of ∠ made by the diagonals of a parallelogram, one with another *,

So is half the product of the diagonals

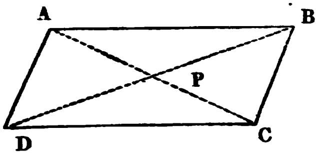
To the area.

---

* The diagonals of the *square* and *rhombus* intersect each other at right angles; therefore their natural sine is 1. The half product of the diagonals will therefore be the area: that is, $\frac{AC^2}{2}$ in the square, or $\frac{AC \times BD}{2}$ in the rhombus, = the area. (See the following fig.)

This rule is common to all quadrilaterals.

Or by the natural sines; $\frac{\text{AC} \times \text{BD} \times \text{nat. } s. \angle \text{P}}{2}$ = the area.



EXAMPLES.

1. What is the area of a rhomboides, the base of which is 360, its slope end 255 links, and the included ∠58°?

By Rule II.

Nat. sine of 58° = ·8480481
360 × 255 = 91800

6784384800
8480481
76324329

77850·8155800=77851, nearly.

Also, by Logarithms.

Radius - - - - 10·
: *s.* 58° - - - - 9·928420
:: 360 × 255 = 91800 - - 4·962843

: 77851 - - - - 4·891263

2. What is the area of a rhombus, whose base is 370, and perpendicular breadth 320·4?

By 1*st Rule.*

370 × 320·4 = 118548 for the answer.

3. What is the area of a rhombus, whose diagonals are 300 and 200?

By the 3d Rule.

$\frac{300 \times 200}{2} = 300 \times 100 = 30000$ for the answer.

4. What is the area of a parallelogram, whose diagonals are 300 and 250, and the angle made by their intersections, equal to 60°?

By Logarithms

As Radius	- - - -	10·
: *s.* 60°	- - - -	9·937531
:: $\frac{300 \times 250}{2} = 37500$	- -	4·574031
: Area, 32476	- - - -	4·511562

By Natural Sines.

Nat. *s.* of 60° = ·8660254

$\frac{300 \times 250}{2}$ - 37500

4330127000
60621778
25980762

32475·9525000 = 32476, nearly.

Examples for Practice.

5. What is the area of a rhombus, whose sides are 700 each, and the included ∠ 60°?

Ansr. 424352·446.

6. What is the superficial measure of a square, whose diagonal is 540? See note to Rule 3.

Ansr. 145800.

7. Required the area of a rhombus, whose base is 181, and perpendicular 156·8. *Ansr.* 28380·8

8. What is the area of a rhomboides, whose diagonals are 540 and 325, and the angle of intersection 61° 20′? *Ansr.* 76994·07.

9. What is the area of a rhomboid, whose sides are 378 and 278, and included ∠ 27° 30′?

Ansr. 48522·389.

PROBLEM II.

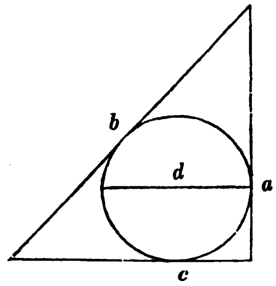
To find the Area of a Triangle.

RULE I.

When one side, and a perpendicular let fall thereon, are given.

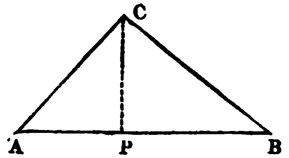
Multiply one of the sides (the base AB, in the following figure), by a perpendicular, CP, demitted from its opposite angle; and half the product will be the area. Or multiply one of those dimensions by half the other, and the product will be the area.

That is, $\frac{AB \times PC}{2}$ = the area.*



---

* If the triangle be right angled, the perpendicular may be thus found, $\sqrt{\overline{b+c}\times\overline{b-c}}$, or $\sqrt{b^2-c^2}=a$; where a stands for the perpendicular, c for the base, and b for the hypothenuse.



By logarithms: Add the logarithm of the base to the logarithm of the perpendicular, and the sum will be the logarithm of double the area.

RULE II.

When the three sides are given.

Add the sides together.

From half the sum of the sides subtract each side severally.

Multiply the half sum and the three remainders together, and the square-root of the last product will be the area.

That is,

$$\sqrt{\frac{a+b+c}{2}\times\frac{-a+b+c}{2}\times\frac{a-b+c}{2}\times\frac{a+b-c}{2}} =$$

the area, when a, b, and c, denote the three sides.

The theorem for the area becomes $\frac{a\,c}{2}$

The area of every triangle is equal to the rectangle under the half sum of the sides and the radius of the inscribed circle; which radius, in the right-angled triangle, may thus be found, viz. $\frac{a\,c}{a+b+c}$

In a right-angled triangle, the difference between the hypothenuse and sum of the remaining sides, is equal to the diameter of the inscribed circle; that is, $a+c-b=$ diameter d.

Or if s be the half-sum of the sides, then

$\sqrt{s \times \overline{s-a} \times \overline{s-b} \times \overline{s-c}}$ will be the area.*

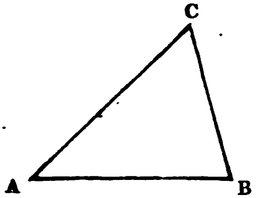
RULE III.

When an angle, and the two sides including the same, are given.

As radius
: the *s.* of any ∠ of a triangle,
: : half the product of the sides, including the angle,
: Area.—Or, by nat. sines.

That is, $\frac{AB \times AC \times \text{nat. } s. \angle A}{2} = \text{area.}$

Or, one side multiplied by half the other, and that product by the nat. sine of the angle, will be the area.



EXAMPLES.

1. What is the area of a triangle, the base, AB, of which is 1100, and perpendicular, CP, 600? — See the fig. to Rule I.

By Rule I.

1100 × 300 = 330000, the area.

---

[*] If all the sides are equal, the rule will become,

$\sqrt{\frac{3}{2}a \times \frac{1}{2}a \times \frac{1}{2}a \times \frac{1}{2}a} = \frac{1}{4}aa\sqrt{3}$: i. e. $\frac{aa \times 1{\cdot}732}{4}$ where a is a side.

2. Required the area of a triangle, the sides of which are 200, 300, and 400.

By Rule II.

```
   200
   300        450          450          450
   400        200          300          400
 2)900        250 1st diff. 150 2d diff.  50 3d diff.
   450 half sum.
   250 first diff.
  22500
  90
 112500
    150 sec. diff.
 5625
1125
16875000
       50 third diff.

. . . . .
843750000 (29047·3, &c. the answer.*
4
49)443
   441
5804)27500
     23216
58087)428400
      406609
580943)2179100
       1742829
        436271
```

---

* The same may be answered by finding a perpendicular demitted from an angle to its opposite side (by Case III. in Trigonometry), and multiplying half the said perpendicular by the side on which it falls.

3. What is the area of a triangle, one angle being 45°, and its including sides 212 and 250 links?

$$
\begin{array}{r}
212 \\
5 \\
\hline
1060 \\
50 \\
\hline
2)53000 \\
\hline
26500 \text{ half product of sides.} \\
\cdot7071068 \text{ nat. } s\ 45^\circ. \\
\hline
212000 \\
159000 \\
265000 \\
185500 \\
1855000 \\
\hline
18738\cdot3302000 \text{ Area.*}
\end{array}
$$

Or,

As Radius - - - 10

Is to *s*. ∠, 45° = - 9·849485

So is $\frac{212 \times 250}{2} = 26500$ - 4·423246

to 18738· = - - - 4·272731

Examples for Practice.

4. What is the area of a triangle, the base being 420, and perpendicular 375? *Ansr.* 78750.

5. Required the area of a right-angled triangle, the hypothenuse of which is 1120, and one of its angles 30°.† *Ansr.* 271585·44.

---

* This also may be solved, by finding a perpendicular demitted to the base from the opposite angle (by Case I. in Trigonometry), and multiplying the base by half that perpendicular.

† It has been heretofore remarked, that if one angle of a right-angled triangle be 30°, the perpendicular will be equal to half

6. The area of a triangle is required, the three sides being 570, 360, and 425. *Ansr.* 76411·97.

7. Given two sides of a triangle, 751 and 604, and their included angle 47° 28′, to find the area.

Ansr. 167126·8

8. If two sides are 37·5 and 49·25, and the included angle 37° 40′, what is the area?

Ansr. 564·281.

9. How many square links are in a triangle, whose three sides are 256, 490, and 502?

Ansr. 61270 sq. links.

PROBLEM III.

To find the Area of a Trapezium.

GENERAL RULE.

Divide the trapezium into two triangles; then the sum of the areas of the triangles, found by the foregoing rules, will be the area of the trapezium.

Particular Rules.

RULE I.

When a diagonal and the perpendiculars are given.

If the trapezium by divided by a diagonal, and perpendiculars be demitted thereon from the opposite angles, the half sum of the perpendiculars multiplied by the diagonal will give the area; see the following fig.

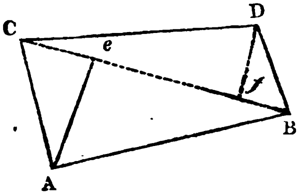
That is, $\frac{Ae + Df}{2} \times \text{CB} =$ the area.*

---

the hypothenuse. The base may then be found by Case I. in Trigonometry.

* The trapezium is evidently equal to the triangles

$$\text{ABC} + \text{BCD} = \frac{Ae \times \text{BC}}{2} + \frac{Df \times \text{BC}}{2} = \frac{Ae + Df}{2} \times \text{BC}$$



RULE II.

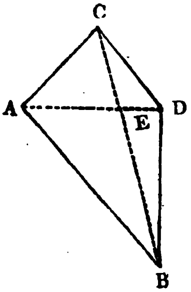
When two diagonals and angle of intersection are given.

If there be drawn two diagonals, crossing each other, the product of the diagonals multiplied by the natural sine of the angle of intersection, will be double the area.

That is, $\frac{\text{AD} \times \text{BC} \times \text{nat. } s. \angle \text{E}}{2} =$ area, as in

Rule 3, Rhomboides.

Or, as radius : $s. \angle$E :: $\frac{\text{AD} \times \text{BC}}{2}$: the area.



RULE III.

When the sides and intersecting angle are given.

Square each side of the trapezium,

Add together each pair of the opposite sides,

Subtract the less from the greater.

Multiply the difference into the tangent of the angle of intersection, and one-fourth of the product will be the area.

That is, $\frac{\overline{\overline{AB^2+CD^2}-\overline{AC^2+BD^2}}\times t\angle E}{4}=$ area.

RULE IV.

When it can be inscribed in a circle.[*]

When the trapezium can be inscribed in a circle, multiply any two adjacent sides together.

Then multiply the other two sides together.

Multiply the sum of these products by half the sine of the ∠ included, by either pair of sides so multipled, and the last product will be the area.

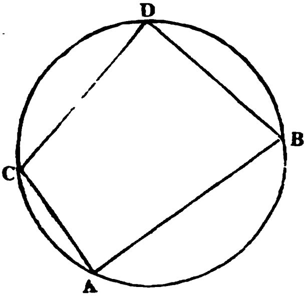
That is, $\frac{\overline{\overline{AC\times AB}+\overline{BD\times CD}}}{2}\times$ nat. $s.\angle A=$ area.

Or, as radius,

: $s.\angle A$ (or $s.\angle D$) :: $\frac{\overline{AB\times AC}+\overline{CD\times DB}}{2}$: area.

---

[*] This happens only when any two opposite angles of the trapezium are = 180°.



Another rule, when it can be thus inscribed, and the sides given.

From half the sum of the four sides subtract each side severally.

Multiply the four remainders continually together, and the square-root of the last product will be the answer.

EXAMPLES.

1. What is the area of a trapezium, the diagonal being 1220, and perpendiculars 510 and 320?

[See figure to Rule I.]

By Rule I.

$$\begin{array}{r} 510 \\ 320 \\ \hline 830 \\ 610 \\ \hline 8300 \\ 4980 \\ \hline 506300 \end{array} = \text{the area.}$$

2. Suppose the diagonals of a trapezium be 1000 and 850, and the ∠ of intersection 65° 25′; what is the area?

By Rule II.

850
1000

850000 product of diagonals.
·9093572 nat. s.

45467860
72748576

2)772953·62

386476·81 = the area.

3. Suppose the four sides of a trapezium, inscribed in a circle, are 600, 550, 750, and 400; what is the area?

By Rule IV.

$$\frac{600 + 550 + 750 + 400}{2} = 1150, \text{ half-sum.}$$

$1150 - 600 = 550$, $1150 - 550 = 600$,
$1150 - 750 = 400$, $1150 - 400 = 750$;
then, $550 \times 600 \times 400 \times 750 =$

$\dot{9}\dot{9}\dot{0}\dot{0}\dot{0}\dot{0}00000$(314642 = the area.
9

61) 90
61

624) 2900
2496

6286) 40400
37716

62924) 268400
251696

629282) 1670400
1258564

411836

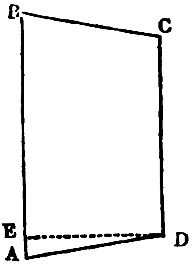
Examples for Practice.

[illegible]

[illegible]

direct distance between them, and the product will be the area.

$$\text{That is, } \frac{AB + CD}{2} \times ED = \text{area.}$$



EXAMPLE.

1. Required the area of a trapezoid, the parallel sides of which are 1080 and 890, and which are distant from each other 610 links.

$$1080 + 890 = 1970,$$

$$\text{Then, } \frac{1970 \times 610}{2} = 600850, \text{ the area.}$$

Examples for Practice.

2. What is the area of a trapezoid, the parallel sides being 920 and 579, and whose direct distance one from another, is 607 links?

Ansr. 454946·5 sq. links.

3. Required the area of a trapezoid, where the parallel sides are 340 and 580 links, and direct distance 486. *Ansr.* 223560 sq. links.

4. How many square links are contained in a trapezoid, the parallel sides of which are 140 and 270, and distance 320? *Ansr.* 65600 sq. links.

Examples for Practice.

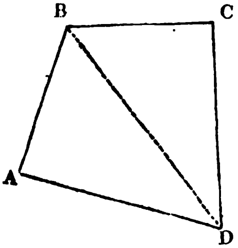
4. If the two diagonals of a trapezium be 920 and 870, and the ∠ of intersection 89° 10′, what is the area? *Ansr.* 400157·658.

5. The two perpendiculars falling on the diagonal of a trapezium are 650 and 810, and the diagonal itself 1140; what is the content? *Ansr.* 832200.

6. The four sides of a trapezium inscribed in a circle are 427, 517, 620, and 810 links; what is the area? *Ansr.* 329918·2 sq. links.

7. The diagonal of a trapezium is 1241, and sum of the perpendiculars 1527 links; what is the area? *Ansr.* 947503·5

8. What is the area of a trapezium, the side AB being = 650, BC = 660, CD = 880, and AD = 915; and whose angle at A = 89°, and angle C = 93°?



Ansr. 587331·716.

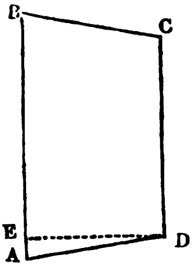
PROBLEM IV.

To find the Area of a Trapezoid.

RULE.

Multiply half the sum of the parallel sides by the direct distance between them, and the product will be the area.

That is, $\frac{AB + CD}{2} \times ED =$ area.



EXAMPLE.

1. Required the area of a trapezoid, the parallel sides of which are 1080 and 890, and which are distant from each other 610 links.

$$1080 + 890 = 1970,$$

Then, $\frac{1970 \times 610}{2} = 600850$, the area.

Examples for Practice.

2. What is the area of a trapezoid, the parallel sides being 920 and 579, and whose direct distance one from another, is 607 links?

Ansr. 454946·5 sq. links.

3. Required the area of a trapezoid, where the parallel sides are 340 and 580 links, and direct distance 486. *Ansr.* 223560 sq. links.

4. How many square links are contained in a trapezoid, the parallel sides of which are 140 and 270, and distance 320? *Ansr.* 65600 sq. links.

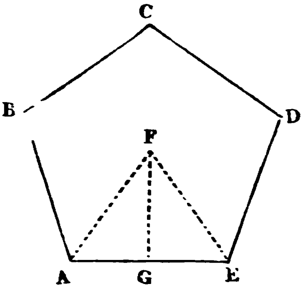
PROBLEM V.

To find the Area of a regular Polygon.

RULE I.

When a side and a perpendicular are given.

Multiply half the perimeter[*] of the polygon by



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[*] The perimeter is the sum total of the sides of any plane figure; and, in a regular polygon, is equal to one side multiplied by the number of sides

The central angles of regular polygons are found by dividing 360° by the number of sides; the quotient shows the number of degrees comprehended in each portion of the polygon. These degrees being taken in the compasses from a sector or plain scale, and successively applied to the circumference of a circle, comprehending the polygon, will determine the points of the external angles of the polygon. The sides may also be set off with proportional compasses: by setting the sliding centre to the number of sides, on the scale marked circles, taking the radius of the surrounding circle in the longer points, and applying the shorter ends to the circumference, the sides of the polygon will be readily pointed off.

the perpendicular demitted from its centre to one of the sides, and the product will be the area.

That is (in the pentagon),

$$\frac{AB + BC + CD + DE + EA}{2} \times FG = \text{the area.}$$

RULE II.

When a side only is given.

Square a side of any regular polygon, and multiply the said square by the multiplier opposite the name of the polygon in the following table; and the product will be the area.

That is, $AB^2 \times$ tabular number = area.

POLYGON TABLE.

No. of Sides.	Names.	Multiplier.
3	Equilateral Triangle	0·433013
4	Square - -	1·000000
5	Pentagon - -	1·720477
6	Hexagon - -	2·598076
7	Heptagon - -	3·633912
8	Octagon - -	4·828427
9	Nonagon - -	6·181824
10	Decagon - -	7·694209
11	Hendecagon -	9·365641
12	Dodecagon - -	11·196152

EXAMPLES.

1. What is the area of a hexagon, each side of which is 470, and perpendicular from the centre to a side 415?

By Rule I.

$$\begin{array}{rr} \frac{470 \times 6}{2} = & 1410 \\ & 415 \\ \hline & 7050 \\ & 1410 \\ & 5640 \\ \hline & 585150 \end{array}$$

585150 the area.

2. Required the area of a pentagon, each side being 25.

By Rule II.

$25^2 = 625.$

Tabular number $1 \cdot 720477 \times 625 = 1075 \cdot 298$, &c.

Examples for Practice.

3. What is the area of a hexagon, the side being 466? *Ansr.* 564187·79.

4. Required the area of a pentagon, the side being 250 links. *Ansr.* 107529·81 links.

5. What is the area of a trigon, the side of which is 769? *Ansr.* 256067.

6. Required the area of a hexagon, the side of which is 850 links, and perpendicular 732.
Ansr. 1866600 square links.

CIRCLE.

Hitherto I have been explaining the principles on which the areas of figures, bounded by straight lines, are founded; and these, it must be confessed, are the figures which mostly occur in the practice of land-measuring: but, as figures of other forms (particularly circular and elliptical) sometimes occur in practice, it is necessary to pay some attention to them also.

The areas of figures, which are bounded by curved lines, are best described by such straight lines as belong to them; knowing the proportion which such lines bear to the curves themselves: such as the diameter of a circle to its circumference; the axis of an ellipse to the circuit thereof, &c.

The proportion of the diameter of a circle to its circumference has engaged the attention of the most renowned mathematicians in the world, for ages past.

Archimedes, who lived 200 years before the Christian era, earnestly sought the true proportion of the diameter to the circumference; but, unable to determine it with accuracy, contented himself with an approximation thereto; namely, that of 7 to 22.

Other succeeding writers carried this approximation to a greater degree of accuracy; particularly *Metius*, who, about the end of the 16th century, laid it down as 113 to 355.

About the beginning of the 17th century, *Ludolph Van Collen* pursued the subject with vast industry, and with greater success than his predecessors; extending the ratio to thirty-six places of decimals; of which the first twenty places are as follows; viz. 1 to 3·14159265358979323846 +.*

In the last century this subject engaged the attention of a *Halley*, a *Sharpe*, and a *Machin*; by whose means the quadrature of the circle was extended to 127 places of decimals; and from thence they also determined the *area* of the circle, to be the square of

---

* Hence, if the diameter of a circle be multiplied by this number (or such portion of the decimal as may be deemed necessary), the product will be the circumference. And if the circumference be divided thereby, the quotient will be the diameter.

By Rule I.

$$\frac{470 \times 6}{2} = 1410$$

```
         1410
          415
         ----
         7050
        1410
       5640
       ------
       585150 the area.
```

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Tabular number 1·720477 × 625 = 1075·298, &c.

Examples for Practice.

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---

* Hence, if the diameter of a circle be multiplied by this number (or such portion of the decimal as may be deemed necessary), the product will be the circumference. And if the circumference be divided thereby, the quotient will be the diameter.

PROBLEM VI.

To find the Area of a Circle.

RULE I.

When the Diameter only is given.

Multiply the square of the diameter by ·7854 †, and the product will be the area.

That is, $D^2 \times m$; where D stands for the diameter, and m the multiplying decimal ·7854.

RULE II.

When the Diameter and Circumference are both given.

Multiply half the circumference by half the diameter, and the product will be the area.

Or, multiply the whole circumference by the whole diameter, and divide the product by 4; the quotient will be the area.

That is, $\frac{D}{2} \times \frac{C}{2} = \text{area}$; when D and C stand for diameter and circumference.

Or, $\frac{DC}{4} = \text{area}$.

---

* Whoever would know more of this curious subject, would do well to consult the deep and ingenious writings of Dr. Hutton, in his Mensuration, his Logarithms, or his Mathematical Dictionary, &c., and also the Baron Maseres's Scriptores Logarithmici, vol. iii.

† The decimal ·7854 is an approximation abstracted from the longer decimal just spoken of; and will be found sufficiently correct for our present purpose.

EXAMPLES.

1. What is the area of a circle, the diameter of which is 7?

By Rule I. $7 \times 7 = 49$.

$49 \times {\cdot}7854 = 38{\cdot}4846$, the area.

2. Required the area of a circle, whose diameter is 113 links, and circumference 355.

By Rule II. $113 \times 355 = 40115$.

Then, $40115 \div 4 = 10028{\cdot}75$ the area.

3. What is the area of a great circle of the earth; the equator, for instance?

The diameter of the earth, in miles, 7964, squared $= 63425296$, which multiplied by $\cdot7854 = 49814227$ square miles for the answer.

Examples for Practice.

4. Required the area of a circle, whose diameter is 327. *Ansr.* 83982.

5. Required the area of a circle, the diameter being 240 links. *Ansr.* 45239 links.

6. What is the area of a circle, the circumference of which is 1110 links? *Ansr.* 98046·3 links.

7. Required the area of a circle, the diameter being 350. *Ansr.* 96211·5.

8. What is the area of a circle, whose diameter is 400 links? *Ansr.* 125664 square links.

PROBLEM VII.

To find the Area of the Surface of a Sphere or Globe.

RULE.

Multiply the circumference by the diameter.

Or, the square of the diameter by 3·141592; and the product, in either case, will be the curve surface required.

That is, $D^2 \times 3{\cdot}141592$; where D stands for the diameter.

EXAMPLE.

What is the superficial area of the earth, supposing it truly spherical, and the diameter 7964 miles?

$7964^2 \times 3{\cdot}141592 = 199256402$ square miles, very nearly, for the answer.

PROBLEM VIII.

To find the Area of a Sector of a Circle.

RULE I.

When the Degrees of the Arc are given.

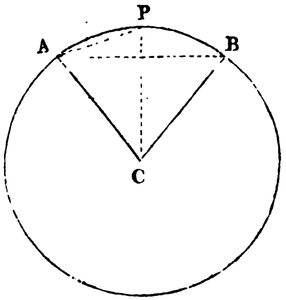
The area of a sector depends on the proportion which its arc bears to the whole circle; therefore, as 360° : area of the whole circle :: the number of degrees in the arc of the sector : to its area.

That is, put a for the degrees in the arc of the sector.

A for the area of the whole circle.

$$\text{Then, } \frac{A\,a}{360} = \text{sector.}$$

See the next fig.



RULE II.

When the Length of the Arc is given.

Multiply the radius of the circle by half the length of the arc of the sector, and the product will be the area.

That is, putting a for the arc, and r for radius $\frac{ar}{2}$ = the area.*

EXAMPLES.

1. What is the area of a sector, whose radius is 200, and whose arc contains 72°?

By Rule I.

$$\begin{array}{lr} \text{For the area of the whole circle} & 400 \\ & 400 \\ \hline & 160000 \\ & \cdot 7854 \\ \hline & 47124 \\ & 7854 \\ \hline & 125664\cdot 0000 = \text{area of the circle.} \end{array}$$

Then, As 360° : 125664 :: 72° : 25132·8 = area of the sector.

---

* The length of the arc of the sector may be found *nearly*, by the following rule; viz. from 8 times the chord of half the arc subtract the chord of the whole arc, divide the remainder by 3, and the quotient will be the length of the arc, viz. $\frac{8\ AP - AB}{3} =$ APB. Or, by another rule, $r \times d \times \cdot 0174524$ = length of the arc: — where r stands for radius, and d the number of degrees in the arc: — the decimal ·0174524 being the *nat. sine* of 1°.

2. What is the area of the same sector, by Rule II.?

The length of the arc is found, by the last note, thus: —

Rad. - 200
Deg. - 72

14400
·0174524

576
288
720
576
1008
144

2)251·31456

125·65728 = ½ length of the arc.
Radius - - 200

25131·456 = the area.

Examples for Practice.

3. Required the area of the sector of a circle, whose radius is 150, and the length of whose arc is 187? *Ansr.* 14025.

4. What is the area of a quadrant, each leg being 286? *Ansr.* 64242·578.

5. The area of a sector is required, whose radius is 260, and the degrees of the arc 74° 15′. *Ansr.* 43801·77

6. What is the area of a sector, whose radius is 240, and the length of the arc 195·5? *Ansr.* 23460.

7. What is the area of a sector, whose radius is 15, and length of arc 30? *Ansr.* 225.

8. Required the area of a sector, whose arc is 149° 15′, and radius 28. *Answ.* 1021·124.

PROBLEM IX.

To find the Area of a Segment of a Circle.

RULE I.

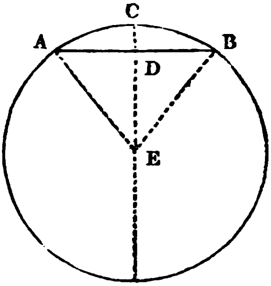
By seeking the Difference between the Sector and Segment.

By the last Problem,

Find the area of the sector CAEB.

Find the area of the triangle ABE.

Subtract the area of the triangle from that of the sector, and the remainder will be the area of the segment, ADBC.



RULE II.

By the help of a Table of Circular Segments.

Divide the versed sine, or height DC, by the diameter of the circle.

With the quotient thence arising, enter the following table of versed sines, and take out the segment area opposite thereto.

Multiply this segment area by the square of the diameter of the circle, and the product will be the area of the segment.

A TABLE

FOR THE AREAS OF CIRCULAR SEGMENTS.

Height of ver. sine.	Segment area.	Height of ver. sine.	Segment area.	Height of ver. sine.	Segment area.
·01	·00133	·18	·09613	·35	·24498
·02	·00375	·19	·10390	·36	·25455
·03	·00687	·20	·11182	·37	·26418
·04	·01054	·21	·11990	·38	·27386
·05	·01468	·22	·12811	·39	·28359
·06	·01924	·23	·13646	·40	·29337
·07	·02417	·24	·14494	·41	·30319
·08	·02944	·25	·15355	·42	·31304
·09	·03501	·26	·16226	·43	·32293
·10	·04088	·27	·17109	·44	·33284
·11	·04701	·28	·18002	·45	·34278
·12	·05339	·29	·18905	·46	·35274
·13	·06000	·30	·19817	·47	·36272
·14	·06683	·31	·20738	·48	·37270
·15	·07387	·32	·21667	·49	·38270
·16	·08111	·33	·22603	·50	·39270
·17	·08853	·34	·23547		

See a more particular Table of Circular Segments, in Hutton's Mensuration.

EXAMPLES.

1. If the versed sine be 5, and the diameter of the circle 25: what is the area of the segment?

By Rule II.

25)5·0

·2

The segment area of ·2 = ·11182

Square of 25 = 625

55910

22364

67092

69·8875 the area.

2. Required the area of a circular segment, whose chord AB = 12, and the radius AE, or CE = 10, as per last figure.

By Rule I.

As AE = 10	- -	1·0000000
: Radius =	- -	10·
:: AD = 6	- - -	0·7781513
: *s* ∠ AED* = 36° 52′	-	9·7781513
2		
73° 44′ = ∠ AEB.		

Now $\overline{2\ \mathrm{CE}}^2$ = 400 × ·7854 = 314·16 = the area of the whole circle.

Then, as 360° : 314·16 :: 73° 44′ = 73° 73

73·73

104720

219912

94248

219912

360 { 6)23164·0640

6,0) 3860·6773

64·34462 area of the sector.

---

* The above proportion to find the ∠ AED is deduced from Case I. in Trigonometry.

Again (by Trigonometry), $\sqrt{AE^2 - AD^2} = \sqrt{100 - 36} = \sqrt{64} = 8 = DE$.

Then, $\frac{AB \times DE}{2} = \frac{12 \times 8}{2} = 48$, the area of the triangle.

Hence, 64·344 (the area of the sector) — 48 (the area of the triangle = 16·344, the segment of the circle required.

Examples for Practice.

3. What is the area of the segment of a circle, whose chord is 16, and the radius of the circle 10?

Ansr. 44·735.

4. What is the area of a segment, whose height is 18, and diameter of the circle 48?

Ansr. 619·8.

5. Required the area of the segment, whose chord is 12, and diameter 36. *Ansr.* 8·28.

APPROXIMATE QUADRATURES, BY EQUIDISTANT ORDINATES.

The segment of a circle, or other curvilineal figure may be found, nearly correct, by means of equidistant ordinates.

RULE.

Let a right line, OP, be divided into any *even* number of equal parts[*], as O*a*, *ab*, *bc*, &c., and let ordinates be raised from these points, as O1, *a*2, *b*3, *c*4, &c.; put A for the sum of the extreme ordinates, O1, and P11; B for the sum of the second, fourth, sixth, and other *even* ordinates, *a*2, *c*4, *e*6, and *g*8; and C for the sum of all the rest.

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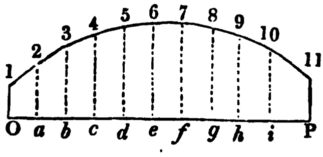
[*] The greater the number of even parts which the line is divided into, the nearer shall we approximate to the exact area.



Then, the common distance of the ordinates *Oa*, or *ab*, being multiplied into the sum arising from the addition of A, 4 times B, and twice C, and the whole sum divided by three will be the area nearly.

That is, $\frac{\text{A} + 4\text{B} + 2\text{C}}{3} \times \text{D} =$ the area; putting D for the common distance between the ordinates.

EXAMPLE.

Suppose that the extreme ordinates are 250, the even ordinates 1250, and the others 1100, and that the common distance between the ordinates is 100, what is the area?

$$\frac{250 + \overline{4 + 1250} + \overline{2 \times 1100}}{3} \times 100 =$$

$$\frac{250 + 5000 + \cdot 2200}{3} \times 100 = \frac{7450}{3} \times 100 =$$

248333·33 for the answer.

Example for Practice.

The length of the base of a field, curvilineal on one side, is 720, and seven ordinates are erected thereon of 200, 225, 230, 248, 260, 280, and 300 links; required the construction and the area.

Ansr. 179680 square links.

PROBLEM X.

To find the Area of a Circular Ring, or Space between two Concentric Circles.

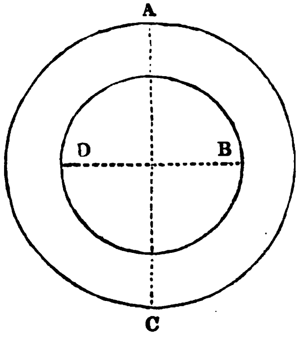
RULE.

Multiply the sum of the diameters by the difference of the diameters, and the product by ·7854; the last product will be the area.*

That is, $\overline{AC + DB} \times \overline{AC - BD} \times \cdot 7854 =$ the area. See the next fig.

EXAMPLE.

1. The diameters of two concentric circles are 100 and 60; what is the area of the ring included between their circumferences?



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* The area of the ring is evidently equal to the difference of the areas of the two circles AC and DB; therefore, if the diameters be denoted by D and d, and the multiplying decimal by m, it will be $D^2m - d^2m = m \times D + d \times D - d =$ area.

$$\overline{100 + 60} \times \overline{100 - 60} = 160 \times 40 =$$

$$6400 \times {·}7854$$

$$\begin{array}{r} 6400 \\ \hline 3141600 \\ 47124 \\ \hline 5026{·}5600 = \text{area.} \end{array}$$

Examples for Practice.

2. Suppose the diameters of two concentric circles are 320 and 240, what is the area of the circular ring? *Ansr.* 35185·92.

3. The diameter of the inner circle is 250, and the radius of the outer circle 250, what is the area of the ring included between those circles? *Ansr.* 147262·5.

PROBLEM XI.

To find the Area of an Ellipse.

RULE.

Multiply continually together the two axes and the number ·7854, and the last product will be the area of the ellipse.

That is, $t \times c \times m =$ area; putting t for the transverse, c for the conjugate, and m the multiplying decimal.*

---

* The ellipse is equal to a circle, whose diameter is a mean proportional between the two axes. And, consequently, its area is a mean proportional between the areas of two circles described on the two axes of the ellipse; that is, between the areas of the inscribed and circumscribing circles.

EXAMPLE.

1. What is the area of an ellipse, whose transverse diameter is 35, and conjugate 25?

35
25

175
70

875
·7854

3500
4375
7000
6125

687·225 the area.

Examples for Practice.

2. Required the area of an ellipse, whose diameters are 450 and 340. *Ansr.* 120166·2.

3. The diameters are 179 and 240 links, required the area. *Ansr.* 33740·78 links.

4. If the diameters be 570 and 450, what is the area? *Ansr.* 201455.

5. Suppose the diameters are 427 and 670 links, quere the area? *Ansr.* 224695 sq. links.

PART VI.

OF THE REDUCING AND EQUALIZING OF FIGURES.

A SQUARE, and its relative the triangle, are the groundwork of the areas of all other right-lined figures. And, as the reducing of the latter to the former is sometimes attended with convenience to the measurer, I will show how parallelograms and other right-lined figures may be reduced and equalized to the square and triangle.

PROBLEM I.

To reduce a Parallelogram to a Square of equal Area.

RULE.

Multiply the length AB of the parallelogram, by the breadth AH, and the square root of the product will be the side of the square equal to the parallelogram.*

Geometrically.

Let the two lines (length and breadth) of a parallelogram be represented by AB and BC; the latter being a continuation of AB.

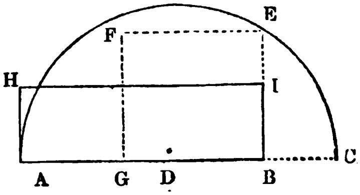
On the centre, D, of this compound line, describe a semicircle.

---

* The square root of the product of any two numbers is the geometrical mean proportional of those numbers.

On B erect a perpendicular, which will touch the semicircle at E.

BE will be the side of the square (BEFG) whose area is equal to that of the parallelogram ABIH.



EXAMPLE.

1. The two dimensions of a parallelogram are 400 and 200; what is the side of the square whose area shall be equal to that of the parallelogram?

$\sqrt{400 \times 200} = \sqrt{80000} = 282 \cdot 84$ = the side of the square.

Examples for Practice.

2. Required the side of a square, geometrically, equal to the foregoing parallelogram.

3. The sides of a parallelogram are 600 and 250; what is the side of a square equal thereto?

Ans. 387·29.

4. Construct a square equal to a parallelogram of 400 and 250 links, and give the area.

PROBLEM II.

To reduce a Rhombus or Rhomboid to a Square equal thereto.

RULE.

Multiply the base by the perpendicular height, and the square-root of the product is the side of a square equal to the rhombus. For, as in the last problem, the side of a square is a geometrical mean between the side of a rhombus and its perpendicular.

The same of a rhomboides.

EXAMPLE.

1. Suppose the side of a rhombus be 10, and the perpendicular 8·33, what is the side of a square which shall be equal?

$$\sqrt{10 \times 8{\cdot}33} = 9{\cdot}13 \text{ nearly.}$$

Examples for Practice.

2. The side 500, and perpendicular 445, what is the side of a square equal thereto? *Ansr.* 471·69.

3. The side 900, and perpendicular 785, what is the side of a square equal thereto? *Ansr.* 840·5.

PROBLEM III.

To reduce a Trapezium to a Square of equal Area.

RULE.

Multiply the diagonal by half the two perpendiculars, and the square root will be the area. For the side of the square is a geometrical mean between the diagonal and the half sum of the perpendiculars.

EXAMPLE.

1. If the diagonal be 60, and the half sum of the perpendiculars 23, what is the side of the equal square?

$\sqrt{60 \times 23} = \sqrt{1380} = 37{\cdot}14$, the side of the square.

Examples for Practice.

2. Suppose the two perpendiculars of a trapezium be 407, and the diagonal 540, what is the side of a square equal to the trapezium? *Ansr.* 331·4.

3. If the diagonal be 50, and perpendiculars 44, what is the side of a square equal thereto?

Ansr. 33·16.

PROBLEM IV.

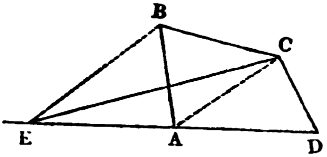
To reduce a Trapezium to a Triangle of equal Area.

Let ABCD be the trapezium.

Draw the diagonal AC, and extend the base line from A to E.

Draw BE parallel to AC.

From C draw CE, and the triangle ECD will be equal to the trapezium ABCD, as will appear by carefully scaling the two figures.



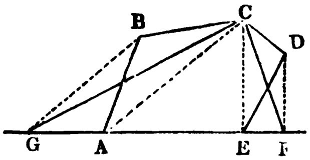
For Practice.

1. In the same trapezium, draw a diagonal from B to D; parallel thereto set off a line from C, which shall fall on the base D, continued, at a point which call *a*; draw B*a*, so will AB*a* be the triangle required.

PROBLEM V.

To reduce a Figure of Five Sides to a Triangle.

Let ABCDE be the given figure. Extend the base each way. Draw CA and CE, and BG and DF parallel thereto; so shall CFG be the required triangle.



PART VII.

LAND-MEASURING.

THE numeral and figural part of Land-measuring depends on the due application of the foregoing principles, to the various forms of inclosures and pieces of land.

Hitherto I have been considering geometrical figures singly; of all which forms, the measurer may sometimes meet with pieces of land; but he will more generally find, that the boundaries of fields constitute figures, *compounded* of *several* of the foregoing; as of a square and triangle, a trapezium and trapezoid, a parallelogram and segment of a circle, &c. These compound figures must be so measured as to reduce them to first principles or simple figures; and the aggregate of their contents will be the measure of the whole compound figure.

The land-measurer's most important instrument is the chain.* With this instrument, and its accompaniments, he is enabled to obtain all such dimensions of a piece of land, or of a whole estate, as may be necessary for ascertaining the quantity contained therein. Besides the chain, the measurer is provided with a small staff or rod, divided into 10 equal parts, answering to 10 links of the chain; wherewith he measures

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* The chain, already described in page 82., was the invention of the Rev. *Edmund Gunter*, who lived about 200 years ago, and is still called after his name. It is peculiarly well adapted to the measuring of land; as 10 square chains (that is, 10 chains in length, and 1 in breadth; 5 in length and 2 in breadth, &c.) are exactly an acre, conformable to the ordinance of Edward I. See also note, page 63.

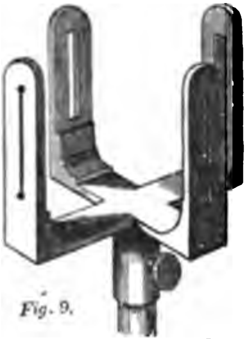


Fig. 9.

off-sets, or short distances between the chain-line and some neighbouring hedge or boundary of the field. To this off-set staff a cross (fig. 9.) is sometimes applied for setting off these at right angles with the chain-line. Arrows and picket-staves are also accompaniments to the chain.*

The various dimensions of a piece of land are taken in *lineal* measure, the foundation of which is a link; but the content thereof is from thence found in *square* measures. The relative proportions of lineal and square measures are shown in the two following tables.

I. TABLE OF LINEAL MEASURES.

Inches.	Link.	Foot	Yard.	† Stat. Perch	Chain	Furlong.	Mile
7·92 =	1						
12	1·5151 =	1					
36	4·5454	3 =	1				
198	25	16·5	5·5 =	1			
792	100	66	22	4 =	1		
7920	1000	660	220	40	10=	1	
63360	8000	5280	1760	320	80	8=	1

---

* See more hereof in the Supplement, where a practical illustration of the chain will be found.

† Perches are sometimes called Poles or Rods. In different

II. TABLE OF SQUARE MEASURES.

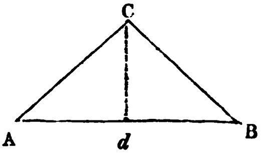
Square Inches. 62·7264 =	Sq. Link. 1							
144	2·2956 =	Sq. Foot. 1						
1296	20·6611	9 =	Sq. Yard. 1					
39204	625	272·25	30·25 =	Sq.Perch. 1				
627264	10000	4356	484	16=	Square Chain. 1			
1568160	25000	10890	1210	40	2·5 =	Square Rood. 1		
6272640	100000	43560	4840	160	10	4 =	Square Acre. 1	
4014489600	64000000	27878400	3097600	102400	6400	2560	640 =	Square Mile. 1

BY THE CHAIN.

Having said thus much in commendation of the chain, it may not be amiss to take some examples of measuring sundry pieces of land by this instrument and its accompaniments.

Triangular Fields.

Measuring along on the base line AB of a triangular piece of ground ABC, it was found (by applying the cross to the staff), that a perpendicular may be raised from *d* to a picket at the angular point C. The base was 7 chains and 50 links (or, in other words, 750 links), and the perpendicular 330 links; what quantity of land does this triangular field contain?



RULE.

By one of the rules for triangles, page 108., find the area in square links; divide the number of square links by 100,000 (which is most readily done by cutting off 5 figures of the product on the right hand), and what remains on the left will be the number of acres. Multiply the remainder (or right-hand figures) by 4, and cut off as before, and the figure on the left will be roods. Multiply by 40, and cut off again, and the left figures will be perches.

---

parts of the kingdom there were formerly (by custom) perches of various lengths: as, of 15 feet, or 5 yards, 7 yards, and 8 yards, &c. All of these are now obsolete, and the statute acre of 160 square perches, of $16\frac{1}{2}$ feet each, is general throughout England.

EXAMPLE *from the foregoing dimensions.*

750
330

22500
2250

2)247500

1·23750
4

·95000
40

acr. rds. per.
38·00000 = 1 0 38 for the answer.

EXAMPLE II.

What quantity of land is contained in a triangular field, the base and perpendicular of which are 3207 and 915 links?

3207
915

16035
3207
28863

*)2934405(

14·67202 square links.

2·68808

acr. rds. per.
27·52 Content 14 2 27

Examples for Practice.

3. What is the measure of a triangular piece of

---

* Land-measurers seldom take the trouble of writing down the divisor 2, or the multipliers 4 and 40.

land, the three sides of which are 720, 609, and 1044? * *Ansr.* 2a. 0r. 21p.

4. Suppose the sides of another triangle are 800, 908, and 1200 links, what is the number of acres, &c.? *Ansr.* 3a. 2r. 21p.

5. Required the content of an equilateral triangle, the side of which is 925 links.† *Ansr.* 3a. 2r. 32p.

6. In a right-angled triangular field, the base and perpendicular are 1570 and 1352, what is the content? *Ansr.* 10a. 2r. 18p.

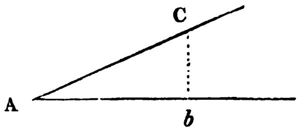
7. How many acres doth a triangle contain, the sides AB and AC being 960 and 872 links, and the included ∠ 44° 30′?‡ See the fig. next page. *Ansr.* 2a. 3r. 29p.

---

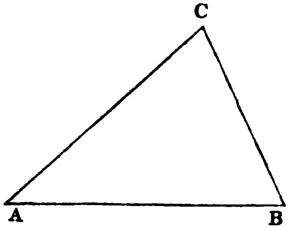
* By Rule II. p. 108, let a, b, and c, represent the three sides; then, $\sqrt{\frac{a+b+c}{2} \times \frac{-a+b+c}{2} \times \frac{a-b+c}{2} \times \frac{a+b-c}{2}}$ = the area.

† Let a represent the length of a side; then the rule is, $\frac{1}{4} aa \sqrt{3}$. See note to page 110.

‡ $\frac{AB \times AC \times \text{nat. } s. \angle A}{2}$ = area. See page 110.



Angles are most readily taken in the field, by a Theodolite, or other instrument constructed for taking angles; but a measurer may do the same, though not with equal facility, with his chain only. For example, let the ∠ A be required. Measure from A to b, 100, or any other certain number of links, where leave a mark: and also, from A to C the same, or any other number of links. Measure the distance from C to the mark at b, and thus the three sides of a triangle being known, by Prob. VIII. IX. or X. in Geo-



Quadrilateral Fields.

EXAMPLE.

1. Two adjoining sides of a field, in the form of a rectangular parallelogram, are 745 and 1740, what is the content?

acr. rds. perches.
745 × 1740 = 1296300 = 12 3 34.*

Examples for Practice.

2. What is the content of a parallelogram, the length and breadth of which are 1317 and 297 links?
Ansr. 3a. 3r. 25p.

3. What is the area of a rhombus, the diagonals of which are 1200 and 690? †
Ansr. 4a. 0r. 22p.

4. What quantity of land does a rhombus contain, the base of which is 1490, and perpendicular breadth 1280? *Ansr.* 19a. 0r. 11p.

5. How many acres are contained in a square, the diagonal of which is 1928? ‡
Ansr. 18a. 2r. 13p.

---

metry, construct the triangle A*b*C. Then, by the 2d method, Case III. Trigonometry, the angle A may be found.

* See page 91.

† The Rule will be found in a note, page 105.

‡ For this case, see note page 105.

6. How many acres are contained in a regular rhombus, the sides of which are, each, 900? [*] *Ansr.* 7a. 0r. 2p.

7. A field in the form of a trapezium has a diagonal of 900, and two perpendiculars 400 and 515, what is its content? *Ansr.* 4a. 0r. 18p.

8. What is the content of a trapezium, the diagonals of which are 1000 and 850 links, and the ∠ of their intersection 65° 25′? [†] *Ansr.* 3a. 3r. 18p.

9. Required the content of a trapezium, the diagonal of which is 780, and the perpendiculars 440 and 300? *Ansr.* 2a. 3r. 21p.

10. Required the area of a trapezoid, whose parallel sides are 1080 and 890, and whose direct distance one from the other is 610 links? *Ansr.* 6a. 0r. 1p.

11. How many acres, &c. does the trapezoid contain, the parallel sides of which are 800 and 700 links, and distance 565? *Ansr.* 4a. 0r. 38p.

Fields in the Form of a Regular Polygon.

1. What is the content of a hexagon, whose side is 740? [‡] *Ansr.* 14a. 0r. 36p.

2. Required the content of a pentagon, whose side is 745? *Ansr.* 9a. 2r. 7p.

3. What is the content of an octagon, the side of which is 525? *Ansr.* 13a. 1r. 9p.

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[*] The perpendicular height of a rhombus may be found by logarithms, knowing the angle to be 60°. Or, square half a side, and subtract it from the square of the whole side; the square root of the remainder will be the perpendicular height of the rhombus.

[†] See Rule II. p. 114.

[‡] See Rule II. p. 121.

Fields of the Circular Form, or such as are, in part, comprehended by Right-lines, and part by some Portion of a Circle.

EXAMPLE.

1. What is the content of a circular piece of land, whose radius is 560 links?

$\overline{560 \times 2}|^2 \times \cdot 7854 = 1120^2 \times \cdot 7854 = 1254400 \times \cdot 7854 =$ 9 acres, 3 roods, 16 perches.

Examples for Practice.

2. What is the content of a circular piece of land, the diameter of which is 367? *Ansr.* 1a. 0r. 9p.

3. Required the content of a circular field, the circumference of which is 1248? *Ansr.* 1a. 0r. 37p.

4. What is the content of a piece of land in the form of the sector of a circle, the radius of which is 450, and the arc of which contains 64°?

Ansr. 1a. 0r. 21p. nearly.

5. The content of a field in the form of a sector is required, the radius of which is 240, and length of the arc 392? *Ansr.* 0a. 1r. 35p.

6. The content of an elliptic field is required, the axes of which are 740 and 568?

Ansr. 3a. 1r. 8p.

Enough has been said of single figures; as the principles of areas in general have been fully given in a former part of this work. I come now to treat of such pieces of land as are compounded of several geometrical figures.

GENERAL RULE.

Divide an irregular piece of land into as many of the foregoing single figures as the nature of the case requires.

Measure such lines and angles as may be necessary, agreeably to one or other of the foregoing cases in Areas, Part V.*

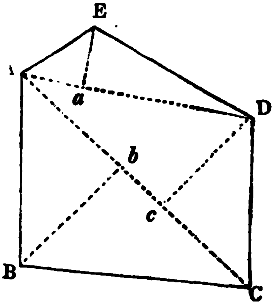
Find the square links of each single part, separately.

Add all such square links together, and reduce their sum to acres, roods, and perches, as taught in page 143.

A land-measurer may divide the same field different ways, and obtain the content thereof by each.

For instance, the field ABCDE may be divided into a trapezium ABCD, and a triangle ADE; as in Fig. I.

Fig. I.



Or, it may be divided into another trapezium, AEDC, and triangle ABC; as in Fig. II.

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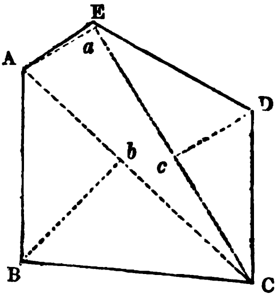
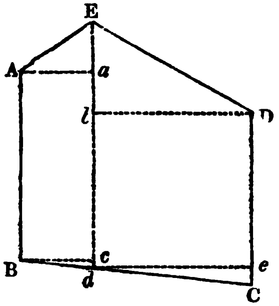
* Land-measurers, for the sake of expedition, are much in the practice of measuring in the field such lines only as will enable them to draw a geometrical plot thereof, by some or other of the Problems in Part I.; and by the same scale wherewith they plot

Fig. II.



It may also be divided into four triangles AE*a*, ED*b*, C*de*, and B*cd*, and two trapezoids D*bde*, and AB*ca*; as in Fig. III.

Fig. III.



Or, into three triangles AE*a*, EDC, BC*e*, and one trapezoid AB*ea*; as in Fig. IV.

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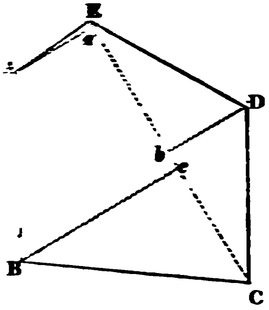
they draw other lines, which, by Part V. on Areas, will enable hem to obtain the content.

Fig. IV.



From hence it is observable, that when a land-measurer enters a field, he has to consider, in the first place, how his work may best be laid out; and also, what lines are most suitable to the purpose of accurate measurement, as well as the least trouble of walking forward and backward.*

The calculations for the quantity of land in the same field, by the four respective methods of taking the dimensions, will stand as follows:

Fig. I. p. 149.

$$\text{Trapezium ABCD} = \frac{\overline{Bb + Dc} \times AC}{2} =$$

$$\frac{\overline{460 + 440} \times 1020}{2} = 459000$$

$$\text{Triang. ADE} = \frac{AD \times Ea}{2} = \frac{780 \times 251}{2} = \underline{97890}$$

$$\begin{array}{r} \underline{5{\cdot}56890} \\ \underline{2{\cdot}2756} \\ 11{\cdot}024 \end{array}$$

acr. rds. perches.
5 2 11 for the answer.†

---

* See Note, p. 150.

† It is unnecessary to divide the square links of each small part by 2; as the double content may be carried on, and the aggregate, from thence arising, be divided by 2, once for all; as in the Examples II. III. and IV.

Fig. II.

Trapezium AEDC = $\overline{Dc + Aa} \times EC =$

$\overline{292 + 330} \times 1020 = 634440$

Triang. ABC = AC × B*b* = 1020 × 470 = 479400

2)1113840

5·5692

2·2768

acr.	rds.	per.		
5	2	11	the answer as before.	11·072

Fig. III.

Triangle AE*a* = A*a* × E*a* = 260 × 180 = 46800

Triangle ED*b* = D*b* × E*b* = 450 × 330 = 148500

Triangle C*de* = *de* × C*e* = 470 × 50 = 23500

Triangle B*cd* = B*c* × *cd* = 320 × 60 = 19200

Trapezoid D*bde* = $\overline{Db + de} \times bd =$

$\overline{450 + 470} \times 560 = 515200$

Trapezoid A*a*B*c* = $\overline{Aa + Bc} \times ac =$

$\overline{260 + 320} \times 622 = 360760$

)1113960

5·56980

2·2792

acr.	rds.	per.		
5	2	11	as before.	11·168

Fig. IV.

Triangle AE*a* = A*a* × E*a* = 330 × 60 = 19800

Triangle EDC = EC × D*b* = 1020 × 292 = 297840

Triangle BC*e* = B*e* × C*e* = 634 × 390 = 247260

Trapezoid AB*ea* = A*a* + B*e* × *ae* =

330 + 634 × 570 = 549480

)1114380

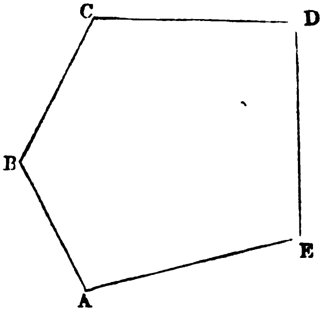
5·57190

2·28760

acrs.	rds.	per.		
5	2	11	as before.	11·5040

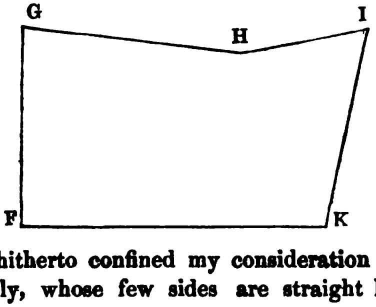
Examples for Practice.

1. The 5-sided field, ABCDE, may be divided into trapeziums, or trapezoids and triangles. Take such measurable lines, within the same, as may be thought most convenient, and from thence calculate its content; the plan being drawn with a scale of 6 chains to an inch.



Answer, 6 acres, 0 roods, 34 perches.

2. Required the content of the field, FGHIK, by a 6-chain scale.

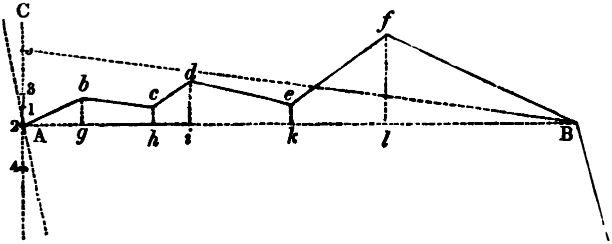


I have hitherto confined my consideration to such figures only, whose few sides are straight lines of considerable length; but, as the general boundaries of many pieces of land consist of short indentations, the student is now to be informed how to avoid the tediousness of computing the content of a multitude of small triangles and trapezoids; and how to find such equalizing lines as shall constitute a triangle, or other figure, of equal area with the sum of all such triangles and trapezoids combined.

In Part VI. he has been taught to equalize a parallelogram, or other four-sided figure, to a square; as well as a figure of four or five sides to a triangle.

Presuming he has been fully informed of the principles on which the subject is grounded, so far as that chapter goes, it will be easy for him to comprehend what follows, on a more extended plan.*

Suppose, then, that an irregular boundary of a field be of the form annexed, composed of two triangles and four trapezoids.



Draw the line AB, and at A erect a perpendicular AC. Lay a parallel ruler from A to *c*, the third point. Move the upper part of the rule to *b*, and note where

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(Note by the Editor.)

* The method here described is at present but little used by surveyors. (See p. 241. and the note in the next page.)—T. G. B.

it cuts the perpendicular, as at 1. From this point 1, lay the ruler to *d;* bring its lower part down to *c*, and note where it cuts the perpendicular at 2. From 2 lay the ruler to *e*, and move it upwards to *d*, and mark the perpendicular at 3. From thence lay the rule to *f*, and bring it down to *e*, and mark the perpendicular at 4. From this point lay the rule to B, and raise it to *f*, and mark the perpendicular at 5. From 5 draw the line 5B; then will the triangle, AB5, be equal in area to the aggregate of the two triangles and four trapezoids.*

Example on the foregoing Figure.

Suppose that, on some well-graduated scale, the base of the triangle, A*gb*, was found to be 185, and perpendicular, *gb*, 110; the base *gh*, of the adjoining trapezoid, 250, and sum of its perpendiculars 160; the base *hi*, of the next trapezoid, 120, and its perpendiculars 180; the base *ik* 325, and the perpendiculars of that trapezoid 190; the base *kl*, of the next trapezoid, 300, and the perpendiculars thereof 349; the base of the latter triangle, *l*B, 630, and its perpendicular, *lf*, 289; what is the content?

And what, also, is the content of the triangle AB5, whose base AB, by the same scale, is 1810, and perpendicular A5 is 238?

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* For expedition, land-measurers are in the practice of laying a bow, of whalebone or cane, strung with horse-hair, over the indentations, so that the hair shall include as much new space as it excludes of the original. The like may be done with a piece of glass or lantern horn having a fine edge; or by simply ruling a pencil line, and afterwards examining, by the eye, the equality of the included and excluded spaces. The last is the most usual method.

First. The double of the

Triangle	*Agb* =	185	×	110	=	20350
Trapezoid	*ghcb* =	250	×	160	=	40000
Do.	*hidc* =	120	×	180	=	21600
Do.	*iked* =	325	×	190	=	61750
Do.	*klfe* =	300	×	349	=	104700
Triangle	*lBf* =	630	×	289	=	182070
						)430470
						2·15235
	acr.	rds.	per.			·60940
Content	2	0	24			24·3760

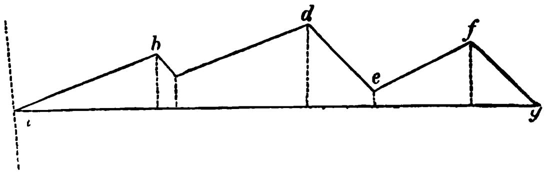
Secondly. The double of the Triangle

AB5 = AB × A5 = 1810 × 238 =				)430780
				2·1539
	acr.	rds.	per.	·6156
Content	2	0	24, as before.	24·624

From whence it appears that the content of the new triangle is the same as the aggregate contents of all the original triangles and trapezoids, to within the decimal of a perch.

Examples for Practice.

1. Set off a triangle, as in the last example, which may be of equal content with the triangles and trapezoids in the figure adjoining, and give the content.*



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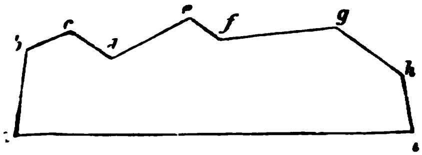
* It is presumed that the pupil is, by this time, so far master of calculations as not to stand in need of answers, which, at best, can only inform him that his computations are correct.

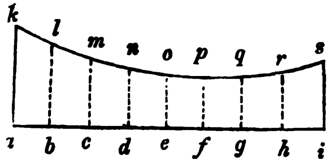
2. It is required to construct a trapezium which shall be equal to the figure *abcdefghi*.



In the course of practice, a land-measurer frequently meets with boundaries composed of curvilineal flexures, some convex, others concave; which the parallel ruler cannot reduce to straight lines. These, however, may be managed with considerable accuracy, by the method of *equidistant ordinates,* described in Part V. page 132.

EXAMPLE.

What is the area of the adjoining figure, in which the sum of the extreme ordinates is 520, that of the even ordinates 770, the sum of the others 560, and the common distance 136?



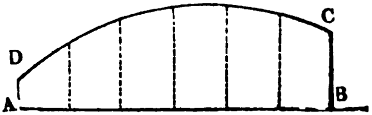
By the Rule, $\frac{520 + \overline{770 \times 4} + \overline{560 \times 2}}{3} \times 136$

$= \frac{520 + 3080 + 1120}{3} \times 136 = 4720 \times \frac{136}{3} =$

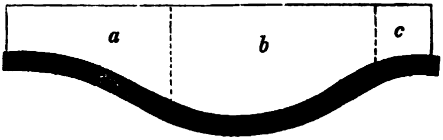
4720 × 45·33 = 2·13973, or 2 acres, 0 roods, 22 perches.

Examples for Practice.

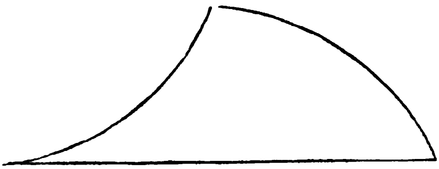
1. Divide the base, AB, of the adjoining figure, into 6 equal parts; erect the proper ordinates; and, from thence, give the content of the figure*: the scale being 6 chains to an inch.



2. The flexures of a rivulet, and the other boundaries of a meadow, being as in the figure, the content of each part, *a*, *b*, and *c*, is required, by the method of equidistant ordinates, by a scale of 6.



3. Required the content of the adjoining inclosure, one of whose boundaries is partly concave, and partly convex; scale as before.



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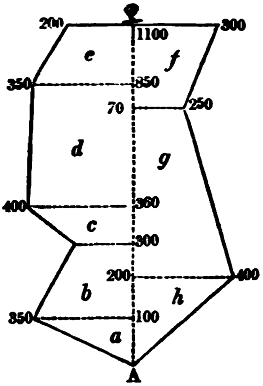
* A nice observer will perceive, that where the ordinates are far apart from one another, there will remain minute segments of the curve, not brought into the calculation; but this may be regulated by adding or subtracting a link or two, at discretion, to

In working with a chain and its off-set staff, a measurer does well in making a rough sketch in his field-book, large enough to admit his writing down the lengths of all the necessary lines, whether for planning, or for casting off the content without a plan.*

Where there is a general base line, with several perpendiculars raised thereon, it may be best to continue the reckoning throughout that line; and, by subtraction, find the intermediate distances between one perpendicular and another.

EXAMPLE.

From the adjoining sketch and dimensions, a true plan and the content of the following figure of a field are required.



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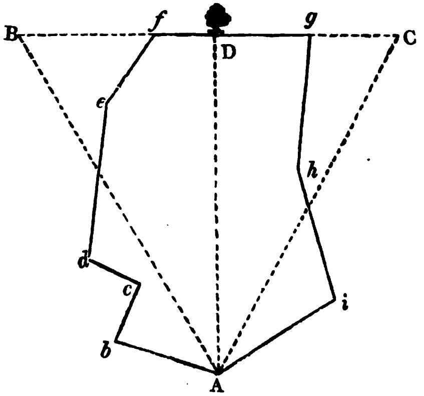
or from the length of the ordinates, according as the curve may be convex or concave; or by taking a greater number of ordinates.

* If he prefers keeping his account in the field-book columns,

Beginning at *A*, draw a line towards the tree at the upper end, and thereon prick off the distances, as in the foregoing sketch.

At the proper points erect the perpendiculars, according to their respective lengths; and the true figure will be as below. The whole content may be found, either by seeking the separate content of each triangle and trapezoid, from the dimensions given in the above sketch; or, with the parallel ruler, by equalizing the true plan, *Abcdefghi,* to a figure of a less number of sides; as, for instance, to the triangle ABC.*



Examples for Practice.

1. Required the true plan and content of the following sketch of a field, from the dimensions given.

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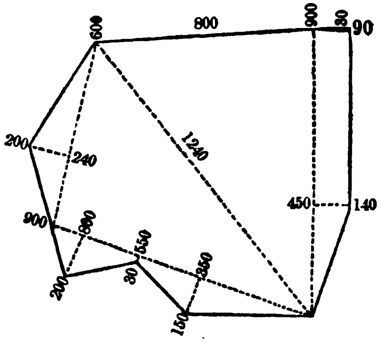
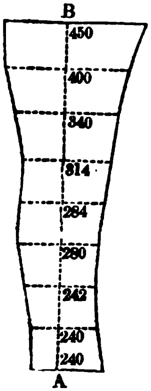
he must be careful so to minute down his dimensions, with reference letters, that one part may not be mistaken for another.

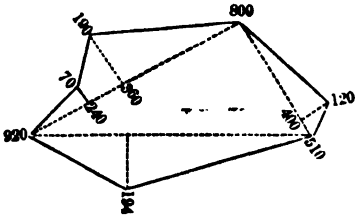
* See another method described at p. 241.



2. What is the content of a dole of land of the annexed form and dimensions, by the method of equidistant ordinates; the line AB being 1152, and the ordinates as in the figure?

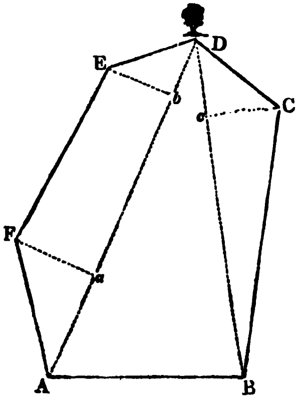


3. Find the content of a field, of the dimensions given in the following figure.

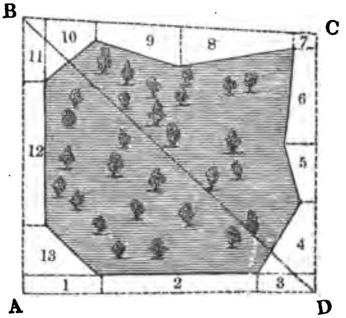


4. Supposing that a field of the form below be measured by a triangle, and perpendiculars of the following dimensions, viz.

AB - -	640
AD - -	1220
BD - -	1130
F*a* - -	260
E*b* - -	200
C*c* - -	190
A*a* - -	400
ab - -	620, what is the content?



Hitherto I have confined myself to such fields as may be measured by lines within; but a measurer sometimes meets with fields so circumstanced, by woody ground, meres of water, &c. as will not admit of the necessary internal lines being taken. Such pieces of land may, however, be measured by taking surrounding lines, making one or more right angles with each other, and raising perpendiculars from those lines to the angular points of the fields; by which a true plan may be constructed, and from thence the content found, either by equalizing the sides with the parallel ruler, as shown heretofore, or by deducting the contents of the small parts without, from the general content of the trapezium or surrounding figure.



EXAMPLE.

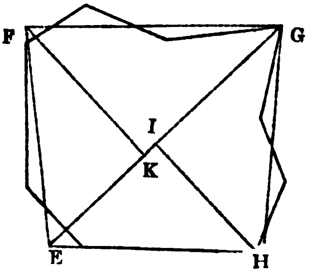
A plan of the piece of woody ground, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, and 13, being drawn by a 6-chain scale, the content thereof is required.

The ∠ A being (by the cross) made a right one, and the sides BA and AD being measured, the diagonal BD is readily found by construction; or else, by extracting the root of $\overline{AB^2 + AD^2}$.

This diagonal being a base to the triangle BCD, and the other sides BC and CD measured, that triangle also is readily constructed; and the trapezium completed.*

The off-sets being made on the lines of the trapezium, the figure of the piece of wood may be correctly drawn.

Its content may then be obtained, either by equalizing (see note page 156.) the lines of the wood, and thus reducing it to a trapezium; or by deducting the content of all the small trapezoids without, from the general content of the outer trapezium. The former method is the one generally used.



First, by reducing the figure to the trapezium, EFGH.

The lines being straightened, as before directed, the diagonal of this new trapezium, found by the scale, will be 1032, and the sum of the perpendiculars 1060.

Then, $\frac{EG \times \overline{FK + HI}}{2} = \frac{1032 \times 1060}{2} = 516 \times 1060 = 546960 =$ 5 acres, 1 rood, 35 perches, for the answer.

---

* This method is imperfect for want of some kind of proof on the construction of the figure ABCD. It will be much better to

Secondly, by finding the content of the surrounding trapezium*, and from thence deducting the aggregate of the outer trapezoids.

To find the triangle ABD, we have $\frac{\text{AB} \times \text{AD}}{2} =$ $\frac{910 \times 930}{2} = 455 \times 930 = 423150$.

The diagonal BD $= \sqrt{\text{AB}^2 + \text{AD}^2} =$ $\sqrt{910^2 + 930^2} = 1301$.

Then, the triangle BCD is found thus (by Rule II. Part V. p. 109.), $\sqrt{s \times \overline{s-a} \times \overline{s-b} \times \overline{s-c}}$, where a, b, c, stand for the sides, and s for the half sum of those sides.

$$\begin{array}{r} a = 1301 \\ b = 970 \\ c = 830 \\ \hline 2)3101 \\ \hline 1550 \end{array}$$

1550 $= s$, the half sum of the sides.

$s - a = 249$

$s - b = 580$

$s - c = 720$

Then, $\sqrt{1550 \times 249 \times 580 \times 720} =$ $\sqrt{161172720000} = 401463$.

The triangle ABD = 423150

The triangle BCD = 401463

824613 = surrounding trapezium ABCD.

---

measure one or two more of the angles, besides that at A, either with a theodolite, or with a pocket sextant. See p. 214. T. G. B.

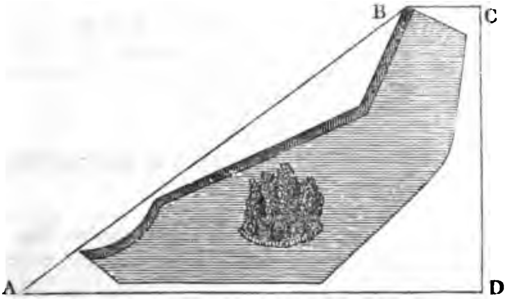
* See the former figure.

		Sum of Perpend.		Lengths.		
The trapezoid, No. 1	=	120	×	250	=	30000
2	=	110	×	520	=	57200
3	=	100	×	160	=	16000
4	=	210	×	250	=	52500
5	=	160	×	210	=	33600
6	=	130	×	320	=	41600
7	=	80	×	50	=	4000
8	=	150	×	380	=	57000
9	=	160	×	280	=	44800
10	=	250	×	164	=	41000
11	=	410	×	110	=	45100
12	=	150	×	460	=	69000
13	=	380	×	170	=	64600
						2)556400
						278200

the aggregate of all the small trapezoids; which taken from 824613 (the content of the surrounding trapezium), leaves 546413 = 5 acres, 1 rood, 34 perches, the content, as before, nearly.

Example for Practice.

The content of the adjoining mere or lake, drawn by a scale of six chains to an inch, is required; both



by straightening the lines with the parallel ruler, and by the surrounding trapezium, ABCD.

Thus far I have applied myself to single fields only; but have employed a sufficient variety for the due understanding of all others, I will now proceed to the measuring of two or more lying contiguously to each other.

It has been observed *, that, in general, land-measurers take only such lines in the field as will enable them to construct an exact geometrical plot thereof; and, by the scale of equal parts †, take such other lines on the plot as will divide it into triangles, trapeziums, or other single figures, such as the case may require; and from thence calculate the content. They sometimes may, however, find it convenient (when only a few fields are to be measured) to take the measuring lines in the field; by which, also, the plan may be made.

EXAMPLE.

From the dimensions given in the following sketch, the contents of the fields A and B are required?

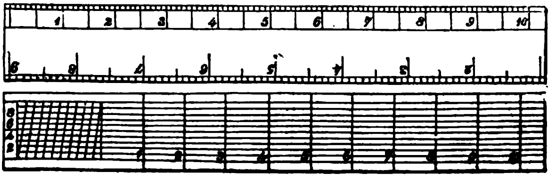
Field A, $\frac{\overline{620 + 290} \times 1060}{2} = 482300 = 4$ acres 3 roods, 11 perches, for the measure of that field.

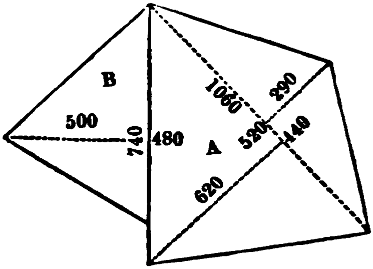
---

* Note, page 150.

† Scales of equal parts are of two kinds: the one having diagonal divisions, for accurately setting off single links, &c.; the other made with feather edges, and having those edges divided into a series of 10 links; and sometimes both scales are graduated on the same piece of brass or ivory. (Fig. 9.)

Fig. 9.

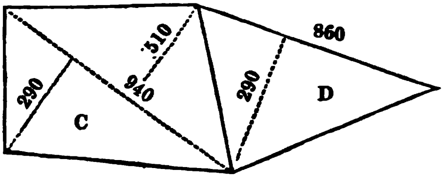




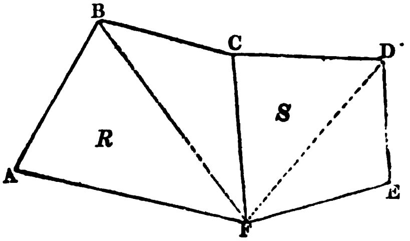
Field B, $\frac{740 \times 500}{2} = 185000 = 1$ acre, 3 roods, 16 perches, for the content of that field.

Examples for Practice.

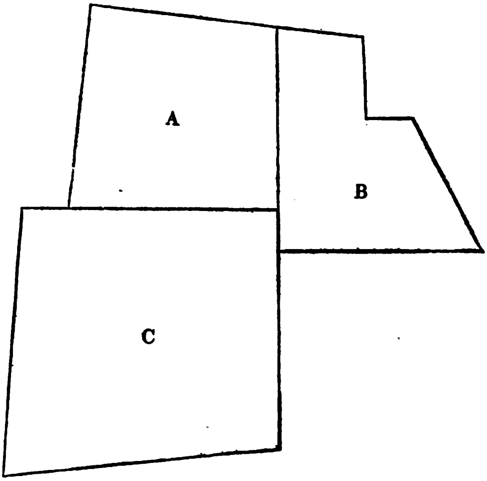
2. From the dimensions given, required the respective contents of the fields C and D?



3. From the following dimensions, a correct plan and content of the fields R and S are required?



Field R.			Field S.		
Side AB	=	520	Side CD	=	500
BC	=	420	DE	=	470
CF	=	580	EF	=	450
FA	=	830	Diagonal	=	730
Diag. FB	=	860			

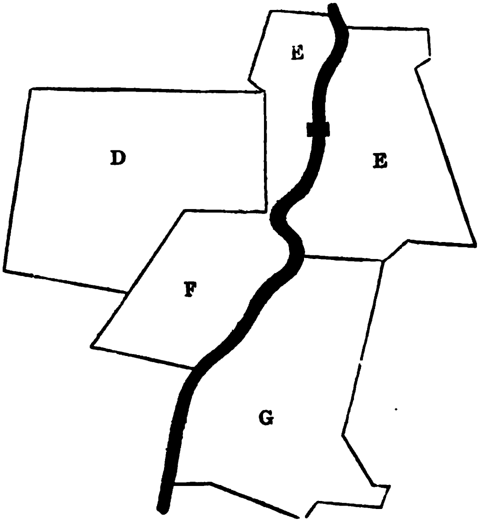


4. The fields A, B, and C, are plotted by a scale of six chains to an inch; quere the content of each field?

			Acr.	rds.	perches.
Answer,	A	=			
	B	=			
	C	=			
			13	0	15

5. The fields D, E, F, and G, are plotted by a six-

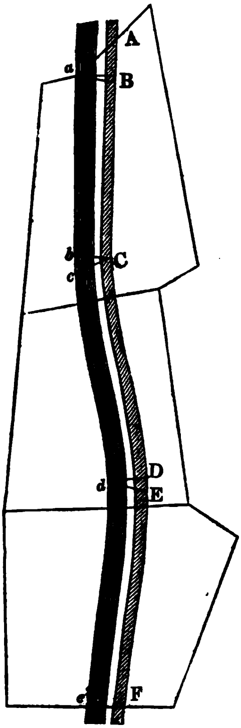
chain scale; what are the respective contents of the said fields, the total content being 12 acres, 3 roods, 11 perches?



Acr. rds. perches.

Answer,
D =
E =
F =
G =

6. The plan of a canal, and the quantity of land taken for the same through the three fields in the annexed plan, are required, from the dimensions given therewith; the towing-path and bank being of equal breadth with the top surface of the water. Scale 12 chains to an inch.



From	A to B	-	200 links.
	B to *a*	-	200
	B to C	-	1260
	C to *b*	-	180
	b to *c*	-	96
	C to *c*	-	180
	C to D	-	1460
	D to *d*	-	180

From D to E	10 links
E to *d*	186
E to F	1460
F to *e*	240

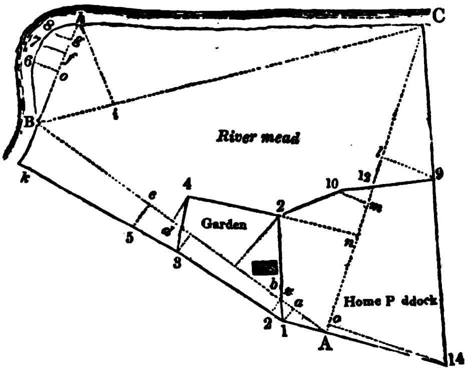
Some land-measurers have adopted the method of normal lines, or the erection of long perpendiculars on a single base line, for their mode of practice, in making plans of estates. The following (by triangles, &c.) is far preferable; being less subject to error, and more easy in operation.

Suppose the small estate fig. 10. was to be measured, by means of a general triangle, and necessary off-sets; the measurer beginning at A, and measuring towards B, from thence to C, and returning to A, making all necessary off-sets as he goes on: required the true plan, and the measure of each field?

From the following dimensions, to plan the estate.

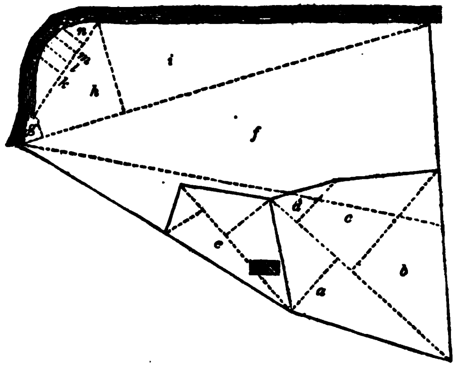
A*a*	=	250	*a*1	=	60
az	=	60	*z*2	=	60
A*b*	=	500	*b*2	=	320
A*c*	=	730	*c*3	=	70
A*d*	=	810	*d*4	=	140
A*e*	=	940	*e*5	=	100
AB	=	1500	B*k*	=	210
B*o*	=	250	*o*6	=	120
B*f*	=	320	*f*7	=	100
B*g*	=	390	*g*8	=	70
B*h*	=	480			
B*i*	=	300			
BC	=	1600	*ih*	=	350
C*l*	=	600	*l*9	=	250
C12	=	710			
C*m*	=	800	*m*10	=	160
C*n*	=	950	*n*2	=	370
C*o*	=	1380	*o*14	=	580
CA	=	1400			

Fig. 10.



The plan of the estate being obtained by these dimensions, other lines must now be drawn in each field, dividing it into such geometrical figures as will most readily give the content; as in fig. 11.

Fig. 11.



By dividing the fields as here directed, the content of each may be found as follows:

HOME PADDOCK.

			Double Areas.	Contents.
Triangle	*a*	$= 1040 \times 350 =$	36400	
Do.	*b*	$= 630 \times 565 =$	355950	
Trapezoid	*c*	$= 270 \times \overline{565 + 230} =$	214650	
Triangle	*d*	$= 140 \times 230 =$	32200	
			639200	
			3·1960 =	3 0 31

GARDEN.

Trapezium	*e*	$= 680 \times \overline{250 + 180} =$	)292400	
			1·462 =	1 1 33

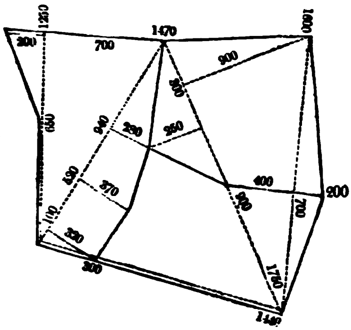
RIVER MEAD.

Triangle *f* (the lower irregular boundary being reduced by the Rule, p. 155.)		$= 1865 \times 860 =$	1603900	
Triangle	*g*	$= 160 \times 115 =$	18400	
Trapezoid	*h*	$= 320 \times \overline{115 + 440} =$	177600	
Triangle	*i*	$= 1275 \times 440 =$	561000	
Do.	*k*	$= 275 \times 110 =$	30250	
Trapezoid	*l*	$= 60 \times \overline{110 + 100} =$	12600	
Do.	*m*	$= 70 \times \overline{100 + 65} =$	11550	
Triangle	*n*	$= 70 \times 65 =$	4550	
			)2419850	
			12·09925 =	12 0 15
			Total	16 2 39

Examples for Practice.

2. Let the following rough sketch (fig. 12.) be regularly planned by the dimensions given therein; and the content of each field found.

Fig. 12.



3. By reason of close hedges, a measurer could extend his triangles only from field to field, in fig. 13. but from thence the content of each field is required.

4. Quære the number of acres in England, from the following dimensions, taken on Cary's Map of England and Wales* (see fig. 14.): —

A triangle, marked A in the sketch, having a base line from Longtown, near Solway Firth, to Sunderland; and its apex near Berwick, the sides of the out-boundaries being equalized; the base = 5250 chains, the eastern side = 5340 chains, and the other side = 5050 chains.

The trapezium B, between the base of the foregoing triangle, and a line from Stockton in Durham, to Barrowhead in Westmoreland, (the western side having a direction towards Dumfries,) whose diagonal from

---

* This Map is 7 feet 3 inches long, and 5 feet 9 inches broad, on a scale of 5 miles, or 400 chains, to an inch; and (from its accuracy and neatness of execution) may justly be esteemed that artist's *chef-d'œuvre.*

Fig. 18.

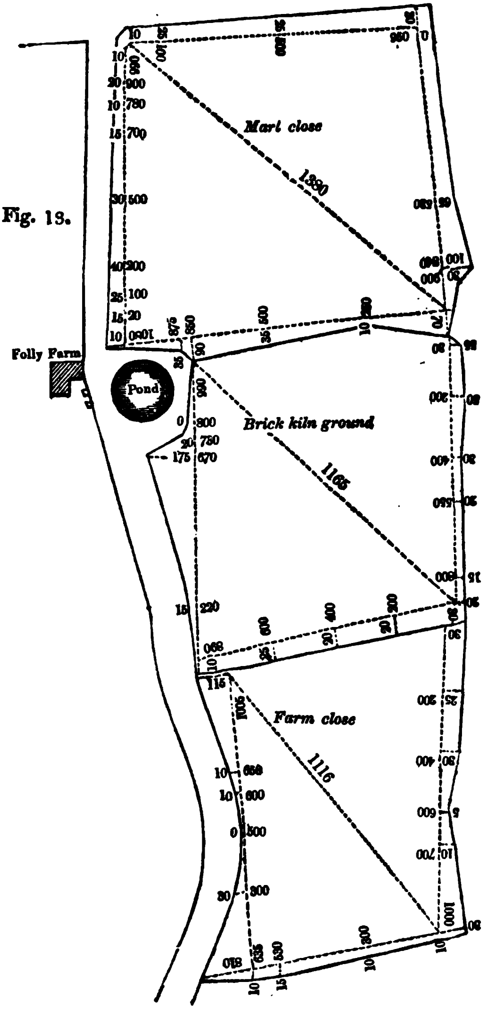
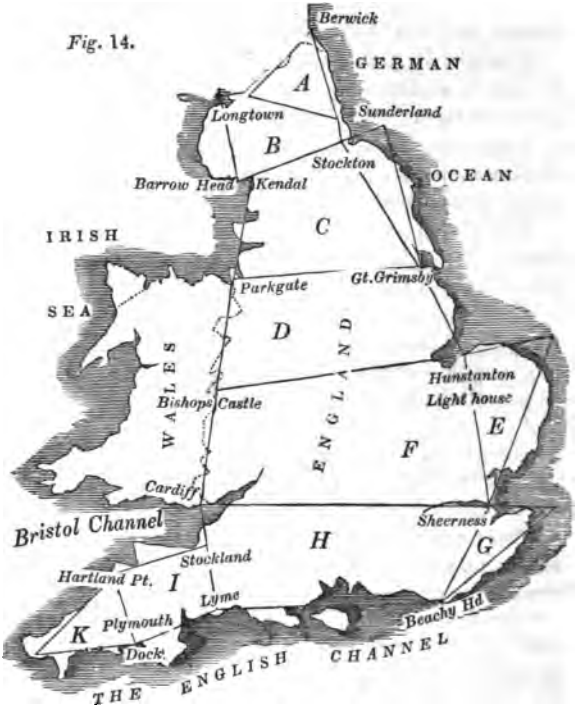

Marl close
1380
Brick kiln ground
1165
Farm close
1116
Folly Farm
Pond
10
20
10
15
30
40
25
15
10
990
900
780
700
500
300
100
20
1080
875
850
35
0
20
175
990
800
780
670
15
220
115
1005
10
650
10
600
0
500
30
300
1000
300
10
15
530
555
810

Fig. 14.



N. W. to S. E. = 7430 chains, and the sum of the perpendiculars = 5830 chains.

Trapezium C, whose southern line extends from Great Grimsby in Lincolnshire, to Park Gate in Cheshire, (its western side extending in the direction of Kendal,) having a diagonal from Park Gate to the N. E. of 10550 chains, and the sum of the perpendiculars 8485 chains.

Trapezium D, whose southern line extends from Bishop's Castle, Shropshire, to Hunstanton Lighthouse, Norfolk; having its diagonal to its S. E. angle 11920 chains, and sum of the perpendiculars 7040 chains.

Triangle E, comprehending Norfolk, Suffolk, &c., having its southern angle at Sheerness, and its sides = 9370, 6990, and 8400 chains.

Trapezium F, having its south line from Cardiff to Sheerness; its diagonal from Cardiff to the N. E. 14100 chains, and its perpendiculars 11540 chains.

Triangle G, east of Sheerness (off Ramsgate) and Beechy Head, having the S. E. side 5600, the N. W. side 4150, and the included $\angle = 33° 45'$.

The trapezium H, having its south side on the English Channel, between Beechy-head and Lyme-Regis, with a diagonal of 13170 chains, and the sum of its perpendiculars 7550 chains.

A trapezium, whose angles are at Lyme Regis, Plymouth Dock, Hartland Point, and Stockland-Bristol, has its diagonal 6050 chains, and perpendiculars 6110 chains.

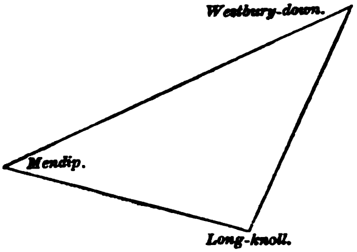
The Cornish triangle has its N. W. side = 6050 chains, and perpendicular 3000 chains.*

---

* Dr. Grew, in the Philosophical Transactions, states the number of acres in England at 46,080,000; Sir William Petty, in his Political Arithmetic, states them at about 28,000,000. — Dr. Hutton, in his Dictionary, under the word " Acre," says, Petty's statement is 39,000,000. Young's Political Arithmetic gives 34,000,000 acres of clear land, and 2,000,000 more for rivers, towns, cities, houses, &c. Depradt's statement is 46,000,000. In the Gentleman's Magazine, July, 1804, is a statement made of the number of acres in England, from Smith's county maps; by which the area is estimated at 32,134,400 acres. Dr. Halley and some others state it at 39,000,000 acres. But Zimmerman, who, perhaps, is much nearer the truth than the others, states, in his Political Survey, the number of acres at 34,831,680. Thus uncertain, at present, is the real quantity of land in England: however, we have much information to expect from the labours of Colonel Mudge of

Answer. By the method I have taken,
the content = acres 31348579

5. In Colonel Mudge's Trigonometrical Survey of the Kingdom, mentioned in the preceding note, he has one of his large triangles between a station on the forest of Mendip, in Somersetshire, another on Westbury-down, and a third on Long-knoll, in Wilts.



The ∠ observed on Mendip, between Westbury-down and Long-knoll, was 37° 48′. The distance from Mendip to Westbury-down was found to be 92954 feet; and that from Mendip to Long-knoll = 61963·5 feet; from whence the number of acres is required.

Let the feet in each of the two given sides be multiplied by 1·515 (the quantity of a link and its decimal contained in one foot) and 140825·3 links will be found to be one side, and 93874·7 links the other.

By Rule III. for finding the area of a triangle, page 109., the area hereof may be found: *i. e.*

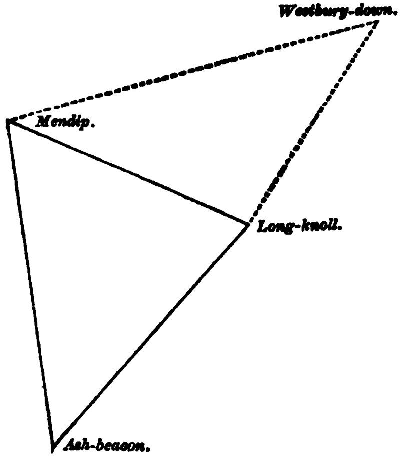
---

the Artillery, in his grand National Survey of the Kingdom, now going on.

$$\frac{140825\cdot3 \times 93874\cdot7}{2} \times \text{nat. } s. \text{ of the } \angle \qquad \cdot6129071$$

= 40512 acres, 3 roods, 33 perches.

6. Another ∠ adjoining the last in the same survey, has its three ∠ 's as follows:—



At Mendip, the ∠ between Long-knoll and Ash-beacon was found to be 58° 16′ 20″. At Long-knoll the ∠ between Mendip and Ash-beacon was observed to be = 71° 34′ 53″. And at Ash-beacon, the ∠ between Mendip and Long-knoll was found = 50° 8′ 45″.

From hence, the number of acres in this triangle is required.

By the Table of Square Measures, page 143., it has been shown, that 100,000 square links are an acre of land.* This is an acre conformable to a statute of 34 H. VIII., as well as to an ordinance of 35th Ewd. I., wherein it is declared, that 160 square perches of 16½ feet each, did make an acre. Formerly the perch varied, by custom, in different parts of the kingdom; consequently so did the acre.— In Devon and part of Somerset, 5 yards (or 15 feet only) were reckoned to a perch †: in Cornwall, 6 yards ‡: in Lancashire, 7 yards: and in Cheshire and Staffordshire, 8 yards were reckoned to the perch.§

In the Isle of Purbeck, and some other parts of Dorsetshire, 15 feet and an inch || were reckoned

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* The *hide* of land seems to have been the origin of land-measure, in the Confessor's time, as the *carucate* was by the Conqueror's new standard; either of which means the same as what we call a plough-land; that is, as much as can be tilled and managed by one plough, having meadow and pasture sufficient to maintain the cattle belonging thereto. The hide consisted of from 100 to 120 or more acres, according to the stiffness or lightness of the soil to be worked by the plough.

A *virgate* was one fourth of a hide; therefore, some were larger than others.

A *bovate*, or *ox-gang*, was one eighth of a hide; also unequal.

The acre was not equal, for the reason that the virgate and bovate are unequal. It contained 160 perches; but the perches were unequal.

† Hence the old Devonshire acre was smaller than the statute, in proportion as the square of 15 is to the square of 16½.

‡ The old Cornish acre was therefore larger than the statute, in proportion as the square of 18 is to the square of 16½; this was anciently called the Woodland perch.

§ The old Lancashire was to the statute, as the square of 21 is to the square of 16½; this was anciently used in measuring church land. And the old Cheshire, as the square of 24 is to the square of 16½; anciently called the Forest perch.

|| This additional inch seems to have been admitted to account

a perch. Abating this deviation, it seems as if the perch originally consisted of some certain number of yards.* The mean of the 5 and 6 yard perches appears to have been taken for the statute measure, *i. e.* $5\frac{1}{2}$ yards, or $16\frac{1}{2}$ feet.

In the common field lands of Wiltshire, and its neighbouring counties, there was a customary measure of a different nature, viz. 120 (instead of 160) statute perches to an acre; so that three roods were accounted an acre.

These ancient customary measures having all become obsolete, the modern land-surveyor will have but little occasion for them. In whatever part of the kingdom he may be employed to measure an estate, he will have to use a chain of four statute perches, or 66 feet in length, and to compute its superficial content in statute acres, containing 160 square perches each.

BY THE PLANE TABLE.

Land-measuring may, in some instances, be expedited by instruments which set off lines in their relative positions, and the angles of their inclination one to another. The Plane Table was formerly much employed for this purpose, and some persons still continue to use it; for which reason the description of this instrument, and some of the directions and examples explanatory of its use, given in former editions of this work, are here retained.

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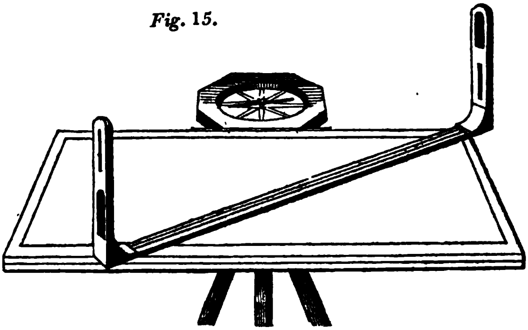
or the diameter of the end of the rod or pole used in measuring, by turning it over on its end.

* By Henry I. it was ordained, that a *yard* should be the length of his *arm*.

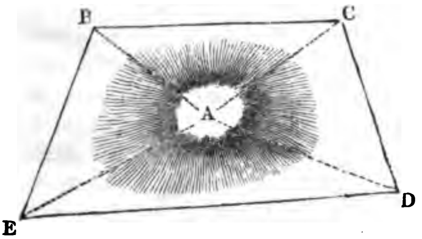
The *Plane,* or *Plain, Table* is composed of a smooth rectangular board, commonly of about 15 inches by 12; around which is a frame, that not only serves to keep the paper smooth on which the plan is to be drawn, but, being graduated into degrees, answering to a central point in the board, the angular bearing of any two lines, issuing from the station where the instrument is placed, may readily be ascertained; or the angle itself may be drawn on the paper. A magnetic needle in a compass-box is fixed to one side of the board, which serves to point out the bearing of any line to the magnetic meridian. There is, also, a brass index-rule, having sundry scales thereon, and also perpendicular sights at the end, used herewith. The whole is supported on a three-legged stand, &c., moveable on a brass ball and socket. (Fig. 15.)

Fig. 15.

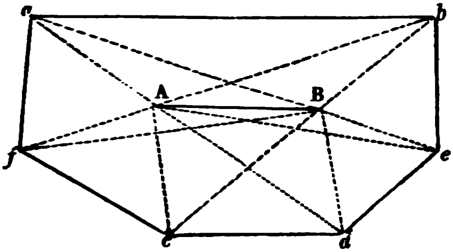


Having set up his pickets, and planted his table at some convenient part, for the beginning of his work, the measurer sets his table level; brings the north part of his compass (360°) directly under the needle, when at rest; and makes a point on some convenient part of his plane-table paper, to represent his station in the field. To this point he brings the fiducial edge of his brass rule, and directs it forwards until, through his sights, he perceives some picket to which he is about to measure. The rule lying in this direction, he draws an obscure line with his pencil, or a point of his compasses, somewhat longer, by his scale, than the supposed length of the line he has to measure; and proceeds to measure the line with his chain. He then must plot, or prick off, on his paper, the same length by the scale, setting it off from the station point, on the obscure line he had before drawn. Every line which is to be measured must be laid down on the plane table in a similar manner.

The plan of a field may be taken with the plane table different ways: *i. e.* by placing it at one of the corners; — by setting it at or about the middle; — by two stations within; — or by sundry stations taken near the boundaries, within or without the fences.



A measurer may place his instrument any where, as at A, about the middle of a field, BCDE (see the above fig.), and take his observations to each corner, measuring the lines, and pricking off their lengths, as before; he may draw the boundary lines, and find the content by either of the methods just mentioned.



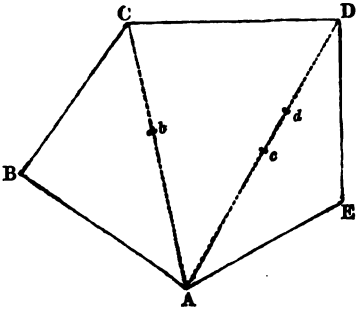
He may, otherwise, by two stations in a field, A and B (see fig. above), plot the same, by measuring the line from one station to the other, thus: at his first station A he must take his observations to each corner of the field, and draw obscure lines along the fiducial edge of his brass rule. He must then direct his sights to the other station B, draw an obscure line, and carefully measure and prick off the distance from one station to the other.

At this second station he must now set his plane table; and placing the scale on the obscure line of stations, he must unscrew the head, and turn it about until the sights are directed to his back station, and there screw it fast. From this second station he must again direct his sights to each corner of the field as before, and draw other obscure lines. The intersections of these obscure lines with the former (if the work be carefully performed) will give the points representing the corners of the field: from whence, by drawing boundary lines, the plot thereof may be obtained, and the content found by the scale.

With all the care a measurer can possibly employ in the use of this instrument, errors will creep in, and one especially which is irremediable: that is, by the expansion of the paper in a moist atmosphere, and the contraction thereof in a dry one. The mid-day sun, and the morning and evening dews, are at daily variance on paper exposed to their respective influences. The lines pricked off under these different circumstances will be somewhat erroneous, when compared with the natural tension of the paper within doors.

EXAMPLE.

A land-measurer having planted his plane table at A, one of the inner angles of the field ABCDE (see the fig. below), and from any assumed point on the paper directed his sights, and drawn lines to B, C, D, and E; and, by measuring these lines on the ground, found them to be as follows, viz. AB = 665, AC 885, AD 1030, and AE 580; he may make a correct plan of the field; and from thence, by drawing other lines on the plan, as heretofore directed, he may calculate the content thereof.



Suppose that, by drawing perpendiculars on the plan B*b* = 424, C*d* = 595, and E*c* = 285, there will be

$$\frac{\text{AC} \times \text{B}b}{2} = \frac{885^{*} \times 424}{2} = \quad - \quad - \quad 1{\cdot}87620$$

= the triangle ABC.

$$\text{and } \frac{\text{AD} \times \overline{cd \div \text{E}c}}{2} = \frac{1030 \times \overline{565 + 285}}{2} = 4{\cdot}53200$$

$$\begin{array}{r} \hline 6{\cdot}40820 \\ \hline 1{\cdot}6328 \\ \hline 25{\cdot} \end{array}$$

giving, for the whole content, 6 acres, 1 rood, 25 perches.

If the other sides of the same field, viz. BC, CD, and DE, had been measured, either on the ground or on the plot, the content may be found by Rule II. for the triangle, page 109.

EXAMPLE.

Let the three sides of the triangle ABC be 665, 575, and 885; those of the triangle ACD be 885, 690, and 1030; and of the triangle ADE be 1030, 600, and 580; from thence the content is required.

For the Triangle ABC.

$$\begin{array}{rrrr} 665 & 1062{\cdot}5 & 1062{\cdot}5 & 1062{\cdot}5 \\ 575 & 665 & 575 & 885 \\ 885 & \overline{397{\cdot}5} & \overline{487{\cdot}5} & \overline{177{\cdot}5} \\ \hline 2)2125 & & & \\ \hline 1062{\cdot}5 & & & \end{array}$$

$\sqrt{1062{\cdot}5 \times 397{\cdot}5 \times 487{\cdot}5 \times 177{\cdot}5} = 191169 =$ $\triangle$ ABC.

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[*] The side AC = 885, as in page 187.

For the Triangle ACD.

$$\begin{array}{rrrr}
885 & & & \\
690 & & & \\
1030 & & & \\
\hline
2)2605 & & & \\
\hline
1302{\cdot}5 & 1302{\cdot}5 & 1302{\cdot}5 & 1302{\cdot}5 \\
 & 885 & 690 & 1030 \\
\cline{2-4}
 & 417{\cdot}5 & 612{\cdot}5 & 272{\cdot}5
\end{array}$$

$\sqrt{1302{\cdot}5 \times 417{\cdot}5 \times 612{\cdot}5 \times 272{\cdot}5} = 301268 = \triangle$ ACD

For the Triangle ADE.

$$\begin{array}{rrrr}
1030 & & & \\
600 & & & \\
580 & & & \\
\hline
2)2210 & & & \\
\hline
1105 & 1105 & 1105 & 1105 \\
 & 1030 & 600 & 580 \\
\cline{2-4}
 & 75 & 505 & 525
\end{array}$$

$\sqrt{1105 \times 75 \times 505 \times 525} = 148230 = \triangle$ ADE.

$$\begin{array}{rcr}
\triangle \text{ ABC} & = & 1{\cdot}91169 \\
\triangle \text{ ACD} & = & 3{\cdot}01268 \\
\triangle \text{ ADE} & = & 1{\cdot}48230 \\
\cline{3-3}
 & & 6{\cdot}40667 \\
\cline{3-3}
 & & 1{\cdot}62668 \\
\cline{3-3}
 & & 25{\cdot}06720
\end{array}$$

Total content, 6 acres, 1 rood, 25 perches, as by the former method.

Having given the angles which the lines bear to each other, taken at the station A, the content of the field may be found by Rule III. for triangles, page 110, viz.

$\angle$ BAC = 40° 30′
$\angle$ CAD = 41 22
$\angle$ DAE = 29 47; see the former figure.

For the Triangle ABC.

$$\frac{\text{AB} \times \text{AC}}{2} \times \text{nat. } s. \text{ of } \angle \text{BAC.}$$

$$\frac{665 \times 885 \times (\cdot 649448 = \text{nat. } s.\ 40^\circ\ 30')}{2} = 191108$$

For the Triangle ACD.

$$\frac{1030 \times 885 \times (\cdot 6608753 = \text{nat. } s.\ 41^\circ\ 22')}{2} = 301210$$

For the Triangle ADE.

$$\frac{1030 \times 580 \times (\cdot 4967215 = \text{nat. } s.\ 29^\circ\ 47')}{2} = 148370$$

$$\begin{array}{llcr}
\text{Triangle} & \text{ABC} & = & 1\cdot 91108 \\
\text{Triangle} & \text{ACD} & = & 3\cdot 01210 \\
\text{Triangle} & \text{ADE} & = & 1\cdot 48370 \\
\hline
 & & & 6\cdot 40688 \\
\hline
 & & & 1\cdot 62752 \\
\hline
 & & & 25\cdot 1008
\end{array}$$

Total content, 6 acres, 1 rood, 25 perches, as by the two foregoing rules.

PART VIII.

(BY THE EDITOR.)

LAND-MEASURING BY THE THEODOLITE.

Of the various instruments employed for the purpose of angular measurement, the Theodolite is the one most frequently used in land-surveying. It has been fitted up in a great variety of forms, but its essential parts are nearly the same in all of them; namely, a compass-box containing a magnetic needle and circle of 360°, and a telescope attached to two graduated circles or circular arcs, placed at right angles to each other, and capable of pointing it in any required direction. This instrument is incomparably superior to the plane table, which it has now almost entirely superseded; being very conveniently adapted for measuring both horizontal and vertical angles, and determining the bearings of lines, or their inclinations to the magnetic meridian.

Fig. 15. represents one form of a small theodolite suited for the ordinary purposes of surveying. A is a horizontal circle of brass, four inches in diameter, divided on its outer edge, or *limb*, into 360 degrees, each of which is subdivided by a nonius into minutes. On the upper concentric plate B are placed the compass-box C, containing a magnetic needle and circle of 360°, and the vertical arch supporting the telescope, to which a spirit-level is attached: a small cross level is also screwed to the upper horizontal plate at *b*. In the focus of the telescope are two very fine hairs or wires crossing each other at right angles; their inter-

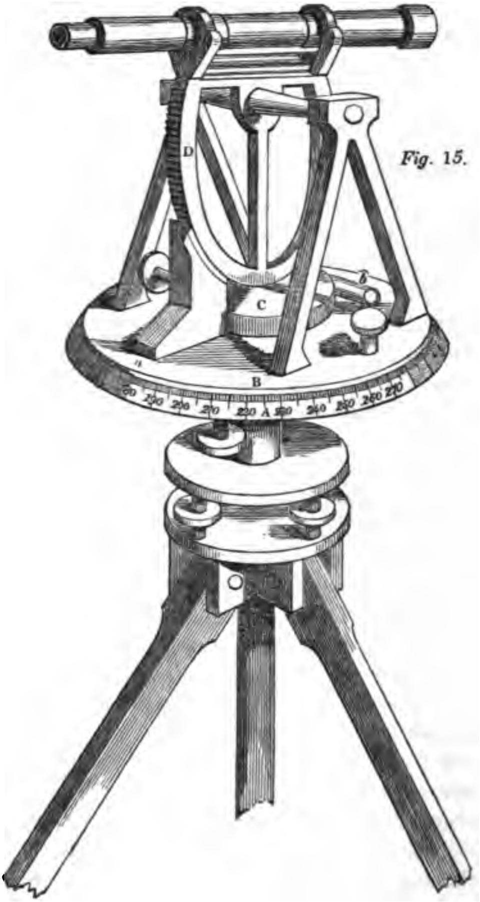

Fig. 15.

THEODOLITE.

section and the centre of the object-glass forming the line of collimation. The principal nonius *a* is engraved on the upper or nonius plate B, immediately beneath the eye-end of the telescope; another nonius *b*, or at least a vertical line, is cut on the opposite part of the circle. The vertical arch D is also divided into degrees, which are subdivided by a nonius. Both of these graduated arcs are moved round by a rack and pinion.

Under the horizontal plates are a ball and socket furnished with adjusting and tightening screws, for levelling and fixing the instrument; and the whole is supported on three legs, folding together by means of a triple joint.

Previously to the introduction of the new system of surveying (which will be the subject of the next chapter), it was customary to employ the theodolite much more extensively than is done at present. The plan of a large estate was usually made by means of this instrument; the first and most important part of the process, namely, the construction of the outer lines of the survey, being entirely dependent upon it. This external figure was generally composed of a great number of sides, all the bearings of which it was necessary to take, in order to lay them down on the plan. The principal roads which intersected the estate were in the next place measured, by another series of lines and angles; and the shorter lines were afterwards filled in by means of the chain only.

This mode of proceeding undoubtedly diminishes the labours of the surveyor, by giving him the advantage of running his lines so as to meet with the fewest obstacles. The plotting of the work also affords the means of proving it; for if every part of both be correct, the lines which complete any particular circuit will intersect each other at the same points on the plan, as those at which they had previously been found to meet on the ground.

It must be confessed, however, that the facilities afforded by the theodolite for conducting a survey, are more than counterbalanced by the inaccuracy which almost invariably attends this method, when the operations are on an extensive scale: it being exceedingly difficult to prevent the errors arising out of so great a multiplicity of angles from accumulating to a serious amount, and at length producing a general distortion of the plan, which it is impossible afterwards to rectify.

For this reason, the theodolite ought never to be relied on for the entire survey of an estate of considerable size, except under circumstances of very peculiar difficulty.

The use of this instrument is now limited principally to such cases as the following: viz.

1. When the width of the tract to be measured is very inconsiderable in proportion to its length: as in the survey of a proposed line of road, canal, or railway, where small errors in the bearings are of comparatively little consequence.

2. When the plan of a city or town is required, which cannot conveniently be comprised within a single triangle, and where the streets are too irregular to be measured and mapped without taking a great number of angles.

3. When an *outline* survey of a large tract of country is to be made, in which only the principal features, such as the hills, roads, rivers, &c. are required; as, for example, in filling up the details of the ordnance survey of this kingdom.

4. When occasional difficulties are met with in com-

pleting certain parts of a survey which has been conducted principally on the triangular system to be hereafter explained; such as hills, water, woods, or buildings, situated so as to render the use of angles necessary. In this case, which is one of frequent occurrence, the theodolite may very properly be resorted to, and, if carefully used, will prove a valuable auxiliary.

In surveying either by the theodolite and chain together, or by the chain only, the mode of keeping the field-book is the same. The page is ruled into three columns, and the writing begins at the bottom of the page, and is carried upwards. In the middle column, which is much narrower than the other two, are contained, the bearing, (if any has been taken,) or angle which the line makes with the magnetic meridian, the station at which the line is begun, and all the distances on the chain at which fences have been crossed, stations made, or perpendicular offsets measured. The side columns are intended to contain sketches of the fences, buildings, and other objects near which the chain passes, which are shown, as the case may require, either on the right or the left side of the page, together with the lengths of the offsets, or distances in links of such objects from the line. A full exemplification of the method of keeping a field-book is given in the engraved specimen, p. 227, which contains all the memoranda required for drawing a complete plan of the estate represented in the frontispiece of this book. The chain lines having been laid out according to the new system of surveying, may be all plotted without the aid of an instrument for measuring angles, simply by the construction of two main triangles on the common diagonal ABCDEF. But in order that the same example may be made to furnish an illustration of the mode of working with the theodolite, we will here suppose that the thicket of trees represented on the plan between C and D had prevented the measuring of that portion of the diagonal, and consequently the construction of the two main triangles; so that, in order to plot the lines, it had become necessary to take their angles or bearings. The line No. 1. is accordingly represented in the margin of the field-book as having been measured in two portions; the first from A to C, a distance of 1840 links, the second from D to F = 1402 links: the latter distance, as well as all the intervening stations, crossings, and offsets, between the points D and F, being found by subtracting 2066 = the length of AD, from each former distance on the line. All the other lines in the field-book remain the same as before.

In order, therefore, to take the necessary bearings, we begin by placing the theodolite over the first station at A, setting 0° on the vertical arch, to 0° on the nonius, and levelling the instrument by means of the parallel plate-screws, and the two cross spirit-levels. Set the nonius *a*, to 0° on the horizontal circle, or *limb*, and turn both the plates A and B together, that is, the entire head of the instrument, until the marked or north end of the needle points to 360° in the compass-box, observing also that the south end cuts the opposite division 180°. Tighten the screw which clamps the limb or lower plate A to the legs, and turn the upper plate B, until the vertical hair in the focus of the telescope cuts the picket erected on either of the stations C, D, E, or F, all of which, together with the foremark, are supposed to have been carefully set out in the same straight line, according to the directions given at page 205., to which the pupil is desired to refer. Read off the degrees and minutes at nonius *a*, on the limb, viz. 74° 20′, and write them down in the field-book. Then read the degrees and fractional parts cut by the north end of the needle, which is the compass bearing of the line, and ought to agree with that already written down from the limb. Write down the reading at nonius *b*, = 254° 20′ underneath the former; observing always that the difference of the opposite readings is exactly 180°.

We are at present supposing that all the lines have been measured (as described) previously to taking the angles, and that fixed marks have been left as stations, showing where to place the instrument, and how to direct the telescope. This mode of proceeding will often be found more convenient than to carry on both lines and angles together, although either mode may be used, according to circumstances.

To obtain the bearing of line 2. (which is the same thing as to take the angle it makes with line 1.) being the theodolite to station E, level it, and turn the entire head until the cross hairs in the telescope cut the picket at the back station A or D. Examine whether the nonius *a* has shifted in bringing it from the former station, and take care that it still show the same reading, 74° 20′, as before. The telescope and limb are now exactly parallel to the position they stood in at A, only having been reversed, the readings are altered 180°. Clamp the limb, and turn the telescope to G. Write down the reading at nonius *a*, (=14° 13′), which will now be found to differ 180° from the compass bearing by the north end of the needle. The opposite nonius, *b*, gives the true reading on the limb, which, for that reason, write down *over* the former, and then see that it correspond with the degrees indicated by the needle.

In all other cases, let the true bearing be distinguished from its opposite, by placing it *above*, to serve as a direction in plotting which way to draw the line, instead of writing N. E., S. W., &c., against the angle in the field-book. A similar distinction is made in surveying with the chain only, by the position of the figures in writing the station at the beginning of the line, as explained at page 225.

We next arrive at station G, where the bearings of both the lines, Nos. 3 and 4., may be taken without removing the instrument. Having levelled and turned both plates until the telescope points back to E, the nonius *a* remaining at 14° 13′, clamp the lower plate, and turn the upper until the cross-hairs cut the picket at B. Write down the readings at each nonius, $\left(\frac{299^\circ\ 30'}{119^\circ\ 30'}\right)$, placing that reading uppermost, which agrees with the north end of the needle. Then turn the telescope to D, read off the angle $\left(\frac{342^\circ\ 52'}{162^\circ\ 52'}\right)$, and insert it in the field-book on the line to which it belongs.

We now remove the theodolite to B, the point of intersection of the 3d line with the 1st, where the angles may be proved by setting the telescope back on G, the nonius *a* standing at 299° 30′, and then clamping and turning about to D or E. If the reading on the limb be now 74° 20′, the angles may be considered as exactly right; and in any similar case where the angles have been but few, and the distances sufficient to fix the position of the limb with accuracy, the error on proving ought not to exceed three or four minutes.

Let the bearings of the lines Nos. 5, 6, and 7., be in the next place taken, beginning at A, and setting the angle 74° 20′ on the first line, then turning the telescope to H, whence the bearing of AH, or No. 5., will be found $= \frac{224^\circ\ 24'}{44^\circ\ 24'}$. At H, setting back this angle on A, take both the bearings HC and HF; the former of which is 161° 2′, and the latter 105° 32′. At F, prove the preceding angles, by setting 105° 32′ back on H, and trying how nearly the bearing of A or D agrees with the first determination of it, viz. 74° 20′.

From these data, the irregular figure AHFEGBA may be laid down in the manner explained in page 228., whence the length of the impracticable part of the main diagonal, contained between C and D, will be found by the scale. It will not be necessary to take the bearings of the remaining internal lines comprised within this principal figure, the positions of all of them being sufficiently fixed by points that have been already determined.

The external lines, from No. 18. to No. 25., may also be laid down without the assistance of angles, yet somewhat uncertainly, both from the want of proof, and from the obliquity of the intersections. The bearings of Nos. 18. 20, 22, 23. and 25., are therefore taken, by which means the positions and lengths of all these lines are more accurately determined, and more satisfactorily proved. (See page 227.)

The method which has been here explained for taking the bearings of lines with the theodolite, is the one best adapted for such a case as that before us, where we have a large figure, consisting of only a small number of sides and angles. This mode of using the instrument is called "working by the limb;" the first bearing only being taken immediately from the needle, and all the others by means of the back observation. When, however, it is attempted to pursue this method through a long series of angles, each of which is made to depend on the one that precedes it, the readings of the limb and of the needle, which ought invariably to correspond, will, after a while, gradually begin to deviate from each other, and the difference will generally be found to go on increasing in proportion to the number of angles taken. Still the most correct method, when the sides are of any considerable length, is to continue working by the limb, or setting the instrument at every station by the back angle; at the same time carefully observing and recording the compass bearing; or, which amounts to the same thing, the *difference* between the two sets of readings. So far as this difference is either permanent or increasing, it ought to be regarded as being most probably due to the accumulated errors of the limb; for, although on single observations, these are much smaller than the errors of the needle, yet, from their tendency to accumulate, they at length exceed them. By carrying on both reckonings separately, it may at last be nearly ascertained what amount of error has gradually crept into the observations of the limb, and how the far greater part of it may be corrected.

In making the circuit of an irregular boundary of moderate extent, where the lines are short and the angles very numerous, it will in general be sufficiently accurate, as well as more convenient, to neglect the back observation, and adjust the instrument, at every station, by the needle only.

THE PRISMATIC COMPASS.

A very useful substitute for the theodolite will often be found in the PRISMATIC COMPASS, which is conveniently adapted, by its portability, for cases like that which has just been mentioned, and any others in which great accuracy in the bearings is not required.

This instrument is a small pocket compass, the card of which is divided into 360°, and, being attached to the needle, travels with it, as in the compasses used for nautical purposes. The sights which determine the line of vision are a vertical slit in a plate of brass at the back of a small prismatic or angular eye-piece, and a fine vertical thread, placed on the opposite side of the centre of the card. The bearing of an object is observed by holding the instrument in the hand, and bisecting it with the vertical thread; at the same time reading the division of the card, brought by the prism into apparent contact with the thread, which is the bearing required. The oscillations of the needle are checked during the observation, by gently pressing a slender spring into contact with the edge of the card.*

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* The following notice of this instrument appears in the Memoirs of the Royal Astronomical Society, vol ix. p. 291. : —

" In the expedition which sailed in 1818, for the discovery of a north-west passage, it appeared that, from their near approach to the magnetic pole, the compasses on board had become nearly useless. This induced Captain Kater to make a series of experiments to ascertain the best kinds of steel, the form of needle most calculated to receive the greatest directive energy with the least weight, and the most effective mode of magnetising it. The results led to the adoption of the *shear* clock-spring steel, and the *pierced rhombus* form, in the proportion of five inches in length to two inches in width. The azimuth compasses thus fitted afforded very accurate deductions, and have therefore been much used in surveying. This instrument is the more valuable, from being fitted with a sight, on which slides (in a frame) the segment of a glass cylinder, ground to a radius of five inches, by means of which a fine line of light is thrown on the index, and may be seen at the same time as the graduations on the card. This ingenious and simple application appears to have been known to Halley; and is, indeed, described by the celebrated Godfrey, of Philadelphia; yet it had been suffered to be forgotten for more than a hundred years."

I have not myself made much use of the above instrument, but I have the authority of Captain W. H. Smyth, R. N. for recommending it, who says, in a note with which he has favoured me on the subject, accompanied by the above extract, it is "an instrument which, from much practice therewith, I venture to pronounce a very valuable one to surveyors." T. G. B.

PART IX.

LAND-MEASURING BY THE CHAIN ONLY.

The system of surveying which is now generally practised, with the exception of occasional instances to which it is not conveniently applicable, is that in which the chain only is used. From its comparatively recent origin, this is sometimes termed the New Method, having been brought into general use chiefly within the last fifty or sixty years. It has also been appropriately designated the triangular system, being founded on the simple principle of constructing one main triangle, or trapezium, from the measured lengths of the sides and diagonal, sufficiently large to comprehend nearly the whole of the tract to be surveyed. The assistance of angular instruments is thus either entirely dispensed with, or resorted to only when an occasional difficulty occurs; and the subsequent operations of the surveyor become little more than repeated subdivisions of the original figure, by lines measured across it in every variety of direction, until as many offsets have been taken from these lines to the flexures and angles of the fences and buildings, as are sufficient for drawing a complete plan of the whole. The simplicity and accuracy of this mode of surveying, and its general applicability to estates of every magnitude, justly entitle it to the preference which it has now almost universally received.

The boundaries of the estate which you are about to measure having in the first place been ascertained, (for which purpose an outline traced from a former map may often be obtained, and will always be found of considerable service,) carefully inspect the ground, and endeavour to find what will be the most convenient form and position of your principal figure, whether triangle or trapezium, that it may be so constructed as to comprehend as much as possible of the lands to be surveyed, without extending very far beyond them. Great care is required in selecting the lines so that they may encounter the fewest possible obstacles, and avoid whatever would render it difficult accurately to measure their length, or preserve their straightness. Some attention should also be paid to the angle of intersection of the sides of the triangle, so as to avoid giving it an undue degree of obliquity.

A transverse line, which ought, in general, to coincide nearly with the perpendicular let fall from this angle upon the base, or opposite side, should also be taken in each main triangle, for the purpose of proving it. The length of this line, found either by construction or by calculation, should agree with its actual measure on the ground.

We will suppose that the figure which you find it necessary to construct, on commencing the survey of an estate, is a trapezium; and that you have ascertained nearly what ought to be its general form, and what will be the actual position of its longer diagonal. Ascend the highest ground over which you find this line will pass, from whence, perhaps a great part, or even the whole of it, may be seen. Choose some conspicuous object, if there be any, for a foremark, nearly in the direction required; taking care that it be something very distinct and definite, and not likely to deceive you while you are approaching it (as will sometimes happen) by insensibly changing its apparent form or position. A tower, a windmill, a house, or a single tree on an eminence, is a proper object to be selected, especially if it be at least as remote as the farthest extremity of your line.

Having walked over the ground, and ascertained that, although a few considerable difficulties may perhaps be encountered, as is generally the case in a long line, yet no insuperable obstacle lies in the way, on arriving at the extremity of the estate, look out for another practicable and convenient line, to form the side of your triangle, nearly in the direction which you have already found to correspond with the general outline of the estate. Pursue this line, in the same manner, to its termination, from whence let a third line be taken, which can be continued until it reach the first, and thus complete the triangle.

The next thing to be done is to set out these lines perfectly straight from end to end, which may often be much facilitated, at the commencement, by moving a little to the right or left of the spot where you had intended to begin, until a point is found at which two or three intervening objects may be seen exactly in the line between that point and the foremark. These alone will, in many instances, when the distance is moderate, be sufficient to preserve the straightness of your course; but on a line of several miles in length it is generally necessary to erect other objects, and make a number of permanent marks at shorter distances from each other.

At the first station, or starting point, let a deep cut be made in the turf, or a few pegs driven firmly into the ground; and if there be no natural object to answer the purpose, let a pole be erected here, or at some more conspicuous point on the line near its commencement. This mark, which is called the back-mark, is of great importance to check the straightness of the line, and not unfrequently becomes the only means of directing it, when the fore-marks happen to be hid for a while behind intervening objects.

Let an assistant be now sent forward with a staff any convenient distance, such as two or three hundred yards, and be directed by signals to fix it up exactly in the line; then go forward yourself to the staff, and, having marked the spot where it was fixed, send it on to be again set up in the line by the assistant, which he will now be able to do of himself, by making it range straight with the last station and the back-mark.

In a country abounding with thick hedges it will generally be necessary, in order to enable you to see your way, to cut a small gap in them here and there, for which purpose the assistant should carry with him either a bill-hook or some other convenient instrument. At the crossing of a hedge, or any other kind of fence, a permanent mark should be left in the line, as a station, which may either be a peg or cut in the ground, or two or three deep notches on the stump of a bush or tree. The latter kind of station is generally the best, being not likely to be removed or obliterated. A good station may also be made on passing the angle of a building, by noting in your book its distance from the line, measured out in the direction of one of its side-walls.

It very frequently happens that a hill lies in your way, which, as soon as you have passed it, hides every object by which you had been directing your course. The greatest care is always necessary in this case to avoid bending the line, especially if it be a long one and the hill nearly midway between its extremities. When the ridge of the hill is abrupt and narrow it greatly increases the difficulty, by limiting the distance between the last two stations, from which the direction of the remainder of the line is to be determined. Various means have been resorted to for overcoming this difficulty, one of which is to take an angle of 180° with a theodolite. A simpler and better method is by means of two plumb-lines, each suspended from three straight wooden rods, about three or four feet in length, fastened together at top, and resembling the stand of a theodolite; the suspended weights being protected from the action of the wind by a few stones placed around them, or in any other simple way. Let the plumb-lines be set up on the summit of the hill, whence you can see along your line in both directions, and, having adjusted them so as to point nicely to the back-marks, send on the assistant to fix up the staff, so that you may see it exactly cut by the plumb-lines, as you stand behind them and look forwards along the line.

It is seldom the case that the narrowness of the ridge will not admit of the plumb-lines being placed as much as twenty-five or thirty feet asunder, and whenever that distance can be obtained a line may be produced, in this manner, with almost perfect exactness.

An angle of 180° may also be measured without sensible error, by means of a small instrument which I constructed, some years ago, for my own use. It consists of three small plane mirrors, two of which are placed at right angles, one over the other, and the third so as to receive the images reflected from them both. Two objects which are exactly on opposite sides of the observer are thus brought into apparent contact, indicating that they form with him one straight line.

All necessary precautions having thus been taken to secure the straightness of the lines, before you begin to measure them, it will be proper to lay down, between two fixed marks, a standard length of twenty-two yards, to which your chain must be kept constantly adjusted.*

The variations of length to which the chain is liable, chiefly from its tendency to stretch in using, are so often the cause of serious error, that there is scarcely anything which requires to be more carefully guarded against. A comparison of the chain with the standard should be made at least once every day until all the main lines have been measured, a slight difference in its length being sufficient to affect these considerably.

You may now commence measuring the lines, beginning at the first station on your main diagonal, and writing 0 in your field-book, to denote that the distances on the chain commence at this point. Take notice, as you proceed, of every object which is proper to be introduced into the map, and write down in your book the distance in links, together with a sketch of the crossing of every hedge, ditch, wall, road, river, brook, or other fence of any kind by which lands are divided from each other. Measure a perpendicular offset to every building or angle of a fence near which the chain passes; enter the same in your book, together with a sketch of the object, on that side of the line on which it lies; and insert in the middle column the distance on the chain at which the perpendicular was taken; carefully note the distance, on the chain, of each of the marks made in setting out the line; in addition to which you will probably have to make several others, it being in general necessary to leave a station at or near the crossing of every fence, or the abutment of one fence against another. From these points you will afterwards have to run other lines alongside of the fences, the offsets from which will enable you to plot them.

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* Care must be taken, in the first instance, that the standard itself be a true one, there being very few scales, or rules of feet and inches, that agree in length with one another.

In measuring over a hill, it is an invariable rule that the horizontal length be taken instead of the sloping length, by reducing the distance actually measured in the proportion of the hypothenuse to the base. This correction is most conveniently made on the ground, by observing its inclination with an instrument which gives also the corresponding reduction, and immediately moving the chain and the arrow *horizontally* forward as many links as are equal to the amount of the reduction. By this mode of proceeding the horizontal distance frow arrow to arrow is in every case = 100 links; the length of the hill, both in the field-book and on the plan, becomes the same as if the line had been measured horizontally through it; and the superficial content of the hill will be reduced to that of the base on which it stands.*

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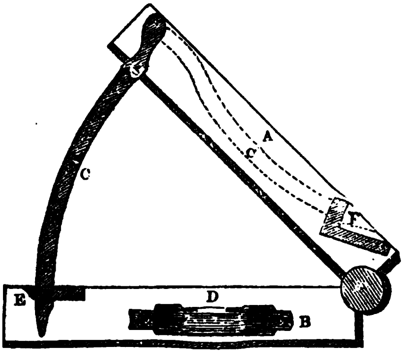
* A little consideration will suffice to show the propriety of making this reduction. Plants of every kind which have an upright stem, require a certain horizontal distance from each other, for which reason the same number of such plants as occupy the surface of a hill might be contained within its base. If a hilly field be laid out for building, it is evident that the number and dimensions of the houses that will cover either its surface or its base must be precisely the same. To contend (as Mr. Crocker has done) that if the base of a hill is to be measured instead of its surface, the area of a section through the earth's equator ought, for the same reason, to be taken for the area of the hemisphere,

The only convenient instrument with which I am acquainted for finding the inclination of hilly ground, and the reduction *per* chain to be made in measuring it, is a clinometer of my own construction, represented here, of which the following is a description. AB is a plain box sector, of 4½ inches radius. C is an arc of brass, divided on one side into degrees from 0° to 50°, and on the other into links and fractions, from 0 to 25, being the horizontal reduction *per* chain corresponding to the inclination. When the instrument is closed, this arc is held by the spring F, in the dotted position cF. D is a small spirit-level for setting the radius B horizontal; E an index for reading off the observed angle and reduction.



Supposing this instrument is to be used in measuring the declivity of a hill, hold the leg B downwards, and

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is to overlook the essential distinction between the two cases; namely, that upright objects, such as trees and buildings, which are parallel to one another on the surface of a hill, are not so over the surface of the terrestrial hemisphere. The few trifling advantages which may attend the small excess of the surface of a hill above its base, are always accompanied with disadvantageous circumstances, by which they are abundantly counterbalanced.

as nearly horizontal as you can, by watching the motion of the bubble; at the same time elevate the other leg A until, by looking along its upper edge to your assistant, who must stand at the lower end of the chain, you see that it is parallel to the surface of the ground. The inclination is then shown by the upper edge of the index E. When you are ascending a hill, invert the instrument, and make B the upper leg, the arc C being in both cases held towards yourself. When this instrument is used as a geological clinometer for measuring the dip of a stratum, the radius A is laid upon it, and the other radius adjusted to a horizontal position. The brass tube is cut open both on the upper and the under side, in order that the bubble may be seen when the instrument is inverted.

The divisions of the clinometer are laid down from the following Table of Reductions of the hypothenusal to the horizontal lengths, calculated by the following proportion, viz.—

As Rad. : cosine of angle of acclivity :: 100 links : the reduced length, or length of the base. The last term subtracted from 100 links gives the tabular reduction.

(*N.B.* The term "*reduction*," usually given to this difference in the length of the hypothenuse and base, more properly applies to the *subtracting* of the tabular difference out of the length of the line, in plotting, when no allowance has been made in measuring it. The method above recommended of drawing the chain forwards on the ground the same number of links, is equivalent to such a reduction, and more convenient in practice.)

A TABLE,

Showing the Number of Links horizontal, *which the Chain must be drawn forwards on Sloping Ground, in order to measure* 100 *Links on the Base.*

If the angle of acclivity or declivity of a hill, be		Every chain must be drawn forwards *horizontally*
Deg.	Min.	Links.
5	45	½
8	10	1
9	55	1½
11	30	2
12	50	2½
14	4	3
15	10	3½
16	15	4
17	15	4½
18	10	5
19	30	5½
19	55	6
20	45	6½
21	35	7
22	20	7½
23	5	8
23	45	8½
24	30	9
25	10	9½
25	50	10
26	30	10½
27	10	11
27	45	11½
28	20	12
28	55	12½
29	30	13
30	5	13½
30	40	14
31	15	14½
31	45	15
32	20	15½
32	50	16
33	25	16½
33	55	17
34	25	17½
34	55	18
35	25	18½
35	55	19
36	25	19½
36	55	20
37	20	20½
37	50	21
38	15	21½
38	45	22
39	15	22½
39	40	23
40	5	23½

The advantages of the triangular system of surveying, above all the others in which a multitude of angles is required, are found in practice to be very great. When proper care is taken for preserving the straightness and obtaining the true horizontal lengths of the lines, the measured length of the transverse line, which proves the triangle, will seldom be found to differ from its calculated length by more than about the $\frac{1}{1000}$th part.

When the main triangle or trapezium has been thus completed and proved, the remaining operations necessary to complete the survey become comparatively easy, especially if all internal *sweepings* are avoided, and such lines only are taken as begin and terminate in the sides already measured. Each successive line will then afford a proof, both on itself and on all the preceding parts of the work; and, should an error in measuring a line occur, it may generally be discovered and rectified with very little trouble.*

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* It may not, perhaps, be amiss here to point out some sources of error in measuring, to which a surveyor is liable, in order that the pupil may guard against them. The following are among the most frequent, viz: —

1. The omission of 1000 links in the field-book, after measuring ten chains.

2. An error of a chain: — in exchanging arrows with the chain-man — by miscounting the arrows in hand in making a station — by omitting to reckon the arrow in the ground at the end of a line.

3. An error of part of a chain: — when the arrow is put down at less than a chain's length, the end happening to reach to the middle of a pool or some other inconvenient place. In this case it is best always to go back and mark the ground at the middle of the chain, measure forward from that mark, and set yourself right again as soon as the obstacle is passed; remembering to make a memorandum in the margin of your book, as a precaution in case of error, that you may know where to detect it. An error of part of a chain is also frequently committed, by not distinguishing the brasses which are equi-distant from the centre of the chain, as 40 and 60, 30 and 70, &c.

4. An error in the main triangle being more easily made, and

THE POCKET SEXTANT.

Although the general principles on which the triangular system of surveying is founded, are such as to render it independent of the measures of angles, so that in a tolerably open country a survey may be completed without them; yet numerous instances occur in practice, in which the use of an angular instrument either becomes indispensably necessary, or is at least attended with important advantages. Of such instruments, the theodolite is undoubtedly the most valuable to the surveyor, and the one which he will generally find it best to resort to in cases like those that have been enumerated in a preceding chapter. The Pocket Sextant may be considered as next in importance, for the frequent assistance it renders in overcoming the minor difficulties of a survey.

This little instrument, which is in principle the same with the well-known Hadley's quadrant, may be conveniently carried in the waistcoat pocket. It is inclosed in a brass box of about three inches in diameter, by which all its parts are so well protected, that they are very little liable to derangement or injury. The

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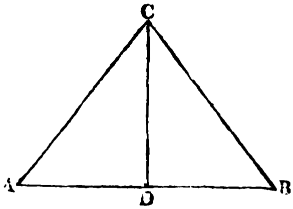
more difficult to detect than any other, it is a very useful precaution to measure as many of the angles as possible, contained either between the sides themselves, or the sides and proof-line, by means of a theodolite or pocket sextant. (See the next chapter.) This will frequently point out at once the side in which the error lies. When the three angles of the triangle have been taken, and their sum differs considerably from 180°, it indicates that one of the sides is not straight; a discovery that may save the labour of remeasuring them.

limb is divided into half degrees, and subdivided by nonius to minutes; the range of angle is about 125°. The observation is read off by a magnifier, and may be depended upon, generally, to a single minute.

So numerous are the cases in which this instrument will be found useful, that the surveyor, when engaged in field operations, ought never to be without it. A specimen only of such cases can be given, it being impossible to enumerate them all.

It is frequently serviceable in the construction of a large triangle; as, for example if it be required to set out a line from C to D, so as to cut the base AB at right angles. In this case, having first measured the angle CAD, let CD be taken so as to make ACD= the complement of CAD, and it will fall perpendicularly on AB, as required.



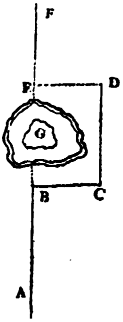
The sextant may be also used for measuring any of the angles of the main triangle, so as to check the length and straightness of the sides. (See note, p. 214.)

If the sides AB, BC, and the included angle B have been measured, the angle ACB may be either calculated or constructed, so that a line may be directed from C, which shall fall on A, although the station cannot be seen from thence. The side AC may also be calculated, and the triangle proved, without measuring the line CD, which, in many instances, may be of no other use than as a proof on the triangle.

When a portion of road, contained between two previous stations, is to be measured, and a bend of the road prevents your running a single line from station to station, two lines, connected by an angle with the sextant, will frequently save much trouble in measuring it.

When a single offset of considerable length, such as three or four chains, is required to complete a fence, or fix the position of a detached building, it may be taken with the sextant, either perpendicularly to the chain, or at any other more convenient angle. And in all cases in which the cross-staff is ever employed, the sextant will answer the purpose much better.

When any considerable obstacle, such as the pool G (see figure), unexpectedly falls in the way, it may generally be avoided, and the line continued, by taking with the sextant a right angle ABC, and measuring towards either side an offset BC. At C take another right angle BCD, and proceed with the line CD, which will be parallel and equal to BE, taking offsets and inserting them in the field-book, where it may be necessary. At D measure back, at right angles to DC, an offset equal to BC, and the point E will be in the original line. The measured distance CD, also, being equal to BE, the length of the line AEF is carried on in the book, as if no interruption had occurred.



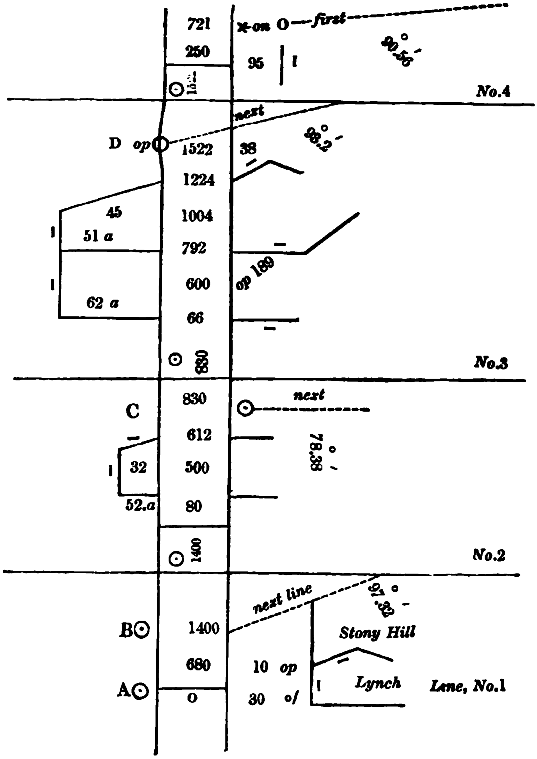
When a village or a small town is included within a triangle, the plan of it may generally be completed without the aid of a theodolite, by means of the sextant only; although there may be no openings between the buildings, through which straight lines might be measured across the triangle.

The assistance rendered by the sextant is, perhaps, greatest of all in finishing the irregular out-boundaries of an estate, by angles taken from any of the adjacent lines of construction. By affording a check on the work, it also frequently saves the necessity of running extra lines merely for proof, which would be otherwise entirely useless.

Thus, in the example given in the frontispiece, and in the field-book, page 227., the line No. 18. is fixed by measuring with the sextant the angle it makes with line 5. (viz. 27° 25′), and is proved by line No. 19. The positions of lines Nos. 20. 22. 25., are also fixed by angles taken with the sextant, and proved by lines subsequently measured.

This little instrument will also be found exceedingly convenient for taking the angles, in order to construct a figure of four or five sides, measured around the boundaries of a piece of ground of any description, in which case it performs exactly the same office as the theodolite, when the method of working by the limb is employed. Of such cases, which frequently occur in practice, the following is an example: —

Let it be required, from the following lines, angles, and offsets, to map and cast the fields called Lynch and Stony Hill, the interior of which is thick coppice-wood.



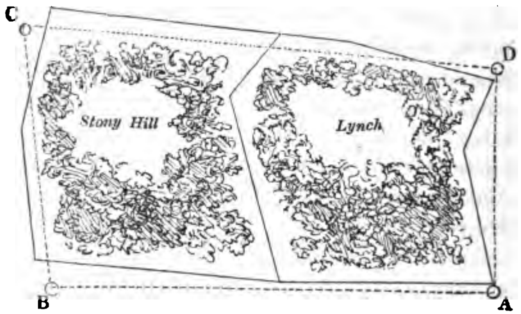
These field memoranda show that at the end of fourteen chains, on the first line, the second line begins, making an angle with the first of 97° 32′. This angle was measured with the sextant held over the station B, by bringing the picket at the first station A, and that at the end of the next line C, into apparent contact, as seen in the instrument. At C, on the second line, the angle measured between the back station B and the fore station D, is = 78° 38′. At D the angle, measured in a similar manner between C and the first station A, now made the fore-mark, is = 93° 2′. At A the lines cross, and here the angle between the last line and the first, that is, between the stations D and B, being measured and found = 90° 56′, affords a proof on the angles; the sum of which, in a quadrilateral figure, should be = 360°. (See page 44.)

	97°	32′
	78	38
	93	2
	90	56
The sum of these four angles is =	360	8
being 8′ more than the truth.		

Let 2′, therefore, be subtracted from each of the angles, by way of making an equal distribution of the error.

All the observed angles, in this instance, are *inward* angles, having been measured from the back station of one line to the fore station, or fore-mark of the next. It is, however, frequently more convenient to measure an *external* angle, viz. from the fore-mark of the line which has just been measured (instead of the back station), to the fore station of the succeeding line. The *supplement* of the angle observed then becomes the internal angle.

From the foregoing data, the plan of these two fields, laid down to a scale of six chains to an inch, together with the lines of construction, will be shown in the following figure: —



The contents of these fields, computed either by the methods which have already been explained, or by that which will be found at page 241., are as follows, viz.

	A.	R.	P.
Lynch =	5	1	29
Stony Hill =	5	2	21

SURVEY OF AN ESTATE.

For the sake of illustrating the foregoing principles we will show how they may be applied to the measuring of the estate represented in the frontispiece. In the former editions of this work directions are given by the author for dividing this estate into three portions, surveying each separately with the theodolite, and afterwards drawing a connected plan of the whole.

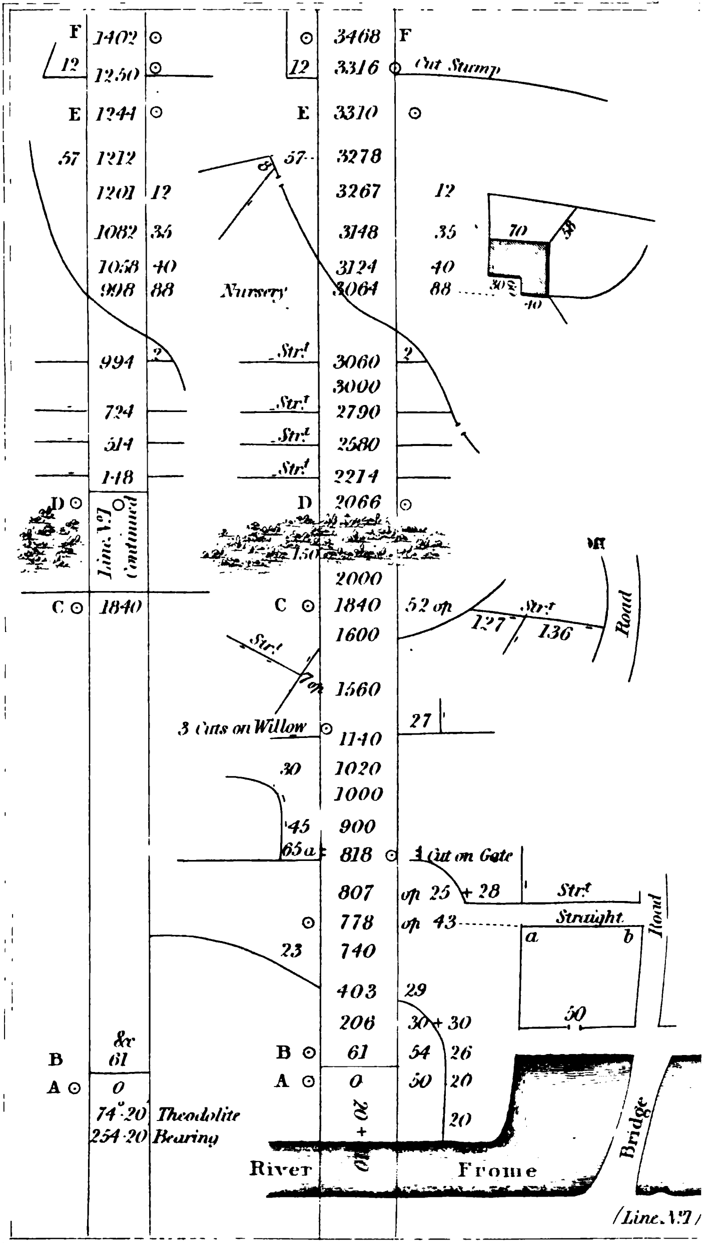
For the reasons that have been already explained, it will be much better to construct a triangle or trapezium with the chain only, so as to include within it as much of the estate as possible, and in order to do so, it will be proper to proceed as follows: —

Having walked over the ground, and observed that its general outline is that of a trapezium, whose longer diagonal is somewhere in the direction AF, begin by selecting the most conspicuous object you can find that may serve as a fore-mark for setting out such a line, taking care to avoid the Plantation No. 20., and the Nursery, No. 27., which, in this instance, is effected by passing between them. Let the directions which have already been given for preserving a straight course, be carefully attended to. Make some good mark on the ground at the first station A, and cut a notch on the gate leading into the field, No. 18., called Little Sandpits. Let other marks be made as stations either on or near all the fences which you cross, carefully observing that all these points are exactly in the line.

Having proceeded as far as F, which lies rather beyond the limits of the estate, and made a station in the last fence by a cut on the stump of a tree, (as marked in the field-book), begin to look out for a second line in a direction nearly parallel to your eastern boundary

This being a road, let such an object be chosen as will enable you to set out the line EG, which lies within the fences of the road for the greater part of its length, and is therefore most conveniently adapted for measuring it. If no suitable object present itself exactly in the required direction, your choice of a fore-mark being confined by the road within narrow limits, send forward an assistant with a staff, and direct him by signals to fix it up at any point G where it may best suit your purpose. Having ranged out the line EG, and left such stations as may be necessary,

To face Page



PLAN OF MANOR OF NORTH HILL, SOMERSET.

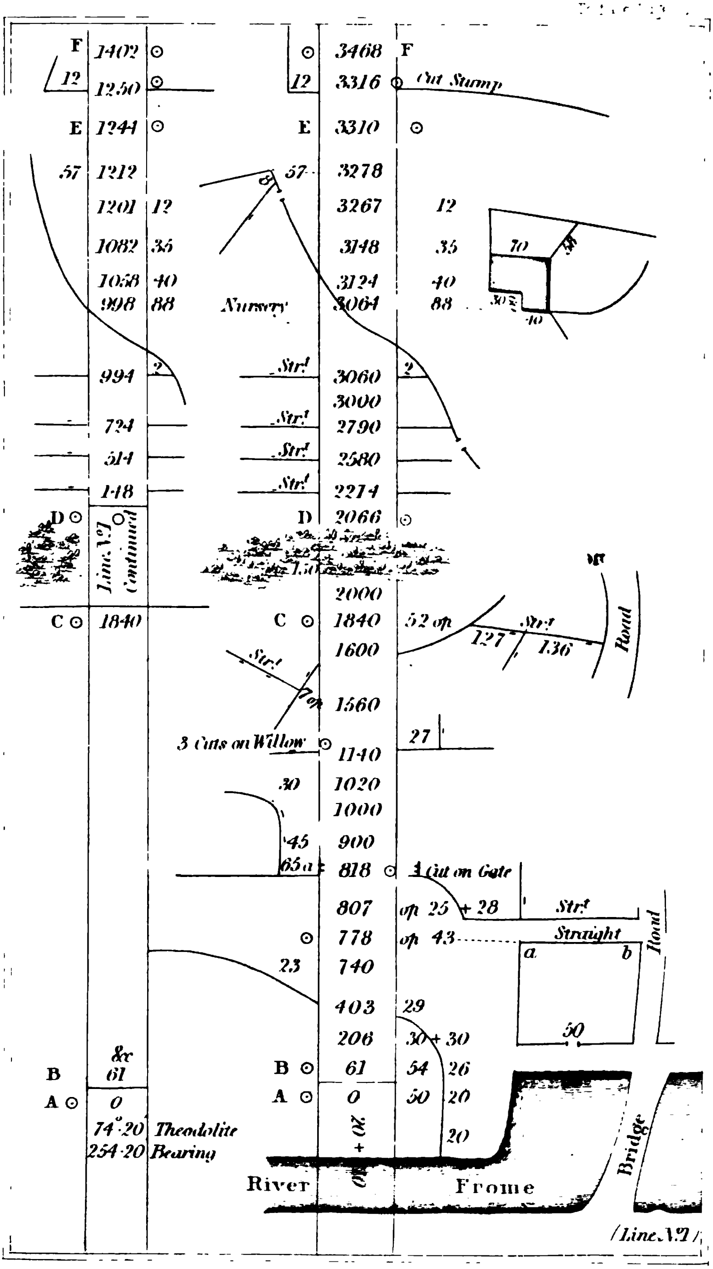
walk on from G towards A, until, by looking alternately forwards and backwards, you perceive how to direct another line so as to fall nearly on the first station A, and just escape the river on the left. Pursue this line until it falls into the first, and carefully mark on the ground the exact point of intersection of the two lines at B.

It now remains to mark out a line across the triangle, both for the purpose of proving it, and of fixing the position of such other lines as may be necessary for measuring the separate enclosures. If the ground admit of your seeing the station G from the first line point C or D, choose some convenient point D, and set out the line DG, cutting the base nearly at right angles, which will be its best position for proving the triangle BEG. The plan, however, represents an intervening hill between D and G, which may perhaps intercept the view; if so, walk forwards as nearly as you can guess in the direction of the perpendicular, until you come to the station G, from which you will now probably be able to direct a line which shall fall on the base near the point at which you are aiming.

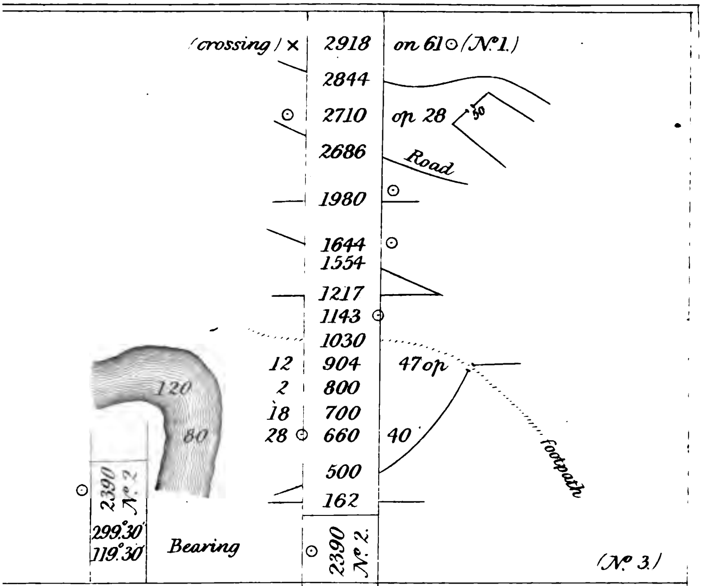
If the angle AEG have been taken with a pocket-sextant, the exact direction of the perpendicular let fall from G on the base will be found, by making the angle EGD = the complement of the angle AEG, (see p. 215.) then set out the line GD, and mark on the ground the exact point of intersection D.

Proceed in the same manner to lay out the triangle AHF, together with the transverse line HC, making stations at H and C, and at the crossings of fences, as before.

The next step is to measure the lines which have



PLAN OF MANOR OF NORTH HILL, SOMERSET.


(crossing) ×
2918
on 61 ⊙ (No. 1.)
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op 28
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2686
Road
1980 ⊙
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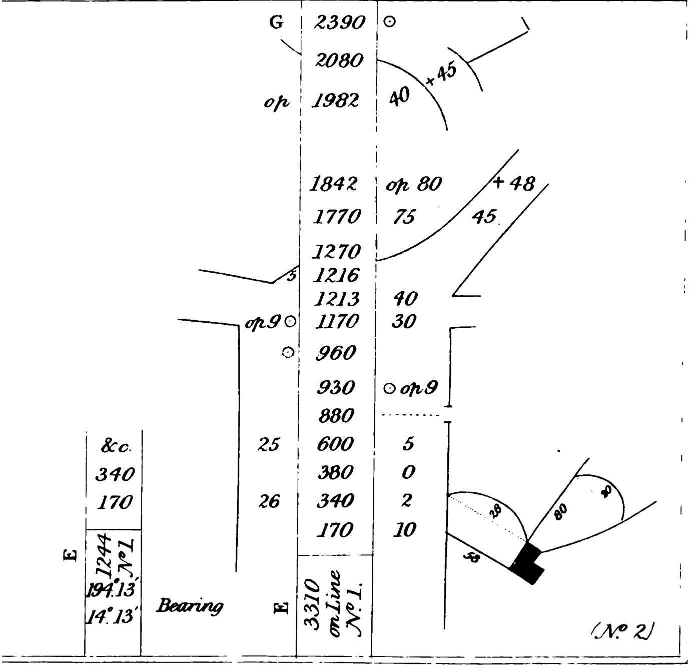
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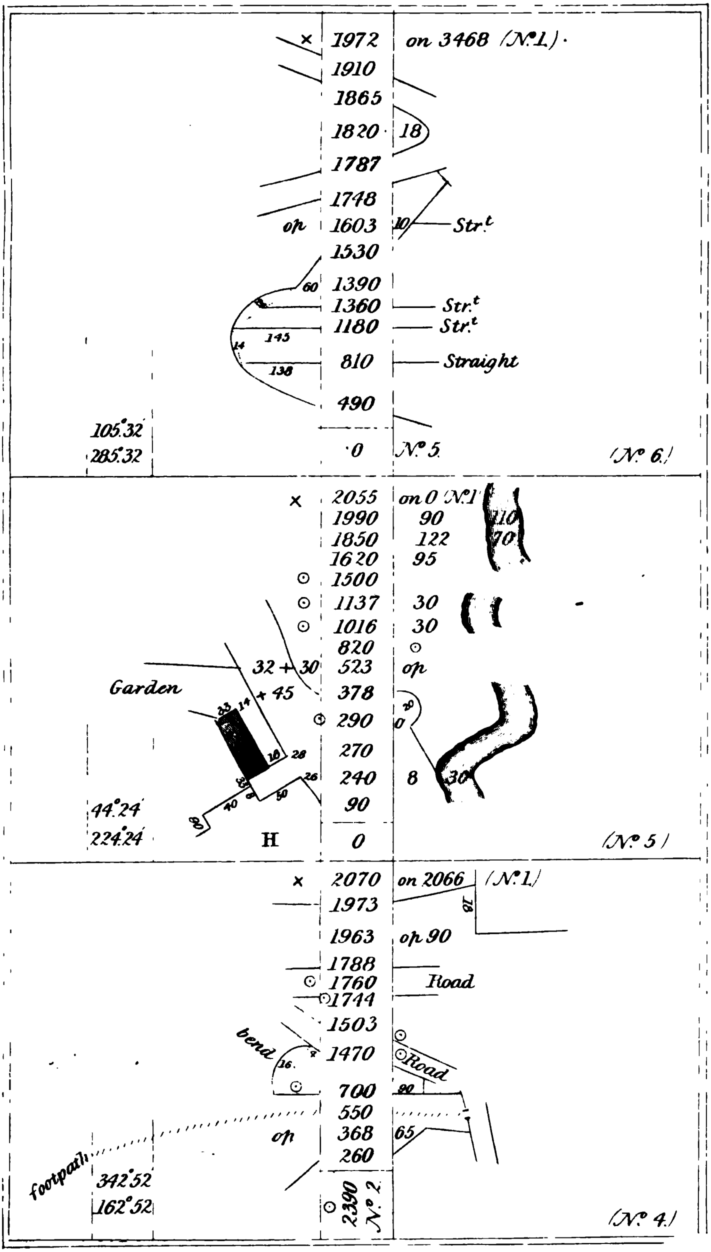
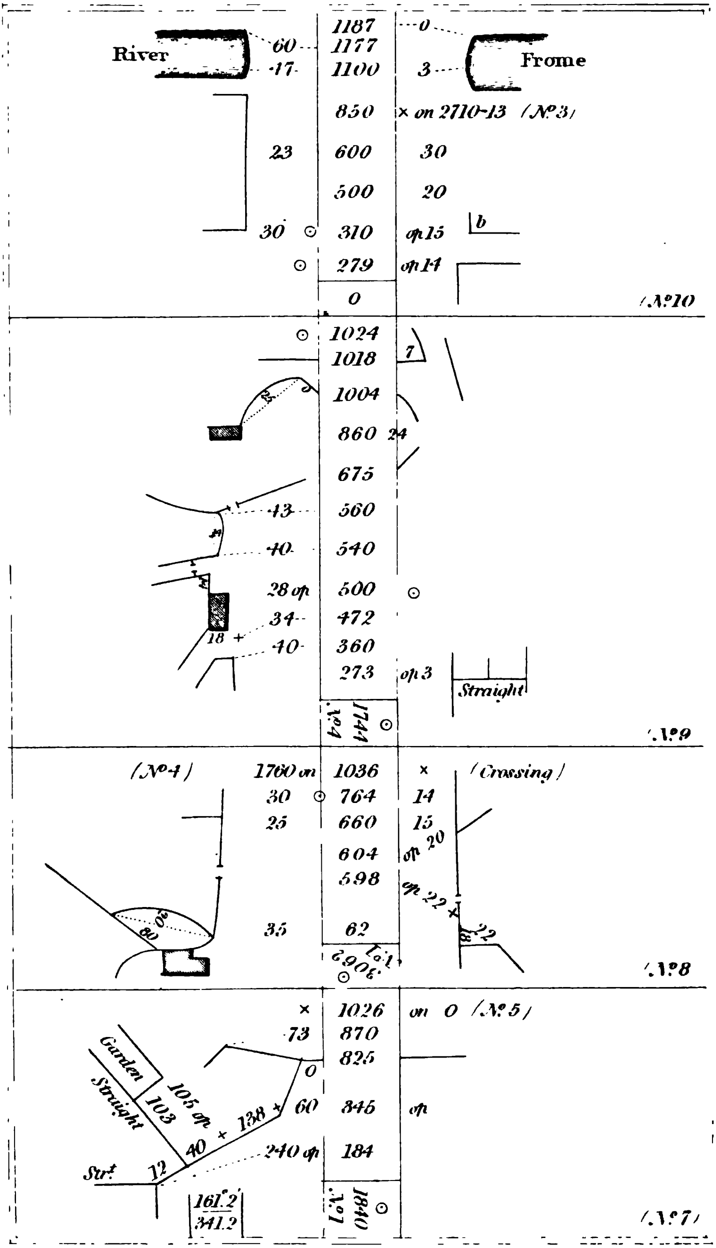
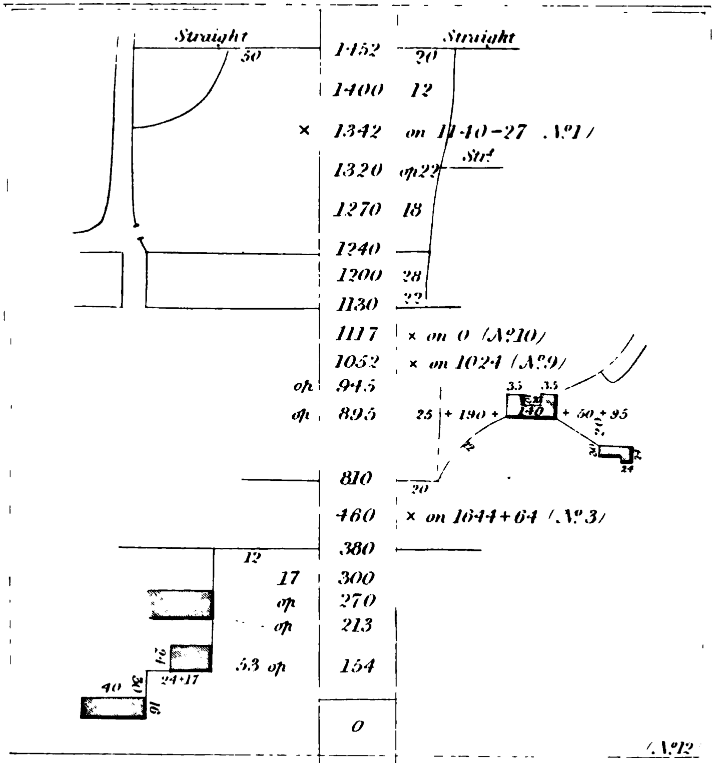
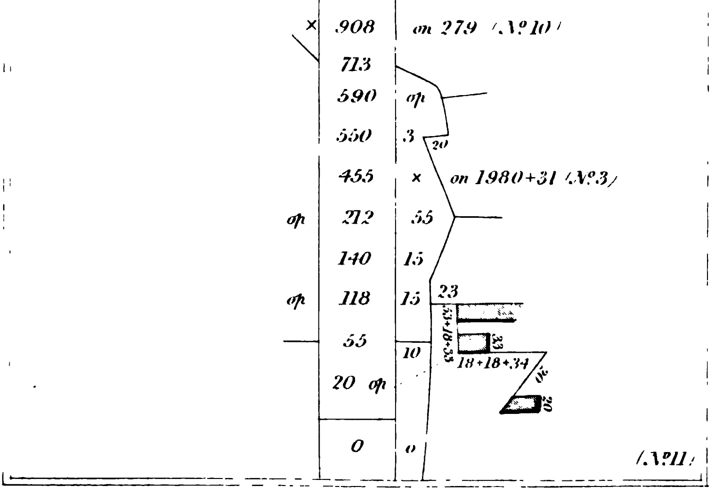
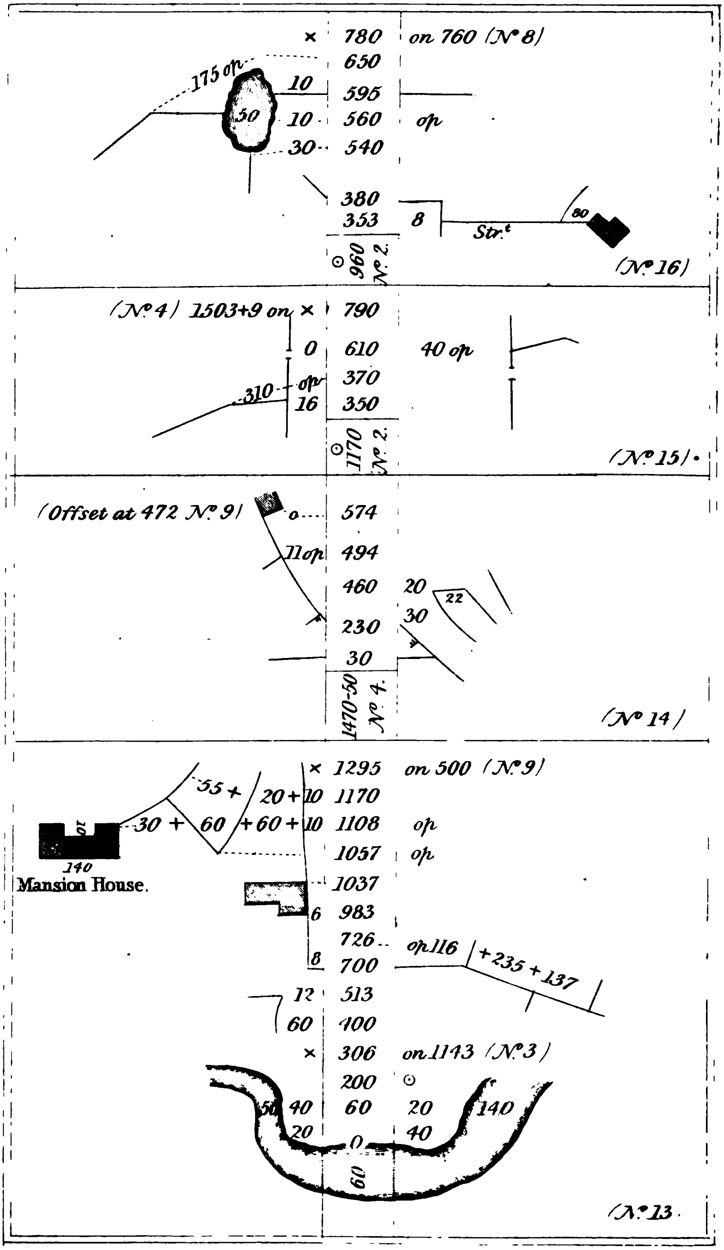
3310 on Line No. 1.
(No. 2.)

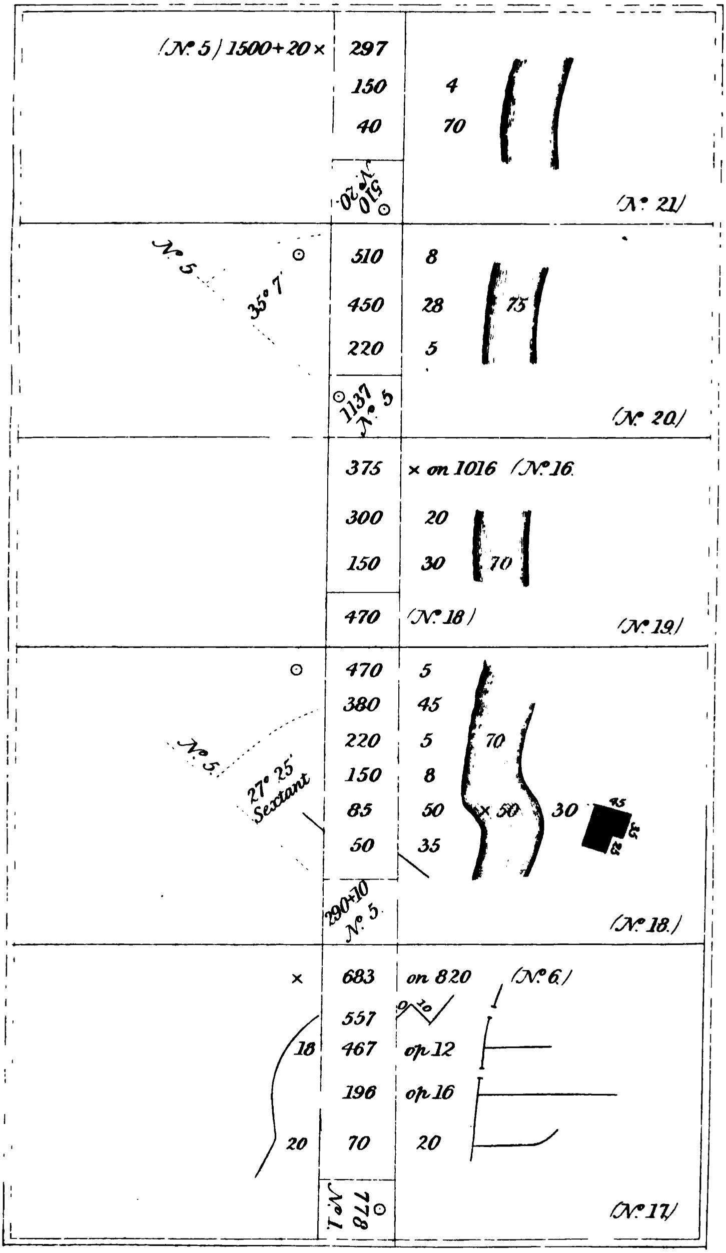

1972 on 3468 (No. 1)
1910
1865
1820
18
1787
1748
op 1603
10 Strt
1530
60 1390
1360 Strt
1180 Strt
145
14
810 Straight
138
490
105°32′
285°32′
0
No. 5
(No. 6)
2055 on 0 (No. 1)
1990 90
1850 122
1620 95
110
70
1500
1137 30
1016 30
820
32 + 30 523 op
Garden
+ 45 378
33
14
290
20
0
18
28
270
33
26
240 8
30
50
8
40
90
44°24′
224°24′
H
0
(No. 5)
2070 on 2066 (No. 1)
1973
18
1963 op 90
1788
1760 Road
1744
1503
bend
1470
16.
8
Road
700
20
550
op 368
65
260
footpath
342°52′
162°52′
2390 No. 2
(No. 4)

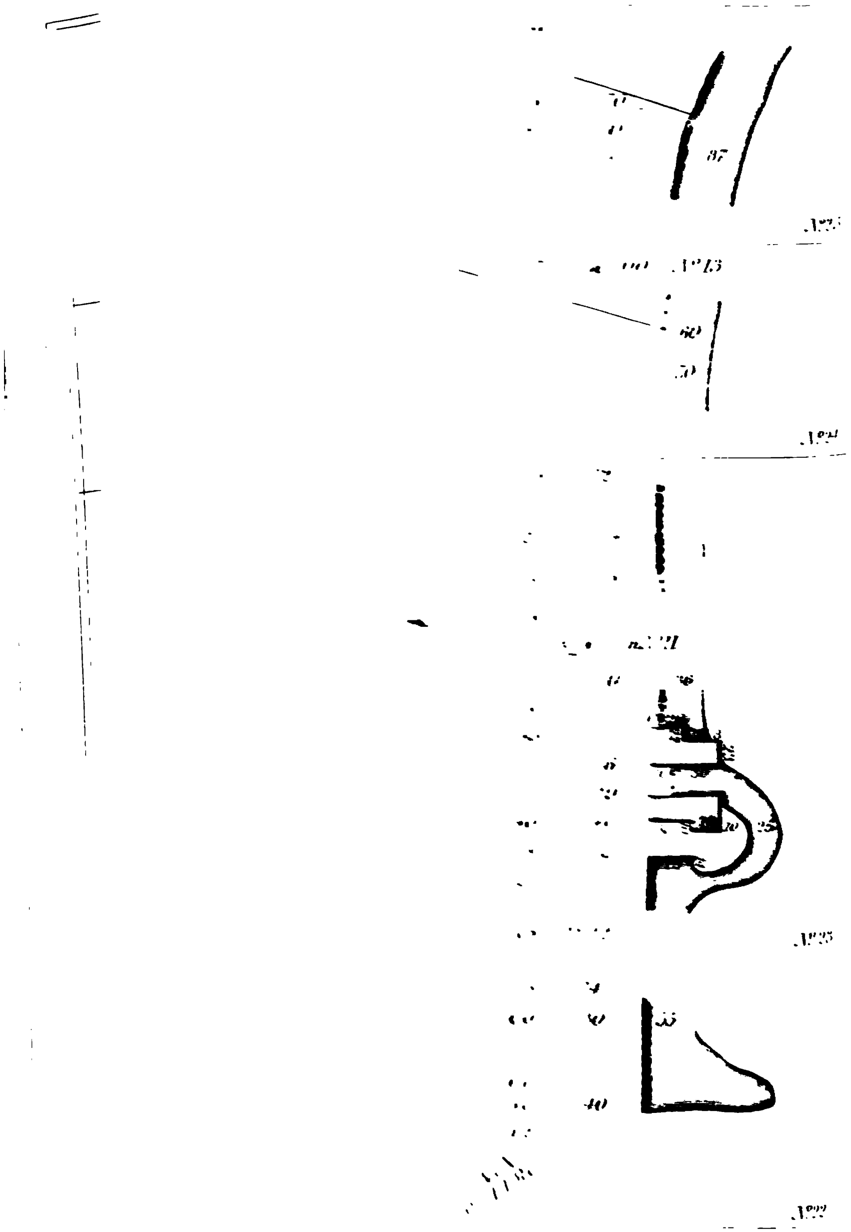

River
60
17
1187
1177
1100
0
3
Frome
850
× on 2710-13 (No. 3)
23
600
30
500
20
30
310
op 15
b
279
op 14
0
(No. 10)
1024
1018
7
1004
860
24
675
43
560
40
540
28 op
500
34
472
18
40
360
273
op 3
Straight
No. 4
1744
(No. 9)
(No. 4)
1760 on
1036
×
(Crossing)
30
764
14
25
660
15
604
op 20
598
op 22
22
80
20
35
62
No. 1
3069
(No. 8)
×
1026
on 0 (No. 5)
73
870
825
0
Garden
105 op
103
Straight
138
60
345
op
40
240 op
184
Str.t
12
161.2
341.2
No. 7
1840
(No. 7)


Straight
50
1452
20
Straight
1400
12
× 1342 on 1140−27 (Nº1)
Strt
1320 op 22
1270 18
1240
1200 28
1130 22
1117 × on 0 (Nº10)
1052 × on 1024 (Nº9)
op 945
op 895
25 + 190 + 140 + 50 + 95
3.5 3.5
24
810
20
460 × on 1644+64 (Nº3)
380
12
17 300
op 270
op 213
53 op 154
24+17
40
16
0
(Nº12)


× 908 on 279 (Nº10)
713
590 op
550 3
20
455 × on 1980+31 (Nº3)
op 212 55
140 15
op 118 15
23
53+18+53
55
33
10
18+18+34
20 op
20
0
0
(Nº11)


× 780 on 760 (N° 8)
175 op
650
10 595
50
10 560 op
30 540
380
353 8
Strᵗ
80
⊙ 960 N° 2.
(N° 16)
(N° 4) 1503+9 on × 790
0 610 40 op
310 op 370
16 350
⊙ 1170 N° 2.
(N° 15)
(Offset at 472 N° 9)
o 574
11 op 494
460 20
22
230 30
30
1470-50 N° 4.
(N° 14)
× 1295 on 500 (N° 9)
55 + 20 + 10 1170
30 + 60 + 60 + 10 1108 op
10
140
Mansion House.
1057 op
1037
6 983
726 op 116 + 235 + 137
8 700
12 513
60 400
× 306 on 1143 (N° 3)
200 ⊙
50 40 60 20 140
20 0 40
60
(N° 13)


(Nº 5) 1500+20 ×
510 Nº 20
(Nº 21)
Nº 5
35° 7'
1137 Nº 5
(Nº 20)
× on 1016 (Nº 16.
(Nº 18)
(Nº 19.)
× 50
Nº 5.
27° 25'
Sextant
290+10 Nº 5
(Nº 18.)
×
on 820
(Nº 6.)
op 12
op 16
778 Nº 1.
(Nº 17)


№15
№25

thus been set out. Begin at the station A, and write O to represent it in the field-book, as the zero-point of your first line. By referring to the field-book at p. 227 it will be seen, that from this point to the edge of the river in the direction of the line continued back, is a distance of 20 links; and 110 links more, in the same direction, is the estimated distance to the opposite bank. At the same point A is a perpendicular offset of 50 links, on the right, to the fence of the road, and 20 more to the edge of the river. At 61 links, the chain falls on the station B, the point of intersection of the first and third lines. A perpendicular from the same point B of 54 + 26 links cuts the angle of the river, and 50 links more in the same perpendicular reaches to the corner of the bridge. At 206 is a perpendicular of 30 to the fence, + 30 to the corner of the inclosure called Road Piece; 50 links more, in the same direction, is the width of the inclosure at that end, in the middle of which stands a gate: 403 crosses the fence into the road.

At 778 we find written, on the right, 43 op. (or opposite), which means, that if the straight fence *a b* were produced 43 links, it would cut the chain line at the distance 778. This abbreviation is always to be understood in the same sense, and distinguishes the offsets which are measured in the direction of a cross fence, from those which are perpendicular to the chain, as all offsets, not having a mark to the contrary, are supposed to be. The circle ⊙ placed against 778 indicates that this distance is to be considered as a station, it being at any time readily found by measuring out 43 links from the angle of the fence *opposite*, or in the direction *a b*, although no permanent mark may have been made in the ground, which on the hard surface of a road is frequently difficult, if not impracticable. In measuring roads, therefore, stations of this kind are often found convenient. The distance 807 on the chain is (op.) opposite the other side of the cross-road, the nearest angle being 25 from the chain : 28 more, in the same direction, that is, measured along the fence, reaches to the abutment of another fence nearly at right angles with the road. Both sides of the cross-road being straight, are marked as such in the field-book, as a direction to be attended to in plotting.

At 818 the chain reaches the station cut on the gate and crosses the edge of the road; *65 a*, written against it on the left, close to the fence, is intended to denote, that at that distance, measured *along* (indicated by *a*) the side of the road, (not perpendicularly to the chain.) is the abutment of another fence. At 900 and 1020 are perpendicular offsets to the same. At 1140 the chain touches a willow, on which three notches are cut, as a station, and 27 measured along the fence, to the right, reaches an abutment. Farther on is the abutment of a straight fence on the left, which, if produced 7 links, would cut the line at 1560. The two short strokes, drawn close and parallel to the line of this abutting fence, indicate on which side of the ditch the hedge stands, the line itself representing the *outer edge of the ditch*, which is, almost universally, the exact boundary line of every inclosure.

At 1840 is the station C, the point where the main proof line of the smaller triangle falls on the base. A straight fence, reaching to the road, points to this station : 52 links is the distance from the chain to the nearer abutment; at 127 farther is the end of another fence running parallel to the chain; and 136 more reaches to the road.

The line having passed through a cluster of trees, reaches at 2066 to the station D, the intersection of the proof-line of the principal triangle with the base. Next follow the crossings of four fences, all of which are straight, as the field-book informs us. The positions of their opposite extremities are all determined by line No. 6, which crosses three of them, and passes near the fourth. On this account no stations are made at the crossing of these fences, it being easily foreseen that they will not be required.

At 40 links perpendicular to 3124 on the right, is the angle or quoin of a building, and at 35 perp. to 3148 is another quoin. The outline of the building is sketched and shaded in the field-book, and the dimensions of the remaining sides are written in their respective places. The abutments of three fences are also sketched, issuing from two of the angles of the house. At 3316 is a cut on a stump in the hedge, and the line terminates at 3468, where it is intersected by one of the sides of the smaller triangle.

The point E 3310, at which the second line commences, being in the road where a mark could not easily be made, is found by measuring back 6 from the station 3316 cut in the hedge. The position in which the figures 3310 stand at the beginning of the line, and in which the book must be held in order to read them, is intended to denote that the line turns towards the right hand, and serves as a direction which way to draw it in plotting.

The first offset fixes the oblique fence, 58 links in length, reaching from an angle of the house to the road. At 340 is a curving fence, extending from another angle of the house to the road; 26, measured from the dotted chord line to the middle of the fence, gives its curvature. The other curving fence, having a bend of 20 links, will be completed by line No. 8. At 930 and 1170 are stations whose places will be found by measuring out 9 links for each from the cross fences to which they are opposite; and 30 links measured along the line from 930 determines the position of station 960, at which the line, No. 16, commences.

At the point G 2390 the second line terminates, and the third begins. The position of the figures, when the station is repeated at the beginning of the third line, shows that it turns to the right hand. 1030 crosses a foot-path which comes from the gate close to the fence opposite 1842 on line 2. At 2918 is the mark (×), which denotes the crossing of a former line, where the chain falls on the station B, 61 links on the first line. The lines being regularly numbered in the field-book, a reference is given to the number of the line whenever a former station is quoted, in order that it may be readily found in plotting.

The sides of the first triangle being thus finished, the main proof-line GD is next measured, which, as the field-book informs us, crosses line No. 1 on the station 2066.

The second triangle and its proof-line being measured in a similar manner, AH is found = 2055 links, HF = 1972, and HC = 1026.*

The practical problem which remains to exercise the skill of the surveyor is, how he may, with the fewest and least obstructed lines, and with the shortest offsets, be able to complete the plan, and make an accurate plotting of every separate inclosure, building, and whatever else it ought to contain.

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* The lengths of all these lines were obtained by applying a four-chain scale to Mr. Crocker's engraved map.

Every new line ought now to begin and terminate at points in those lines which have been previously measured, avoiding as much as possible all internal sweepings, or the formation of small triangles within the principal figure, by the intersection of the newly measured sides. An inspection of the engraved plan will show that all such cases have been avoided, and that the only new triangles formed are exterior to the principal lines of construction, where the irregularities of the boundary rendered them necessary. The lines No. 18, 20, 22, and 25 constituting the sides of all these outer figures are proved by angles taken with the pocket sextant.

PART X.

(BY THE EDITOR.)

PLOTTING.

In Part VIII. I have supposed the estate represented in the frontispiece to have been measured with the assistance of the theodolite; in Part IX. the chain only is supposed to have been used. In the former case the principal lines are to be plotted by means of their bearings, or angles with the magnetic meridian; in the latter, by the construction of triangles from the lengths of the sides only.

In order to lay down the lines from their bearings taken with the theodolite, begin by drawing a vertical line on your sheet of paper, to represent the magnetic meridian. Mark a point near the middle of this line, on which the centre of the protractor * is to be laid; and let the opposite divisions 0° and 180° be made to coincide with the line.

Mark off from the protractor on the plan a point corresponding to every bearing that has been taken;

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* The protractor is a circle of brass, divided on its outer edge into degrees, from 0° to 359°, for laying down on the plan the angles which have been taken with the theodolite, or any other angular instrument. An ivory protracting scale, six inches in length, usually contained in a case of instruments, is often used for the same purpose. See figs. 3. and 4. p. 25. and 26.

A very useful protractor may be made by dividing into degrees a circle of from 5 to 10 inches diameter, on transparent paper.

make a small pencil circle round each point, and write against it its number of degrees and minutes. Let the opposite bearings be also marked on the plan in a similar manner.

Turning to the field-book (p. 227.,) we find the true bearing of line 1. to be 74° 20′ from the north point of the magnetic meridian, and its opposite bearing 254° 20′. These are inserted in a column in the margin, intended to represent the middle column of the page. The remaining bearings (part of which only are given in the field-book) are as follows, viz.,

	True Bearing.	Opposite Bearing.
Line No. 2	194° 13′	14° 13′
3	299 30	119 30
4	342 52	162 52
5	224 24	44 24
6	105 32	285 32
7	161 2	341 2
18	251 49	71 49
20	259 31	79 31
22	162 32	342 32
23	121 32	301 32
25	143 7	323 7

Having considered whereabouts the work should begin, so that the whole may come within the compass of the paper, make a mark thus: ⊙ to represent the first station A. Apply the edge of a parallel ruler, on the accuracy of which you can depend, to the points marked 74° 20′ and 254° 20′, and see that the central point also falls in the same line. Move the edge to the point A, and draw a pencil line in the direction of the true bearing 74° 20′, somewhat longer than the sum of the distances AC and CF, there being also an unknown portion to be added for the distance CD. Apply your scale to this line, and mark off from A the distances 61 and 1840 for the stations B and C.

In the next place lay the ruler to the bearings of line 3., viz. 299° 30′ and 119° 30′, noticing also the central point; move the edge to the second station B, and rule a pencil line of sufficient length to be able to mark off upon it the distance BG = 2390.

Apply the parallel ruler to the bearings of line 2., viz. 194° 13′ and 14° 13′; move it to the last station G; rule a line, and set off the station E at the distance of 2390 from G. This point E ought to fall on the first line. Measure back from it the distance 1244, which gives the position of the point D. The scale being now applied to the points C and D shows the distance which it was found impracticable to measure = 226; and the whole line AE becomes (= 1840 + 226 + 1244) = 3310.

Set off from G a line parallel to the points marked with the bearings 342° 52′, 162° 52′, and measure on it by the scale a distance of 2070, being the length of line No. 4. If the extremity of this line fall exactly on the station D (2066) on line 1., the triangle is correct.

In a manner precisely similar may the lines No. 5, 6, and 7. be laid down. The extremities of the lines No. 6 and 7. ought to coincide with the stations 3468 and 1840 (viz. F and C) on the first line.

Proceeding with the lines on which bearings have been taken, we come next to No. 18. It appears by the field-book, that this line starts from a point which is ten links beyond station 290 on line 5., or 300 links from the station H. From that point draw a line parallel to the bearings 251° 49′, 71° 49′, in the direction of the former, and make its length by the scale = 470. Also from 1137 on the same line, No. 5., set off No. 20., with a bearing of 259° 31′, and a length of 510. Let No. 22., 23., and 25. be also laid down in their true positions, as the field-book directs, and all the lines dependent on bearings will be then completed.

The plotting of the remainder of the lines and offsets will now be exactly the same, whether we suppose the theodolite to have been employed or not.

When the length of the diagonal is given, as well as the lengths of the sides, the construction and proof of the triangles BEG, AHF, are exceedingly simple. Draw the line AF = 3468 by the scale, and mark also the distances 61, 1840, 2066, and 3310 for the stations B, C, D, and E. From E with the length EG = 2390, and from B with the length 2918, sweep two small arcs, the intersection of which will give the point G (Prob. X., p. 35.). Be careful to measure the lengths of the sides with your scale after sweeping, and see that the point of intersection is accurately determined; then lay the scale from G to D, and see if the distance of those points on your plan agrees with the measured length of the line, viz. 2070 links. Should a small difference, such as four or five links, be found, it is best to adjust the station G to the *mean* between the point of intersection and the extremity of the proof-line.*

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* The sides BE, EG, GB, and the distance ED, being given, the length of the line DG may be calculated either by the 47th proposition, 1st bk. Euclid, and the proportionality of similar triangles, or by trigonometry. The problem is an exceedingly interesting and valuable one to a surveyor, especially when the sides of the triangle which he is desirous of proving, (as is often the case,) are several miles in length. This mode of proof is far

Lay down in the same manner the triangle AHF, and prove the distance HC. The angle AHF being rather

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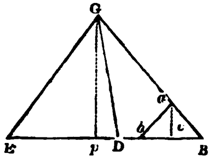
superior to the one which is afforded merely by construction, being free from the practical difficulties and imperfections to which that process is liable.

Let us first calculate the length of GD, without trigonometry, from the properties of right-angled, and similar triangles, only.



In our first main triangle we have the following data:—

BE (= 3310 − 61) = 3249
BD (= 2066 − 61) = 2005
EG = 2390
GB = 2918

First, to find the segments of the base Ep and pB, made by the perpendicular let fall from G to p.

By Case III. Trig. p. 76. As $\mathrm{BE} = 3249 : \mathrm{EG} + \mathrm{GB} = 5308 :: \mathrm{GB} \mathbin{\dot{-}} \mathrm{EG} = 528 : 862{\cdot}6 =$ the difference of the segments. Whence $\mathrm{B}p = \dfrac{\mathrm{EB} + 862{\cdot}6}{2} = 2055{\cdot}8$, and $p\mathrm{E} = \dfrac{\mathrm{EB} \mathbin{\dot{-}} 862{\cdot}6}{2} = 1193{\cdot}2$. The perpendicular G$p$ is therefore $= \sqrt{2390^2 - 1193{\cdot}2^2} = 2070{\cdot}8$; pD being $= \mathrm{B}p - \mathrm{D}p = 2055{\cdot}8 - 2005 = 50{\cdot}8$; and G$p$ being $= 2070{\cdot}8$, GD is found $= \sqrt{\mathrm{G}p^2 + p\mathrm{D}^2} = 2071{\cdot}5$, which exceeds its measured length only by 1·5 links.

The calculation is much longer when the proof-line is taken from a point on one of the sides, instead of the angular point itself. This, from various causes, is often unavoidable in practice; for which reason, we will show by another example from the field-book, how such a line is to be calculated.

Line No. 12. falls on line No. 3., at 1644 + 64 from G, = 1708 obtuse, the point H may, perhaps, be more correctly determined by sweeping with the sides AH and HC, in which case the side HF becomes the proof; or rather, if a small difference appear in the distance of H from the base, after finding that point by the intersection of the sides AH and HF, let it be altered on the paper so as to make the length HC, by the scale, *very nearly* equal to the measured length, which, on account of the obliquity of the angle, will but slightly affect the lengths of the sides.

Lay your scale along the line AF, setting 0 against the station A, and divide it into furlongs, by marking a point at every ten chains. Surround each point with a small circle, and number the furlongs thus: ⊙ 1., ⊙ 2., ⊙ 3., &c. to the end of the line. Divide into furlongs, in a similar manner, all the other main lines, and, as a general rule, any subsequent ones that may be of sufficient length; and whenever there has been occasion, on proving the lines, to alter their lengths

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$= Ga$; whence $aB = 1210$. It also cuts the first line BE at 1113 from the first station A, making $Bb = 1052$.

By similar triangles we have, As $BG = 2918 : Gp = 2070{\cdot}8 :: Ba = 1210 : ac = 858{\cdot}7$. Also, As $BG = 2918 : Bp\ 2055{\cdot}8 :: Ba = 1210 : Bc = 852{\cdot}5$, $Bb - Bc = 1052 - 852{\cdot}5 = 199{\cdot}5 = bc$. The measured distance ab ought therefore to be $= \sqrt{ac^2 + bc^2} = \sqrt{858{\cdot}7^2 + 199{\cdot}5^2} = 881{\cdot}5$. By the field-book, it appears that line 12. has a length of 460 where it falls on line 3., and 1342 when it reaches line 1. The measured length of ab is, therefore, $= 1342 - 460 = 882$, exceeding the calculation by 0·5 links.

The same result may be obtained trigonometrically, by resolving the problem into the two propositions which form the 2d and 3d cases in trigonometry, p. 64. By Case III., the angle EBG is found from the three given sides; and by Case II. the side ab, or measured proof-line, is obtained from the given sides aB, Bb, and the included angle EBG.

slightly, let each furlong be increased or diminished in its due proportion. This precaution is intended to guard against the effects of the expansion and contraction to which paper is liable, especially if it has been pasted down on canvass. The errors arising from this cause, on a large survey, when proper means are not taken to compensate them, are often considerable; and, if combined with the improper practice of sweeping, or constructing internal triangles by new intersections, they can scarcely fail to occasion a great distortion of the plan.

The remaining lines in the field-book may now be laid down without difficulty by marking the points on the plan at which they begin and terminate, trying by the scale whether the distances between those points agree with the distances measured with the chain, and then ruling in the lines.

The order in which the lines succeed each other in the field-book need not be regarded in plotting, and can seldom be exactly followed. Thus, after laying down No. 8., we find that No. 9. terminates in No. 12., from which line also No. 10. commences. On turning to No. 12., we perceive that the position of this line is fixed by passing through two points already determined. The first of these is in the line No. 3., which line 12. reaches with a length of 460, where we find this memorandum in the right hand column, × on 1644 + 64 (No. 3.); which means, that the station 1644 in line 3. lay to the right of line 12., and that the distance from the intersection to the station 1644, measured along line 3., was 64 links; also, that, in consequence of the direction in which line 3. was measured, these 64 links must be *added to* 1644, to give the distance of the crossing from the origin of the line at G, viz. 1708 links. Had line 3. been measured in the opposite direction, viz., from B towards G, the distance, at the crossing, from its origin would have been *less* than at a station lying to the right of line 12., and the entry would have been 1644–64. The other fixed point through which this line passes is in line 1., where the length of line 12. is 1342 links: here we find it written × on 1140—27 (No. 1.), which means, that the crossing is on that point in line 1. which is 27 links *less* than 1140 (or 1113) from its origin at A. The same thing (viz., that the 27 links must be subtracted, not added), is also implied by the station 1140 being shown in the field-book *to the right* of the line. (See the calculated length of this line between the crossings of No. 3. and 1. in the note at page 232.)

Whenever one line crosses another without falling exactly on a station, a similar memorandum will be found in the field-book. The above explanation, it is hoped, will render every such case intelligible.

Line 12. being thus laid down in position, must be produced at both extremities. 460 links being measured towards the south gives the point at which it begins. Here 0 is written in the field-book, which always denotes that the line begins at a new station, and not at a point in a former line. Lines 1., 5., 6., 10., 11., 13. begin in the same manner. Set 0 on the scale to this point of commencement of line 12., or 460 to the first crossing, and see that the second crossing agrees with the length 1342 on the scale. Then mark off the distances 1052 and 1117, at the former of which No. 9. terminates, and at the latter No. 10. begins. Find the station 1744 on line 4. (measured from G towards D), and lay the scale from this point to 1052 on line 12. If the distance be found to agree with the book, viz. 1024, rule in the line on the paper. Write the number of the line against each, when it is laid down, as shown in the engraved plan.

No. 10., which begins at 1117 on No. 12., cuts, with a length of 850, the point in line 3. which is 2710 − 13 or 2697 links from its commencement at G. Find this point, and, if the distance from the former agree with the book (850 links), rule in the line, and produce it to the length of 1187, at which the last offset was taken. No. 11. must be produced southward 455 links from the crossing of No. 3., and No. 13. 306 links in a corresponding direction.

No. 14. terminates at the angle of a building, which had been previously fixed by an offset from No. 9. Let this offset (viz. 34 to the left of 472) be laid down, and line 14. may then be plotted.

No 18. begins at 300 links on line 5., from which it diverges, as the field-book indicates, 27° 25′ to the right; the angle being measured with the sextant. Lay the centre of the protractor on the given point in line 5., set 0° or 180° to the line, and mark a point at 27° 25′ towards the right, remembering that line 5. was measured from H towards A. Draw the line 18. at its proper angle, lay down the last station 470, and prove it by No. 19.

The last line, No. 25., having been laid down by an angle from No. 3., of 23° 37′, is proved by measuring from its extremity, at 560, 90 links + 132 = 222, in the direction of the fence, crossed by 162 on line 3.

Having thus laid down the lines, we proceed with the field-book, explaining whatever may present a difficulty in the plotting of the offsets.

Apply a scale, having its divisions on a bevelled edge, to the first line, setting 0 on the scale to the first station A ; and make a fine pencil-dot on the line at each of the lengths in the middle column of the book. From 0 measure back, in the direction of the line 20 links, where make a mark to show the edge of the river; 110 more, in the same direction, gives the width of the river. Then turn the scale at right angles to the line, and at 0 mark off, to the right, 50 for the fence, and 20 more (=70 from 0) for the edge of the water. At 61 draw a faint perpendicular line to the right, on which mark three points at 54, (54 + 26 =) 80, and (54 + 26 + 50 =) 130 links from 61 on the line. Join these points or offsets, as the sketching in the book directs. At 206 are also three points in the same perpendicular, the last being (30 + 30 + 50 =) 110 from the chain. The latter point or angle is better fixed by an offset from line 3.; 28 links, measured from the angle in the direction of the fence, falling on 2710 on that line, as indicated by the word op. (opposite). 403, crosses the fence into the road; an offset of 29 to the right reaches to the opposite fence. At 740 is an offset of 23 to the fence, on the left.

In order to lay down the fence opposite 778, look forward in the field-book to line 10., where we find that its extremity *b* is 15 links from that line, and points to the station 310. Draw a line, therefore, from 310 on No. 10. to 778 on No. 1., measure off 15 links from the former and 43 from the latter, and the position of the fence *ab* is accurately determined. The other side of the road is laid down by a line joining the distances 279 and 807, measuring off 14 links from line 10., and 25 from line 1. At 28 links farther, or (25 + 28 =) 53 from the line, is an abutment.

The cross fence pointing towards station 1840, also requires the assistance of another line, in order to plot it. At 273, on line No. 9., was an offset of 3 links to its other abutment. Join 273 and 1840; set off 3 links and 52, respectively. The remaining part should be equal to the length of the fence, which was found = 127 + 136 = 263 links. Mark these distances separately, and show the abutment which divides them.

At 3124 the line passes a house, which is distinguished in the sketch from an open court by the shading lines. In order to plot this building, lay down the two offsets from the chain of 40 and 35, and join them by a line. Let all the angles of the building be assumed as right angles (to which form they generally approximate), and connect the remaining sides, drawn to their respective dimensions, with the line first laid down. Mark an abutment of a fence against its eastern, and two others against its southern angle. Two out of these three abutments are completed by the first two offsets on line 2., where the house is again sketched in the book. From the offset at 340 draw a faint chord-line to the angle of the house, mark a point at 26 from the middle of it, through which draw a curve to join the two abutments. The third abutting fence is completed by an offset at 353 on line 16. This being drawn, measure 80 along it from the house, and mark a curving abutment. An offset of 32 perpendicular to 62, on line 8., gives the other abutment, and a line connecting them, having a bend of 20 towards the west, completes the fence.

Another case occurs at line 7., where several straight fences are laid down, by noticing the point in the chain-line which is *opposite* to each. Thus, a line joining 184 on No. 7., and 196 on No. 17., gives the position of a straight fence between the fields called Moon Close and Cornered Cap; 16 and 240 links are the distances of its abutments, from No. 17. and No. 7. respectively. From the abutment towards line No. 7. is a straight fence pointing towards 1560 on line 1., and terminating 7 links from it. From the same abutment, another straight fence points to 345 on No. 7., and terminates 60 links from the line. Draw in this fence; mark a point in it at 138 + 60 = 198 from 345; at 40 links farther (= 238) mark an abutment, which ought to be just 12 links from the angle where three fences meet. From this latter abutment, draw the line of a fence towards 523 on No. 5., and let it terminate at 32 + 30 = 62 from the point 523. The remaining offsets here are easily understood. After a careful perusal of these explanations, I hope that the whole of the field-memoranda will become sufficiently intelligible to the pupil. I would then recommend him to endeavour, by the aid of the field-book alone, to construct a complete map for himself; and refer to the engraving, in case of his meeting with any difficulty, which will immediately remove it.

The contents of the separate inclosures constituting the Manor of North Hill, are afterwards cast up on the plan. The number, name, state of cultivation, and statute measure of each inclosure are as follows: viz.

No.	Name.	State of Cultivation.	Statute Measure.		
			A.	R.	P.
1	The Manor House, Outbuildings, and Gardens	—	1	2	36
2	Lucern Paddock	Past.	0	2	38
3	Mount Pleasant	Past.	5	2	19
4	Thornhill	Past.	2	0	37
5	Wheathill	Ara.	2	3	13
6	Hawthorn Wood	Wood	1	1	36
7	Great Meadow	Past.	3	2	8
8	Farm Close	Past.	1	3	10
9	Farm House, Yards, and Outbuildings	—	0	1	29
10	Bridge Mead	Past.	3	3	12
11	Road Piece	Past.	0	2	17
12	Long Meadow	Past.	4	3	31
13	Orchard	Orch.	1	0	33
14	House and Garden	—	0	2	4
15	Cornered Cap	Past.	1	0	35
16	Moor Close	Past.	1	2	30
17	Stone Acre	Past.	0	0	36
18	Little Sand Pits	Past.	0	1	33
19	Paddock	Past.	0	1	18
20	Plantation	Plant.	1	0	19
21	New Close	Past.	0	3	25
22	Rack Close	Past.	0	3	33
23	Summer Lease	Past.	5	2	18
24	Oatlands	Ara.	3	0	8
25	Long Paddock	Past.	1	0	31
26	Sweet Nap	Past.	0	3	11
27	Nursery	Nurs.	0	0	30
28	Dwelling House and Gardens	—	0	1	14
29	Road Field	Past.	1	2	7
30	East Field	Ara.	1	1	31
31	Crossway Paddock	Past.	1	2	9
32	Three Cornered Ground	Past.	1	1	34
	* Total		55	2	25

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* See note p. 236.

NEW METHOD OF CASTING.

In addition to the general rule for casting the contents of fields, and the numerous directions and examples which Mr. Crocker has given at pp. 144. et seq., I subjoin the following method, which after many comparative trials, and many years' experience, I have found to be decidedly preferable, both for expedition and accuracy, to any of those which are in general use.*

Take a piece of transparent tracing paper, and rule it very accurately into squares of one chain each, by the scale to which your map is plotted. Lay this paper on any field in the plan which is to be cast, making one of the lines on the paper coincide with the longest straight fence, or straight part of a fence, which the field may have; or if the fence, though crooked, do not deviate far from a straight direction, you may substitute for it, in casting, one of the lines on the tracing paper, by so placing it over the fence, that the small spaces taken into the field, on one side of the line, shall be equal to those excluded from it on the other. If the field have two fences lying nearly at right angles with each other, a vertical and a horizontal line of the squares may be made to coincide with them both.

The tracing paper being kept in its place by a weight laid upon it (or rather by being pasted to the under side of a frame of lead), begin by counting the complete squares in the upper horizontal line; at the same time measuring with a pair of compasses the average width, either horizontal or vertical, of each fraction of a square, and adding them together by successive openings of the compasses. When the width to which the compasses are extended exceeds one or two inches, you may begin to close them, by deducting the complements or deficient parts of squares, remembering, whenever this is done, to reckon 1, as for an entire square.

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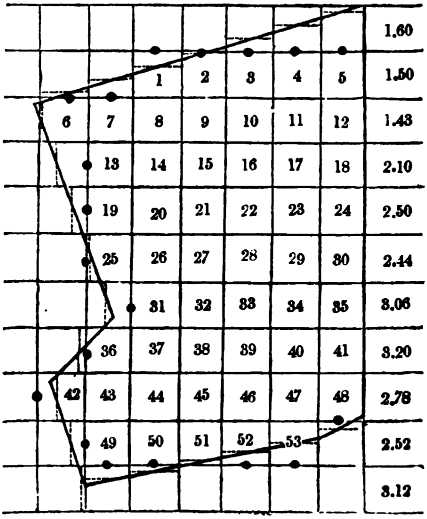
* A modification of this method is said to have been lately introduced and adopted at the Tithe Commission Office.

When you are arrived at the end of the field, measure from the scale the number of squares, and fractional parts of a square, contained in the compasses, add these to the number of complete squares counted, and the sum, divided by 10, is the area of the field in acres and decimals.



As an example of this method of casting, let us take the field called Summer Lease, No. 23., in the frontispiece, which is represented in the above figure with the squares laid upon it: the eastern fence, which is straight, coinciding with one of the lines of the squares.

Begin at the bottom of the square above No. 3., and extend the compasses, in a vertical direction, to the dotted line, which shows the mean width of that fractional part of a square. Still holding the compasses so as to measure vertically, set one foot on the dot at the bottom of the next square, the other foot resting in the square No. 4., and extend the compasses again to the dotted line, thus adding the mean width of the second fractional part of a square to the first. Add the width of the next fraction, which completes the upper line, and the sum, if measured by the scale, (which is not necessary,) will be found = 1·60 squares, or 160 links. Then descend to the second row, add the two fractional parts next above the squares Nos. 6 and 7., and *complete* the next square by closing the compasses so as to *deduct* the width of the deficient part, at the top of that square, for which you therefore reckon 1. Deduct also the small fraction wanting in the next, and count 2, then the complete squares, 3, 4, 5.; deducting a fraction, count 6, and so on to 12; in the next line add a large fraction, measured horizontally, and count on to 18. The width retained in the compasses may be seen at the end of each line, although it need not be measured until you come to the last square, when there will be 53 complete squares counted, and 3·12 more (or 312 links) in the compasses; making together 56·12 squares, or, (removing the decimal point one place to the left,) 5·612 acres = 5A 2R 18 P.

Every field should be cast a second time, as a necessary check on the first casting; and always with some variation in the mode, lest by falling twice into the same error, you suffer it to escape undetected.

If the second casting be made with the squares, vary their position, by making one of the straight lines coincide with a different fence of the field; or you may cast three or four fields together, as one, disregarding the fences which divide them, and compare the total content with the sum of the separate castings. When this kind of check is employed, the enclosures, which are measured together as one, ought not to differ from one another in size, by any very great disproportion.

Two successive castings of a field of three or four acres ought not to differ more than about one perch per acre, or the $\frac{1}{160}$th part. When the fields are large, the difference of the castings should be in a smaller proportion.

PART XI.*

DIVISION OF LAND.

One part of the Land-measurer's profession is that of dividing a piece of land between sundry claimants, in proportion to their respective claims. In some instances, the division relates only to the quantities that each claimant is entitled to; in others, the quality as well as quantity must be considered. In doing which, the measure of the whole must be first accurately ascertained, by some or other of the rules which have been heretofore laid down in this work.

For the more ready performing of such divisions, the following Table is inserted.

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* This chapter is reprinted from the former editions of this work, with a few slight alterations.

A TABLE,

Showing the Square Links in any number of Acres, Roods, or Perches.

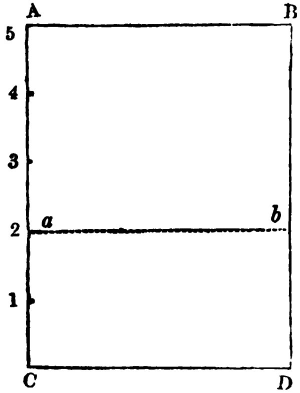
Acres.	Sq. Links.	Per.	Sq. Links.	Per.	Sq. Links.
1	100000	1	625	21	13125
2	200000	2	1250	22	13750
3	300000	3	1875	23	14375
4	400000	4	2500	24	15000
5	500000	5	3125	25	15625
6	600000	6	3750	26	16250
7	700000	7	4375	27	16875
8	800000	8	5000	28	17500
9	900000	9	5625	29	18125
10	1000000	10	6250	30	18750
20	2000000	11	6875	31	19375
30	3000000	12	7500	32	20000
40	4000000	13	8125	33	20625
50	5000000	14	8750	34	21250
60	6000000	15	9375	35	21875
70	7000000	16	10000	36	22500
80	8000000	17	10625	37	23125
90	9000000	18	11250	38	23750
100	10000000	19	11875	39	24375
		20	12500		
Roods.	Sq. Links.				
1	25000				
2	50000				
3	75000				

PROBLEM I.

To cut off a certain Portion of a Square or Parallelogram, parallel to one Side.

RULE.

If a part is to be cut off parallel to AB or CD, divide the other sides (AC and BD) in the proportion required, and draw the line of division from one point to the other, and it will be done; for parallelograms of the same altitude are to one another as their bases. (EUCLID, B. 6. prop. 1.)



EXAMPLE.

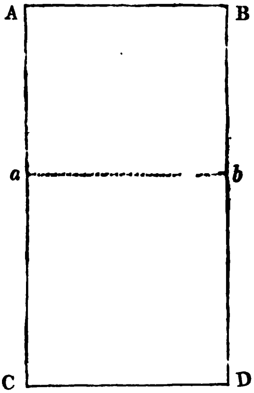
In the parallelogram ABCD, containing 5 acres, cut off a part parallel to CD, which shall be in proportion as 2 is to 5.

The sides AC and BD being divided into 5 parts, two of such parts will reach from CD to *a* and *b*. Draw the line of division from *a* to *b*. The scale being 4 to an inch.

If it were to be divided parallel to AC, or BD, it is evident that AB and CD must be previously divided in the proportion required.

RULE II.

If a parallelogram, whose length and breadth are given, is to be divided by cutting off any given quantity, parallel to one side, divide the square links in that quantity by the links in the side, and it will give the hight on the other sides, where the line of division is to be drawn.



EXAMPLE.

Two acres, 1 rood, and ten perches, are to be cut off from the parallelogram ABCD, parallel to CD; which is 460 links.

By the table of square links, page 241., we find

2 acres	=	200000
1 rood	=	25000
10 perches	=	6250
		231250

Then, $\frac{231250}{460} = 502{\cdot}7$ links; which set off from C and D to *a* and *b*, and draw the line.—Scale 4 chains.

Examples for Practice.

1. From the figure to Problem I., page 247., cut off a portion parallel to CD, which shall be to the whole as 3½ is to 5.

2. From the same figure cut off a portion parallel to AC, which shall be also to the whole in proportion as 3½ is to 5.

3. From the figure in page 248., cut off 1 acre, 2 roods, 17 perches, parallel to the side AC; supposing that side to be equal to 700 links.

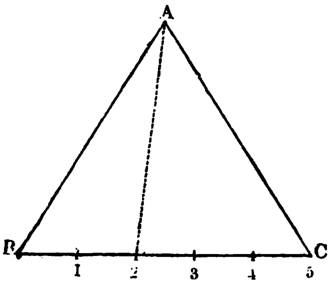
4. From the same figure, cut off 2 acres, 20 per. parallel to AB, which is 460 links.

PROBLEM II.

To cut off a Portion from a Triangle, in any assigned Proportion; by a Line drawn from the Vertex to the Base.

RULE.

Divide the base in the required proportion, and draw a line from the vertex to the division point, and it will divide the triangle, as required.



EXAMPLE.

To divide the △ ABC in proportion as 2 is to 3.

Divide the base into 5 equal parts; at two of these parts set up the dividing line to A.

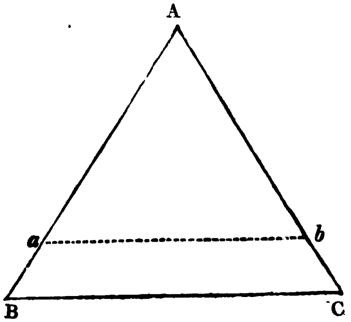
Example for Practice.

2. Take the same triangle, and divide it from the angle C into three parts, which shall be in proportion as 3, 4, and 6.

If it be required to cut off any quantity of land from a △, by a line parallel to its base, take the following

RULE.

Find the content of the whole triangle, and subtract from thence the quantity to be cut off.—As similar △s are in proportion to the squares of their like sides, the △ A*ab* will be to the △ ABC as the square of A*a* is to the square of AB. Then say, as the whole content : the square of the side AB :: the remaining quantity : square of A*a*. Extract its root, and the distance from A to *a* will be found.—Scale of 6 to an inch.



EXAMPLE.

1. Suppose the foregoing triangle ABC, contains 5 acres, and it be required to cut off 2 acres parallel to BC, and that the length of AB is 1060 links; what distance from A must be the point *a*? Scale 6 to an inch.

```
Acres.             Acres.
  5   :  1060  ::  3
         1060
        ------
          636
        1060
       -------
       1123600

       -------
    5)3370800
        674160(821 from A to a
        64
      ------
    162)341
         324
      -------
    1641)1760
          1641
        ------
```

Examples for Practice.

2. Suppose from the same △ 2 acres were to be cut off parallel to AB, and that BC is 1110 links; in what point of BC must the line of division begin?

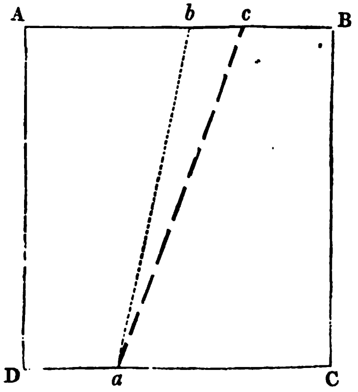
PROBLEM III.

To cut off from any assigned Point in the Side of a Field any number of Acres, &c.

RULE.

Set off, from the given point as nearly as can be guessed, the quantity proposed; and accurately measure the quantity to set off.

Find the difference in square links between the quantity proposed and the quantity set off; measure the length of the guess line, and take the half thereof, for a divisor to the square links of the difference; and the quotient will be a perpendicular to set off, on one side or the other of the guess-line, as the quantity found may be more or less than the quantity proposed. To that perpendicular draw a new line from the point assigned, and it will cut off the quantity required.



EXAMPLES.

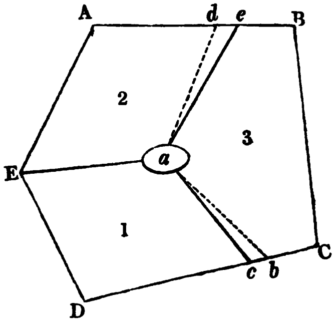
1. From the point a, in the figure ABCD, it is required to set off 5 acres, 1 rood, 15 perches (which by the table = 534375 square links) towards the side AD; scale, 6 chains to an inch.

First draw, by guess, the line ab, and measure the quantity $AbaD$, which will be found to be only 442950 square links; being too small a portion by 91425 square links. This sum divided by 565 (the half of the line ab) will give 162 for the height of the perpendicular, from *b* to *c*. Thus *ac* is found to be the line of division.

2. The field ABCDE, which contains 6 acres, 3 roods, 28 perches, is required to be divided equally between three claimants; in such manner that each may possess the benefit of the pool of water therein; scale, 6 chains to an inch.

By the table, 6 acres, 3 roods, 28 perches, =692500 square links.

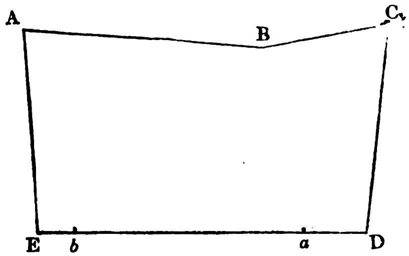
692500 ÷ 3 = 230833, for each share.



From *a* set off a line to E, and also a guess line to *b*; the content of the trapezium ED*ba* will be found 233750; too much by 2917; which excess, divided by 225 (half *ab*), will give 13 for the perpendicular from *b* to *c*. The trapezium E*ac*D will be the first share.

From *a* set off a guess line to *d*; the content of 2 will be found = 225250; too little by 5583; which divided by 250 (half *ad*), will give 22 for the perpendicular from *d* towards *e*; the trapezium EA*ea* will be the second share.

And the five-sided figure 3, will be the remaining share; = ·230833, as will be found by casting it.



Examples for Practice.

1. In the field ABCDE (in the above *fig.*) set off 1 acre, 1 rood, on the side CD, by a line from *a* to BC.

2. In the same figure set off 3 roods, 27 perches, from *b*, on the side AE, by a line to AB. Scale 6 chains.

PROBLEM IV.

To divide a piece of ground, of uniform value, between two or more claimants in proportion to their respective claims.

RULE.

As the sum total of the proportional claims : to the whole quantity of land to be divided :: each respective proportional claim : its respective share.

EXAMPLES.

1. Divide 50 acres of land between A and B; the claim which A has being as 7, and that of B as 9.

	Acr.			Acr. rds. per.	
As 7 + 9 = 16 :	50	:: 7 :		21 3 20	for A's share,
Then, as 16 :	50	:: 9 :		28 0 20	for B's share,
				50 0 0	

2. Divide 108 acres, 2 roods, 20 perches, between A, B, and C., in proportion to the value of their respective estates; that of A being 150*l.* per annum; that of B 120*l.*; and that of C 340*l.* per annum.

The sum total of the claims is as 610*l.*

The quantity (by the foregoing table) is 10862500 square links.

Links.
Then, as 610*l.* : 10862500 :: 150*l.* : 2671106 links, or 26A. 2R. 34P. for A.
And, as 610 : 10862500 :: 120 : 2136885 links, or 21A. 1R. 19P. for B.
And also, as 610 : 10862500 :: 340 : 6054508 links, or 60A. 2R. 7P. for C.

108A. 2R. 20P.

Examples for Practice.

1. Divide 15 acres of land between F, G, and H, whose claims are in proportion as 19, 37, and 43.

2. A, B, and C, have estates of the respective values of 800*l.*, 500*l.*, and 440*l.*, per annum, and have a right of commonage in a piece of waste of 176 acres, 2 roods, 35 perches. What are their respective rights, on a division thereof?

3. Thirteen commoners, whose estates are as below, are to divide 150 acres, 1 rood, 19 perches, between them.

A's farm	*L.* 240	H's farm	*L.* 100
B -	200	I -	95
C -	180	K -	90
D -	140	L -	8
E -	130	M -	7
F -	120	N -	6
G -	110		

What is each person's share?

PROBLEM V.

To divide a piece of common which is not of uniform value, between sundry claimants, in proportion to their several claims, regard being had to the quality of the respective allotments.

RULE.

Divide the numbers which express the proportions of the claims, by the sums denoting the qualities of the land which each claimant is to take. Then, as the sum of the quotients thus obtained : the whole common :: each respective quotient : the quantity thereto belonging.

EXAMPLE.

It is required to divide a common of 195 acres, between A, B, and C; A's claim being for his estate of 100*l.* per annum, B's claim being for 300*l.* per annum, and C's claim being for 400*l.* per annum.—Moreover, A takes that part of the common which is worth but 5*s.* per acre; B, that which is worth 8*s.* per acre; and C, that which is worth 10s. per acre.

A, 100 ÷ 5 = 20
B, 300 ÷ 8 = 37·5
C, 400 ÷ 10 = 40
97·5 Sum of tne quotients.

Then, for A, as 97·5 : 195 :: 20 : 40
For B, as 97·5 : 195 :: 37·5 : 75
And for C, as 97·5 : 195 :: 40 : 80
195

Example for Practice.

Divide a common of 140 acres, 2 roods, between H, I, K, and L; whose estates, on which their claims are founded, are respectively 300*l.*, 250*l.*, 70*l.*, and 45*l.* per annum: the land allotted to the several claimants being also of the respective values of 15*s.*, 14*s.*, 16*s.*, and 13*s.* per acre.

PROBLEM VI.

The foregoing Problems include the principles of the whole which has been hitherto published on the subject; but as it seldom happens that a common is of uniform quality, or that the respective qualities extend exactly so far as the share which each claimant is entitled to, land-surveyors frequently find it expedient to allot land of different qualities to one claimant; in which case it is necessary to adopt the following

RULE.

Measure and plot each portion of the common, according to the various qualities thereof; and let the surveyor set the respective annual valuations on those parcels, and from thence find the annual value of the *whole common.*

Let the annual values of *all the estates,* which have rights in the common, be ascertained.

Then, as the *annual value of all the estates* : the the annual value of the *whole common* :: each *respective estate* : its *annual share* of such common.

Again, as the price per acre of each portion of the common : 1 acre :: each *annual share* : the *quantity* due thereunto.

The quantities, thus found, must be set off (by Problem I, II, or III.) in such parts of the common as may be most eligible to the estates which they belong to.

> *Note.* If one portion of the common be of less quantity than a claimant's share, a part of the adjoining (or some other) portion, of different value, must be taken, for completing such claimant's share.

EXAMPLE.

It is required to divide, between four claimants, a common of the form in the next page*, which is of three different qualities. The portion *a*, being 5 acres, 1 rood, 24 perches, valued at 16*s.* per acre; the portion *b*, being 6 acres, 1 rood, 8 perches, valued at 14*s.* per acre; and the portion *c*, being 6 acres, 3 roods, 0 perches, valued at 12*s.* per acre. The claimants' estates being as follows; viz.

A's	farm	200*l.*	per annum.
B's	farm	150	
C's	estate	80	
D's	lands	30	

The portions and values of the common are as follow; viz.

	Quantities.					Values.			
	Acr.	rds.	per.			*l.*	*s.*	*d.*	
a	5	1	24	at 16*s.*	=	4	6	4¾	per annum.
b	6	1	8	at 14*s.*	=	4	8	2¼	
c	6	3	0	at 12*s.*	=	4	1	0	
Total	18	1	32		Total	12	15	7	

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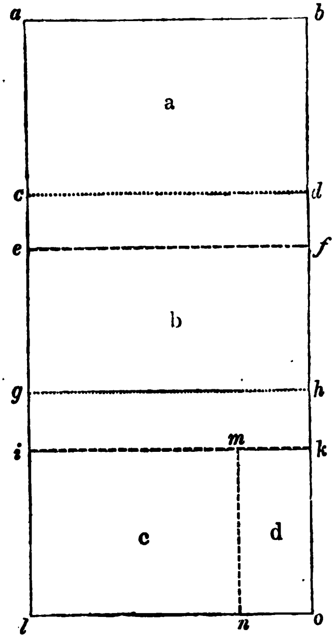
* Drawn by a scale of 6 chains.



Calculations for A's share.

Total of all the Estates.		Total value of the Common				A's farm.		Val. of A's sh.		
l.		*l.*	*s.*	*d.*		*l.*		*l.*	*s.*	*d.*
As 460	:	12	15	7	::	200	:	5	11	1¼
Value of the portion *a*								4	6	4¾
Value of the quantity to be taken from *b*								1	4	8½

Val. per acre.		Acre.		*l.*	*s.*	*d.*		Acr.	rds.	per.
And as 14s.	:	1	::	1	4	8½	:	1	3	2¼

	Acr.	rds.	per.		Sq. links.				Length of *cd*.	
Then,	1	3	2¼	=	176250	÷	900	=	195¾	for

the perpendicular from *c* to *e*.

Calculation for B's *share.*

									Val. of B's share.		
	l.		l.	s.	d.		l.		l.	s.	d.
As	460	:	12	15	7	::	150	:	4	3	4

	Acr.	rds.	per.
The portion *b*, measured	6	1	8
That part taken to complete A's share. - -	1	3	2¼
Remains -	4	2	5¾ at 14*s*. =

3*l*. 3*s*. 6*d*. This taken from the value of B's share, (4*l*. 3*s*. 4*d*.) leaves 19*s*. 10*d*. to be taken from portion C.

	s.		Acre.		s.	d.		Acr.	rds.	per.
Then, as	12	:	1	::	19	10	:	1	2	24½

Acr.	rds.	per.		Sq. Links.				
1	2	24½	=	165312	÷	900	=	184, nearly,

for the perpendicular from *g* to *i*.

Calculations for C's *share.*

									Val. of C's share.		
	l.		l.	s.	d.		l.		l.	s.	d.
As	460	:	12	15	7	::	80	:	2	4	5½

	s.		Acre.		l.	s.	d.		Acr.	rds.	per.
Then, as	12	:	1	::	2	4	5½	:	3	2	33

Acrs.	rds.	per.		Sq. Links.				
3	2	33	=	370605	÷	566*	=	654¾, for the

perpendicular from *i* to *m*.

Calculations for D's *share.*

									Val. of D's share.		
	l.		l.	s.	d.		l.		l.	s.	d.
As	460	:	12	15	7	::	30	:	0	16	8

	s.		Acre.		l.	s.	d.		Acr.	rd.	per.
Then, as	12	:	1	::	0	16	8	:			22½

Acr.	rd.	per.		Sq. Links.				
1	1	22	=	138906	÷	566	=	245½, for the

perpendicular from *n* to *o*.

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* 566 is the length of the line *mn*.

PROOF.

	Quantities.			Values.		
	Acrs.	rds.	per.	*l.*	*s.*	*d.*
A's share, in *a*	5	1	24	4	6	4¾
Ditto, in *b* (to *cf*)	1	3	2¼	1	4	8½
B's share, in *b*	4	2	5¾	3	3	6
Ditto, in *c* (to *ik*)	1	2	24½	0	19	10
C s share, in *c*	3	2	33	2	4	5¾
D's share, in *c*	1	1	22½	0	16	8
	18	1	32	12	15	7

PART XII.*

AN EXTRACT FROM MR. CROCKER'S SUPPLEMENT, CONTAINING THE FIELD PRACTICE, WITH THE CHAIN, ETC.

[At the solicitation of several respectable Teachers, into whose academies this work has been introduced, I here subjoin some of the most general instructions for field-practice; thereby endeavouring to render the whole work more pleasurable to their young Students.]

LAND-MEASURING, BY THE CHAIN

AND ITS ACCOMPANIMENTS.

The accompaniments to a measuring-chain are, an off-set staff, or rod, shod with pointed iron at one end, and a small crook at the other, of the length of ten links, divided into ten equal parts; by which short distances may be measured more readily than by the chain itself: to which may be affixed occasionally (by an arm fitted thereto) a rectangular cross, for the purpose of observing points which are perpendicular to any general line. Small picket staves (with white paper on their tops), of five or six feet in length, to be set up at or near the angles of a field, are also accompaniments. So also are ten arrows of strong wire, pointed at one end, and turned to a ring at the other, for marking the chain's length on the ground, as well as counting the number of chains in a line.

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* A few alterations only have been made in those parts of the Author's Supplement, which are here retained.

Preparatory to the measuring of lines in any field, it may be necessary for the surveyor to set up his pickets at the angles of the field, or at some more convenient points, and to observe whether or not there be any flexures, or small indentations, in the fences which lie near his general measuring lines.

Having determined on such general lines as are necessary to be measured, and made a sketch thereof in his field-book, he observes, at his first station, the distance, if any, which he is at from the fence or boundary of his field, and notes the number of links on the proper side of his sketched line. Having thus done, and standing at his first picket, he sends his assistant forwards towards the next picket, with the fore end of the chain and one arrow in his right hand, and the others in his left; who having gone his chain's length, as nearly in a straight line as he could guess, towards the next station, holding his arrow perpendicularly at the end of the chain, looks back to the measurer for instructions; whose business it now is to direct the chainman to the one side or the other, by waving his left hand, until he be exactly in a line with the picket to which he is measuring; then, straining the chain so as just to prevent it from touching the ground, he stoops, and brings the handle of the chain into contact with the picket; and instantly afterwards the leader also stoops and thrusts his arrow into the ground, placing it exactly opposite the middle of his feet. Having done so, he proceeds forwards another chain's length, where he will be enabled readily to bring himself into a proper direction, by placing himself in a line with the last arrow and the first picket-staff; but still observing and obeying any signal which the master-measurer may give; thus proceeding until the chain-man is arrived at the end of the line; when the measurer must count the number of arrows in his hand as so many chains, or hundreds of links, to which he must add the odd links, between the last arrow and the picket or end of the line. Should the line be more than 10 chains, the assistant waits at the 10 chains' end until the measurer comes up to him, who gives him all the arrows again, makes a memorandum of his 10 chains; and then proceeds as before.* If there be any distance between this picket and the boundary of the field, the measurer must note the number of links, as was directed at his first station. If there should be any bends, gates, stiles, or other objects in the boundary near which the measurer passes, worthy of notice, he should (by his offset staff, or otherwise) measure off to them as he proceeds, provided, in general, that their distances do not exceed a chain, and set down their distances from the general line, and also note down the points of the line to which they were perpendicular. The points of these perpendiculars may usually be found with sufficient accuracy, by holding the offset-staff, as nearly as the eye can judge, at right angles with the chain, the hand being held immediately over it, then moving a step or two either to the right or to the left, until the staff point straight to the object. When the perpendicular considerably exceeds a chain in length, or when greater exactness is required, its place may be found by fixing the staff and cross in the ground close by the chain, and directing one pair of its sights to either picket, and observing if the cross sights are then directed to the bend, indentation, or object. If not, the cross must be moved forwards or backwards until it be placed at the point directly opposite to the bend, &c.

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* An error is frequently committed on exchanging the arrows, when only ten are used in measuring, the leader being apt to go forward, after receiving the arrows, with nine only in his hand, leaving the tenth in the ground. *Eleven* is a more convenient number of arrows, and not so likely to cause a mistake. When this number is used, the leader having put in the eleventh arrow, waits until the measurer comes up, receives *ten* arrows, and then goes forward, the eleventh remaining always in the ground until the next chain is measured.—T. G. B.

In like manner must a measurer proceed with all other lines which may be necessary for obtaining the content of any field. How such content is found by the pen has been fully explained in p. 148. and subsequent pages of this work.

After the surveyor has made a fair plot of his work, he will have to draw a line to represent the true meridian, to the eastward of the magnetic meridian, according to the variation of the needle, where the estate lies.* On this true meridian line he must place

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* *To find the Variation of the Needle.*

Find a meridian line, either by observation of equal altitudes of the sun, by the theodolite; or by concentric circles. Let these be drawn on some smooth level surface open to the sun; place an upright pin or wire of about four inches in height, in the centre of the circles; observe in the forenoon, when the shadow of the top of the wire touches either of the circles, and there make a mark; observe also when the shadow touches the same circle in the afternoon, and make another mark. Draw a line from one mark to the other, and it will be an east and west line. Bisect this line with another, perpendicular thereto, and a *fleur de lis*, or some other device, as a north point. He will also have to give a title to his map: to draw a scale of the proportion he has plotted by; and to give the whole a margin line.

As the measuring of a town or village frequently occurs in the practice of a surveyor, I deem it expedient to give the student a particular example; taking, for that purpose, a part of the city of Bath.*

Having taken an ocular survey of his intended work, the surveyor must choose some convenient spot, whereat to place his instrument to advantage; from whence he may look *into*, if not *through*, several streets; in the present instance, let it be supposed to be that part of the city named the *Circus*.†

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this last will be the true meridian, or real north and south line. Remove the pin or wire, and in its place put another pin, very short and nicely pointed; on which place the needle of the compass, and let it rest. Make a mark on the plane over which the north end of the needle rests, and from thence draw another line to the centre. The angle formed by these lines is the variation of the needle.

Otherwise,

By the help of some annual ephemeris, find the sun's right ascension, in time, for the day of observation, and also the right ascension of the polar, or other known star for that year. Then from the right ascension of the star (adding 24 hours, if need be,) subtract the right ascension of the sun. The difference will show the time when the star will culminate, or be on the meridian. Having the theodolite nicely adjusted, especially with respect to the limb and the needle, set the telescope by the limb, to observe the star at the time of its culminating; and the limb will then precisely show the variation of the needle.

* See Plate.

† The measurer should be at his street-work long ere the votaries of pleasure and dissipation are awake, or the bustle of commerce is begun; that neither the prying eye of idle curiosity, nor the "busy hum of men," may interrupt his progress.

Here, having fully adjusted his theodolite, he takes his first observation down through *Gay Street,* which he finds to bear from the north end of his needle 190° 30′. Measuring down this street, making all necessary offsets, entering into his survey-book the breadth of each house, &c. as he goes on, until he comes to *a,* opposite *George Street,* he will find his line to be 740 links.

Hither he brings his theodolite, and adjusts it, as at an original station, by setting the needle and limb to their respective 360°, checking the angle by the back station: taking here his observation through *George Street,* he finds the bearing to be 101° 15′ from the north point; measuring along this line, making offsets as circumstances require, he finds that from *a,* in *Gay Street,* to *b,* by *York Buildings,* the general line is 700.

At this ⊙ *b,* he brings his theodolite, and adjusts it, as before directed, and takes his observation to ⊙ *c,* in *Bennet Street,* the bearing of which he finds to be 7° 15′. This line he measures (taking the necessary off-sets as he proceeds) and finds the length thereof to be 970.

At ⊙ *c,* he finds the bearing to ⊙ *d,* near an entrance to the *Circus,* to be 270° 25′, and the length 335.

At ⊙ *d,* he takes the bearing to the centre of the *Circus* (from whence he set out,) and finds it to be 254° 15′, and the length 370.

Having finished this circuit, he may proceed from ⊙ *a,* in *Gay Street,* on the continuation of his first line to *e,* 180 links forwards, where a small street branches off, at an angle of 282 N. W., which he has now to measure in manner before directed.

At *f*, 320 further forward in his first line, he will find himself in front of the upper part of *Queen's Square;* this line also bearing 282° N.W., is now to be measured.

Proceeding 360 links further, he comes to the lower part of the square, which line, at the right, also bears 282° N.W.

When at ⊙ *f*, in front of the upper part of *Queen's Square*, he was also opposite to *King Street*, on the other hand, which bore 102° S.E. He may now proceed to the measuring of this street, noting every particular offset in his progress, as before directed. So also must he proceed with every other street comprehended in his survey: to advert to the particulars of all which, in a work of this kind, would be deemed a tedious repetition.

Having, on his survey-book, all the street-work, wherein the front of each house is noted down, he must go within, and set off the back of each house, and its garden-plot, in which he would do well to have a light chain of 50 links only; or a tape line and box, divided into links.

The survey of a large tract of country, as a county, &c., must be conducted on similar principles; the surveyor of which would, however, do well in making frequent observations with his theodolite, from various stations, on particular distant objects; the which he may plot off by intersections, or by some of the rules of inaccessible distances, laid down in a former part of this work.

By distant objects are meant such as are found within the limits of the survey; as towers, castles, beacons, windmills, remarkable houses, &c. These being set off with the plotting instruments, as before directed, will serve as useful checks to the other parts of the work. In the progress of the road-survey of a county, regard must be had to the passing of every hill, dale, wood, park, brook, river, bridge, ford, sea-coast, rock, sand, and other objects deserving of notification; from all which a faithful delineation of the face of the country is to be drawn.

In all these points, nothing can exceed the accuracy and elegance of the general survey of the kingdom, now carrying on by the honourable Board of Ordnance, and placed at first under the direction of General Roy, afterwards of Colonel Mudge, and now of Colonel Colby; an undertaking that will constitute a monument of the improved state of our practical science, and reflect high honour on the abilities, the industry, and the taste of those who are engaged in it.

Thus, by precepts and examples, have I led the pupil through the various parts of land-measuring; and may his judgment, his industry, and his integrity, go hand-in-hand through his whole practice, and so entitle him to a liberal remuneration of his labours.

PART XIII.

(BY THE EDITOR.)

ON LEVELLING.

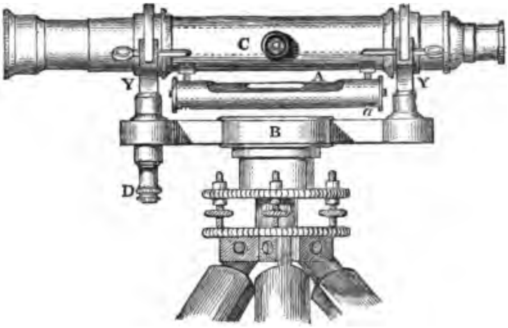
INSTRUMENTS USED IN LEVELLING.

THE two principal instruments required in levelling are, a spirit-level, or glass tube, partially filled with alcohol, or some other fluid, and attached to a telescope, having fine hairs or wires stretched across its focus; and a levelling staff divided into feet and hundredths, and furnished with a sliding vane, for marking the height at which the telescope points, and measuring the difference of level between that instrument and the station on which the staff is rested.

The spirit-level differs in nothing essential from the telescope with its attached spirit-tube, which have been already mentioned (at page 191.) in the description of the theodolite. That instrument is, in fact, frequently employed as a substitute for the spirit-level, although it is, in general, very inferior to it in steadiness, power, and accuracy.

Figure 1. represents the most common form of the spirit-level, which is denominated the Y level, from the forked or angular shape of the two supporters of the telescope. A is a glass tube, partly filled with spirit, the bubble or space occupied with air, indicating, by its position in the middle of the tube, the hori-

Fig. 1.



zontality of the instrument. *a* is a screw for making the visual axis of the telescope parallel to the glass tube. B is a compass-box, which is useful for taking the bearings of the course of levelling, although it is not essential to the operation itself. C is a milled-head, by turning which the object-glass is adjusted to its proper distance from the cross-wires, dependent on the distance of the object under observation. Y Y are the two supporters of the telescope, one of which admits of a vertical motion, by turning the milled-head D.

Fig. 2.

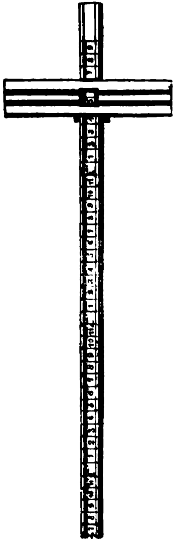


Figure 2. represents the common levelling-staff and vane. The staff is usually made of two pieces of mahogany, one of which slides on the other by means of a dove-tailed groove, so as to admit of being drawn out to a length of about eleven feet.

The face of the staff is divided from the bottom upwards into feet and hundredths as far as six feet, which is the usual length of the staff before being drawn out; the graduations are afterwards continued on the back of the staff downwards.

The vane is made to slide up and down the staff, with just sufficient friction to retain it in any situation in which it may be placed. A white horizontal line, bounded by two black ones, is placed across the middle of the vane, as the mark to be bisected by the horizontal wire of the telescope in taking an observation. In the centre of this line is an aperture, through which the divisions of the staff may be seen, for the purpose of reading off the height of the vane.

There are three principal adjustments required in the spirit-level itself, besides that which must be made in taking an observation.

The first is to make the line of collimation, or optical axis, coincident with the axis of the brass tube of the telescope. This is necessary in order to adapt it for observing objects at very different distances from itself. When the object viewed is near the telescope, the divergency of the rays incident on the object-glass causes the image to be formed farther back towards the eye than when they come from a greater distance, and are consequently more nearly parallel. The object-glass must be moved outwards, by turning the milled-head C when the object is a near one, and in the contrary direction when it is a remote one, in order to bring its image into contact with the cross wires, which ought to be made quite perfect in every observation. It is necessary, therefore, that the intersection of the cross wires, and the optical centre of the object-glass, which determine the line of collimation, should be coincident with (or at least parallel to) the axis of the brass tube in which the object-glass is made to slide, in order that the parallelism of the line of collimation may not be affected by the motion of the object-glass.

To make the adjustment for collimation, draw out the eye-piece until the cross wires are distinctly seen, direct the telescope to some rather distant object, and notice the point with which the intersection of the wires coincides. Turn the telescope half round its own axis, in the Y's or supporters, and if the wires still cut the same point the adjustment is perfect; if not, turn the screws, near the eye-end of the telescope, which move the cross wires, until they are brought back half way towards the object, and the error will be corrected, provided the half of the deviation has been exactly estimated, which may be known on repeating the trial.

The next adjustment is, to set the line of collimation parallel to the spirit-tube, so that it may be exactly horizontal when the bubble settles in the middle of the glass, on which two scratches are made to mark the position of the extremities of the bubble when of its ordinary length, or when the air is at a mean temperature.

This adjustment may be approximately effected in the Y telescope, by merely reversing its position in the Y's, and correcting one half of the error of the bubble, on change of position, by the parallel plate screws, and the other half by the screw (*a*) at the end of the tube.

It will be much better, however, to make the final adjustment, after the telescope is firmly fixed in its supports, by pressing it down tightly with small elastic plugs or wedges, introduced under the semicircular staples by which it is confined in the Y's. This may be done in different ways, one of which is as follows: — Drive into the ground, say at the distance of three chains from each other, two wooden pegs as stations, distinguishing them by the letters *a* and *b*. Place the level exactly midway between them, and adjust the bubble to its central position in the tube by means of the parallel plate screws. Let the levelling staff be held by an assistant over each peg in succession, and the vane elevated or depressed, until its central line or mark be exactly bisected by the horizontal wire of the telescope. Let the reading of the staff at the station *a* be supposed = 2·07 feet, and at *b* = 3·07; the difference, = 1 foot, is the height of *a* above *b*. The want of exact adjustment in the level is in this case of no consequence; for since it is equidistant from the two stations, its error will either increase or diminish both readings equally, so that their difference will not be at all affected. Then place the level at a small distance from one of the stations, say at 30 links from *a*, and read off the staff, first on *a*, and then on *b*. If the difference of the readings remain the same as before, the level is correctly adjusted. Supposing the case to be otherwise, and that the reading on *b* is 3·16, while that on *a* is only 2·00, the error (0·16 too much at the farther station) shows that the telescope points upwards, and that the eye-end must be raised by turning the screw (*a*) which connects it with the spirit-tube. The amount of the correction may be found at once by this proportion: —

As the difference of the distances from *a* and *b* (270 − 30) = 240,

Is to the difference of the errors at *a* and *b*=0·16,

So is the whole distance from *b*=270,

To 0·18, the whole error made by the instrument in reading the staff on the station *b*, at the distance of 270 links.

Let the vane be lowered that quantity, or set at 3·16 − ·18=2·98, and again held over the station *b*. Also let the eye-end of the telescope be raised above the spirit tube, by turning the screw (*a*), until the horizontal wire cut the centre of the vane, when the bubble stands in its true situation, and the level will be in adjustment. The reading at *a* will now be found reduced from 2·00 to 1·98, giving 1 foot as the difference of level of *a* above *b*, as at first. Then examine the adjustment, by repeating the observations with the distances nearly reversed, that is, with the instrument placed near the station *b*. If the reading at *b* still exceed that at *a* by 1·00, the adjustment may be considered as perfect.

The third adjustment is to set the telescope at right angles to the vertical axis, so that in turning it horizontally round, the position of the bubble shall remain unaltered. Adjust the bubble by the parallel plate screws, turn about the telescope so as to reverse the positions of the object-end and eye-end of the instrument, and if the bubble deviate towards either end, correct half the error by the parallel plate screws, and the remainder by turning the milled-head D, which connects the telescope with the horizontal bar underneath.

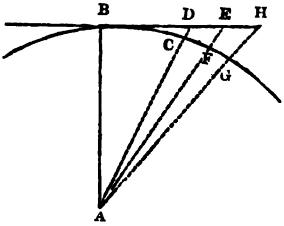
PRINCIPLES AND PRACTICE OF LEVELLING.

LEVELLING may be defined as the art of carrying a line over the surface of the earth in a horizontal direction; or in a plane parallel to the surface of the sea, or of any extensive body of water, when in a tranquil state, and subject to no other influence besides that of terrestrial gravity.

In the practice of levelling a series of such lines are taken, the lengths, bearings, and vertical distances of which from each other are varied in any manner which the nature and inequalities of the surface levelled over may render necessary.

Two or more places are on a true level, when they are equally distant from the centre of the earth. Also one place is higher than another, or out of level with it, when it is farther from the centre of the earth: and a line equally distant from that centre, in all its points, is called the *line of true level*. Hence, because the earth is round, that line must be a curve, and make a part of the earth's circumference, or at least be parallel to it, and concentrical with it; as the line BCFG, which has all its points equally distant from A, the centre of the earth, considering it as a perfect globe.

But the line of sight BDEH, given by the operation of levels, is a tangent, or right line perpendicular to the semidiameter of the earth at the point of contact



B, rising always higher above the true line of level the farther the distance is; and is called the *apparent line of level.* Thus, CD is the height of the apparent above the true level, at the distance BC or BD; also EF is the excess of height at F, and GH at G, &c. The difference, it is evident, is always equal to the excess of the secant of the arch of distance above the radius of the earth.

Now the difference CD between the true and apparent level, at any distance BC or BD, may be found thus: — By a well-known property of the circle 2AC + CD : BD :: BD : CD; or because the diameter of the earth is so great with respect to the line CD, at all distances to which an operation of levelling commonly extends, that 2AC may be safely taken for 2AC + CD, in that proportion, without any sensible error, it will be, As 2AC : BD :: BD : CD, which therefore is $= \frac{BD^2}{2AC}$, or $\frac{BC^2}{2AC}$ nearly; that is, the difference between the true and apparent level is equal to the square of the distance between the places, divided by the diameter of the earth; and, consequently, it is always proportional to the square of the distance.

The diameter of the earth being nearly 7958 miles; if we first take BC = 1 mile, then the excess $\frac{BC^2}{2AC}$ becomes $\frac{1}{7958}$ of a mile, which is 7·962, or almost 8 inches, for the height of the apparent above the true level, at the distance of 1 mile. Hence, proportioning the excesses in altitudes, according to the squares of the distances, the following table is obtained, showing the height of the apparent above the true level, for every 100 yards of distance, on the one hand, and for every mile on the other.

Dist. or BC.	Diff. of Level, or CD.	Dist. or BC.	Diff. of Level, or CD.	
Yards.	Inches.	Miles.	Feet	In.
100	0·026	0¼	0	0½
200	0 103	0½	0	2
300	0·231	0¾	0	4½
400	0·411	1	0	8
500	0·643	2	2	8
600	0·925	3	6	0
700	1·260	4	10	7
800	1·645	5	16	7
900	2·081	6	23	11
1000	2·570	7	32	6
1100	3·110	8	42	6
1200	3·701	9	53	9
1300	4·344	10	66	4
1400	5·038	11	80	3
1500	5·784	12	95	7
1600	6·580	13	112	2
1700	7·425	14	130	1 *

The above table shows the correction for reducing

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* Hutton's Mathematical Dictionary, art. LEVELLING.

the apparent to the true difference of level, depending on the distance of the station from the levelling instrument. Thus, if the reading on the staff, or height of the vane, at the distance of 500 yards, be 5 feet 6 inches, it must be reduced by subtracting 0·643 inches, which gives 5 feet 5·357 inches for the true height on the staff, or that which is on the same line of true level with the telescope.

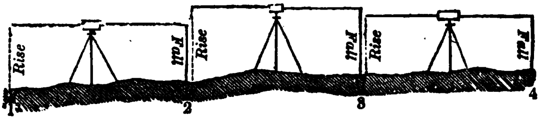
In the operation of levelling it is usual to drive into the ground a number of wooden pegs, along the line to be levelled over, as stations on which the staff is to be successively rested. The distances of the pegs from each other may be varied according to circumstances, such as the inequalities of the ground, the power of the telescope, and the degree of accuracy and minuteness of detail which may be required in the operation. When the ground is so nearly level that the lengths of the sights may be chosen at pleasure, an interval of from 6 to 10 chains between the pegs will usually be found most convenient. In every instance in which it is practicable, the place of the level should be midway between the station-pegs, so as to make equal distances from the staff to the instrument, at the back and the fore observation. By attending to this very important rule, which is too much neglected in general practice, all the instrumental errors arising from imperfect adjustment, whether in the line of collimation itself, or in the parallelism of that line with the spirit-tube, are entirely destroyed; the effects of curvature and refraction are also much better compensated than can be done by the introduction of tabular corrections.

The first two station-pegs having been driven into the ground, and the level placed between them, let the assistant hold the staff perpendicularly on the first station-peg; then level the telescope for two positions at right angles to each other, so that the bubble may remain nearly in its place while the telescope is turned in any direction. Direct the assistant by signals to raise or lower the vane, until its central line is bisected by the horizontal wire. The vane is then to be clamped by a tightening screw, and the staff and vane brought forward to the observer at the level, who reads off the height in feet and hundredths, and inserts it in his book, in the column headed "Back station." The number of the station, its compass bearing, and distance in links from the level, are also noted down in separate columns.

The assistant now goes on, and sets up the staff on the second station-peg; the telescope is turned about so as to point to the staff, and again accurately adjusted for level. The fore observation being made by again bringing the vane to agree with the horizontal wire, and the bearing and distance written down, the level is taken up by the principal observer, and carried forward to the fore station, where he reads off the height of the vane, and inserts it in the column headed "Fore station." He then proceeds and sets up the level between stations Nos. 2 and 3., the assistant remaining with the staff still resting on station 2., and merely turning it round to face the telescope, which is now adjusted for a second back observation.

The process which has just been described is again repeated, and the same is continued throughout the whole extent of the line. The reading of the staff at the first station is always a back observation; that at the last, a fore observation. Every remaining station is made both a fore and a back station successively, and consequently affords two readings, to be inserted each in its appropriate column.

This system of observation may be further illustrated by the following diagram, in which the pegs or stations are marked with the numbers 1, 2, 3, 4.



The series of heights taken at the back observation are represented by the broken or dotted vertical lines, and those taken at the fore observation, by the unbroken or full lines. Each term of the former series is a *rise*, or ascent from the peg to the level; and each term of the latter, a *fall*, or descent from the level to the peg. It is evident, therefore, that if the lengths of all the *dotted* lines are cast up into one sum, and the lengths of all the *full* lines into another, the difference of those sums will be = the difference in the level of the first and last stations; and that the height of the last station will either *exceed* or *fall short* of the first, according as the sum of the dotted lines, or back observations, is *greater* or *less* than the other sum.

The following is a specimen of the mode of keeping a Levelling Book. [See top of next page.]

In order to avoid the introduction of negative signs into the heights of those stations which happen to lie below the first, it is usual to assume the height of the first station above an imaginary line any convenient number of feet, such as 100, 500, &c.

Back Station.				Fore Station.					
No. of Station.	Feet.	Bearing.	Dist. Links.	No. of Station.	Feet.	Bearing.	Dist. Links.	Height of Station.	No.
1	4·23	272°	350	2	3·53	98°	350	100·00 +4·23	1
								104·23 −3·53	
2	4·73	250°	400	3	4·50	75°	400	100·70 +4·73	2
								105·43 −4·50	
3	4·11	220°	420	4	3·90	42°	420	100·93 +4·11	3
								105·04 −3·90	
4	3·10	201°	400	5	3·58	26°	400	101·14 +3·10	4
								104·24 3·58	
Sum Add Subtr. Proof	16·17 100·00 116·17 15·51 100·66		1570	Sum	15·51		1570	100·66	5

In the foregoing example the height of the first station is assumed = 100 feet, from which the height of each successive station is deduced, by *adding* the reading of the staff at the back station, and *subtracting* that at the fore station, as shown in the two right-hand columns, where the reduced or actual height of each station is seen, with its number placed against it.

When the levels of the intermediate stations are not wanted, and only the difference of level between the first and last is required, it is sufficient to cast up the heights observed at the back and fore stations, respectively, into two sums, the difference of which will be the difference of level between the last station and the first. The same method also affords a check on the alternate additions and subtractions required in reducing the levels of the successive stations when the height of each is necessary; as shown in the example above, where the height of the last station, or No. 5., comes out by each method = 100·66 feet, No. 1. being supposed = 100 feet in height.

Having been employed, about two years ago, by a Committee of the British Association, to connect, by a course of very exact levelling, different points on the English and Bristol Channels, which had been selected as stations for observing the heights of the tide at high and low water, with a view to the determination of some important questions relating to the level of the sea, I I shall here introduce from Professor Whewell's report of that undertaking the detailed account which I was then requested to furnish, of my levelling instruments and operations. (See *Eighth Report of British Association.*)

The instruments made for this undertaking were a spirit-level and brass levelling-staff, by Simms, London. The telescope, though only 14 inches in length, was found to bear the high magnifying power of 26 so well, under all circumstances, that the other eye-piece (of inferior power) with which it was furnished, was never employed. The glass spirit-tube is so nicely ground, that the position of the air-bubble is found to be sensibly altered by raising or lowering either end of the telescope only the $\frac{1}{100000}$th part of an inch. In the focus of the telescope are a horizontal and two vertical hairs, which latter afford a very convenient means of measuring the distance of a station, within about the $\frac{1}{100}$th part of the truth, by counting the number of intercepted divisions of a scale made for the purpose, and held horizontally over the station by an assistant.

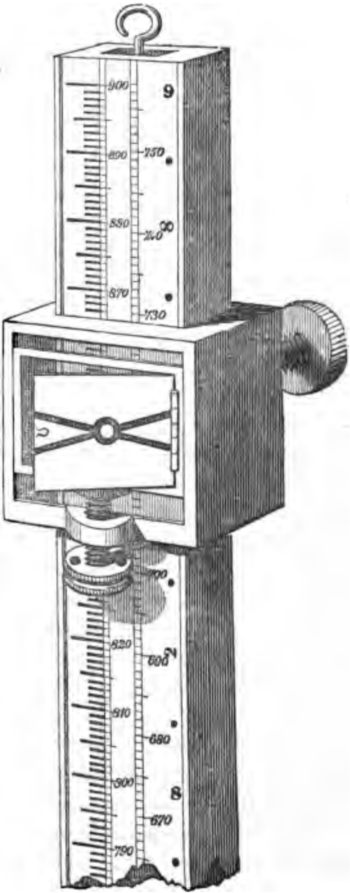
The legs which had been made to support the level, although very strong, were found to vibrate so much from the action of the wind, as to render it difficult to take a correct observation, except in perfectly calm weather. It was also next to impossible to level the spirit-tube, unless by accident, for want of a slower and more delicate motion than that afforded by the parallel plate screws. I therefore ordered a very strong stool to be made by a carpenter, the top of which was a thick board, 12 inches in diameter. The level was then detached from its former support, and fastened to a circular piece of mahogany, which rested on three foot-screws on the top of the stool, and was firmly secured to it by a stout wooden screw, with a nut at bottom, passing through both the circular boards. On trying this apparatus, I found that a more delicate vertical motion was still wanted, which was at length perfectly attained by causing one of the three foot-screws to rest on a small brass lever, at a very short distance from the fulcrum, while the farther end, furnished with a fine screw and milled-head, communicated about $\frac{1}{12}$th of its own vertical motion to the foot-screw of the level, affording a very simple and delicate means of adjustment.

The level, although now incomparably steadier than before, was still found liable to disturbance from the wind, when it blew with any considerable force; to protect it from which we carried with us a piece of canvass, 6 feet square, nailed to two poles, which were sharpened at bottom to enter the ground. This screen, being held firmly by two men on the windward side of the instrument, sheltered it so completely, that I was able to proceed in windy weather with but little interruption.

The brass levelling-staff having been found liable to get out of repair, I substituted for it one of wood, 9 feet long, and 2 inches wide, made of a single piece of straight-grained oak. On the face of this staff are laid down two different scales of equal parts. One scale is the common one of feet and hundredths of a foot; the other has larger divisions, in the proportion of 19 to 16 nearly, or more exactly as 1·18702 to 1; an aliquot ratio having been purposely avoided. Both of these are reckoned upwards from a common zero at the bottom of the staff. The centesimal divisions of the foot are produced in strong black lines towards the left, and large figures, denoting feet and tenths, are placed opposite to them, so that the height may be read off at the telescope to the $\frac{1}{100}$ part of a foot, at a distance of 150 or 200 yards. These marks were principally useful for directing the assistant at what height, nearly, to fix and clamp the vane, by calling the division to him, especially when the reading was near the top of the staff, after which a few touches of the vertical screw were sufficient to adjust it exactly to its true height; a stud of wire, about half an inch long, projects from the bottom of the staff, and a hole is bored to receive it in the top of the peg, which is driven into the ground at every station, and on which the staff rests during the observation. A small spirit-cup, with a glass cover, screwed to the lower part of the staff, serves to adjust it to a vertical position, in which it is held fast by a clamp attached to three strong legs, jointed and made to fold together in the usual manner.

The vane is a small mahogany box, about 3 inches in each dimension, open at the ends to admit the staff which slides through it. Two large wooden screws at the back of the vane clamp it very firmly to the staff, and preclude all danger of shifting. In front is a frame of brass, about 2 inches square, sliding within an outer frame of brass screwed to the vane, which has a range of motion of about half an inch, either upwards or downwards, being moved by a large vertical screw with a milled-head, which works through the lower part of the outer frame. A square aperture, corresponding with the inside of the frame, is cut through the mahogany, in order that the divisions of the staff may be seen. A small ivory door, moving on a hinge, is fitted into the sliding frame, on which are drawn two stout black lines, crossing each other at an acute angle, and having a black ring with a white circular spot within at the centre or intersection. Attached to the inside of the sliding frame, and exactly behind the centre of the white circle, is a vernier, nearly in contact with the face of the staff, which divides the hundredth of a foot into five parts, of 20 ten-thousandths each, so that the observation is read off and recorded to four decimal places, the fractional parts of these smaller divisions being estimated by the eye. The divisions of the larger scale are subdivided by another vernier. In favourable weather, I have usually found the average error, or the difference of a single reading from the mean of the number of readings taken to be about $\frac{1}{250}$th of an inch on a distance of four chains, which is equal to about a quarter of a second of angle. For a representation of this staff and vane see figure 3.

Fig. 3.



Whenever the vane is raised so near the top of the staff as to be out of reach of the hand, the adjusting-screw is worked by means of a long fork of stout wire, thrust into two holes made in the milled-head to receive it. A groove made in the upper part of the staff receives the fork when it is not in use.

In levelling, I proceeded regularly in the following manner:— Two equal distances, usually of four chains each, having been measured forwards from the last station, the level was placed at the end of the first distance, and at the second, a strong wooden peg driven firmly into the ground for the fore station, the level being exactly midway between the stations. Where (as it happened in a very few instances only) I was prevented from making the fore and back distances equal, compensating unequal distances were immediately afterwards taken, so that the sums of the two sets of distances were kept equal throughout. Thus, supposing the back and fore distances between stations 5 and 6 had been 350 and 450 links respectively, the distances between the stations 6 and 7, or perhaps between 7 and 8, would be 450 and 350, or the same distances reversed.

The staff being held vertically over the back station-peg, by the means before described, and the first observation taken, the height was read off, and written down by the assistant in a rough minute-book which he carried for the purpose. The vane was then purposely thrown out by turning the screw at random, the level re-adjusted, and a second reading taken. If these readings agreed within $\frac{20}{10000}$ths (or about $\frac{1}{40}$th of an inch) of each other, the staff was brought forward to me, when I read off and inserted the last reading, according to both scales, in separate columns of my book. If the difference exceeded that quantity, a third observation was taken. The mean of the readings was inserted in another column, after my assistant and myself had called over and compared the last reading. The assistant then read off and called to me the last reading from the large scale, as a check on what I had entered in my book. The magnetic bearing and the distance in links being also inserted in their respective columns, completed the back observation.

The process in taking the fore observation was the same, except that, instead of having the staff brought to me to be read, I then carried forward the level to the staff.

A rigid adherence to this system rendered it improbable that a wrong reading could be written down without immediate detection:—in fact, such an instance does not appear to have occurred. Had it even been so, a discrepancy would have been introduced into the columns of different scales, which must have been detected on casting up, and comparing the totals of the respective columns, at the end of the day. From erroneous readings, therefore, it is evident, that there was little or nothing to fear; but these are far from being the only or even the principal sources of error. On one or two occasions, we were very near committing a mistake, in beginning at a different station from the one on which we had previously closed. This would have occasioned an error, perhaps of large amount, which could only have been detected by the second and independent series of levels, taken over the ground in an opposite direction. For this reason alone, I should not consider it safe to depend on any *single* course of levels, whatever may have been the precautions used to guard against error.

The total length of my line of levelling between Portishead (near Bristol) and Axmouth (Devon), besides the branch-lines to Bristol and East Quantockshead, is about seventy-four miles. This distance was divided into separate stages, of an average length of about ten miles; each of which was twice levelled over, first in one direction, and then in the opposite, before the next stage was commenced.

It is very remarkable, that with a few partial exceptions, the heights of all the points touched upon by both series, came out *less* by the levels *returning* than by the levels *going*: so that the first station, or starting point, appeared to be lower at my return than it was at my setting out. But the height of this point being really the same in both cases, the error must, of course, be thrown on the distant point, or the station at which the returning levels commenced, by which the apparent differences are reversed, and the heights in the second series rendered progressively greater than those in the first, the most distant point having the greatest error.

The following are the differences which were thus found at twenty points, nearly equidistant, along the line between Portishead and Axmouth, the height, in every instance, coming out greater in the series of levels returning towards Portishead. [See the Table at top of next page.]

After a careful examination of every circumstance which could possibly tend to occasion these curious differences, I am very much inclined to believe that they arise principally from rapid variations in the amount of atmospheric refraction, which occur during the time that elapses in making a single observation, and that the progression of the error is in some way or other connected with the progressive changes of the average temperature during the course of the day, from about eight in the morning till six or seven in the evening, — the usual limits of my working hours. These variations in the refraction are much greater and more sudden in summer than in winter, especially during the forenoon of a hot and sultry day, when there are frequent alternations of cloud and sunshine, and copious exhalations of moisture from the ground.

No. of Station.	Miles from Portishead.	Height greater by 2d than by 1st levels.
		Feet.
1683	0	0·0000
1681	3	0·0633
1593	6	0·1557
1562	9	0·2703
1527	12	0·3501
1278	15	0·3796
1229	18	0·4591
1178	23	0·5339
1128	27	0·5734
759	30	0·6352
1	33	0·6888
45	37	0·6956
63	39	0·7170
114	43	0·7532
177	49	0·8237
210	52	0·8622
246	56	0·9021
248	59	0·9208
402	63	0·9373
462	68	0·9714
656	74	1·0294

On such occasions I have sometimes known the sudden clearing away of a cloud from the sun followed almost in an instant by a change in the apparent height of the vane, amounting to $\frac{1}{10}$th of an inch, or more, on a distance of only 88 yards. At other times the change has been more gradual, so that several successive readings, taken at intervals of two or three minutes, have all either increased or diminished progressively. Different seasons or states of weather may therefore be sufficient to account for the more rapid increase of these differences at certain times than at others, such as the above table presents, in which the errors are found proportionably greater between Portishead and Bridgwater, than between Bridgwater and Axmouth; the latter distance having been levelled over in the summer of 1837, and the former in that of 1838. For the same reason it appears better to divide the distance into stages, and finish them one at a time, than to complete the whole in one direction, before returning over any part of it; it being much more probable that errors, depending on the state of the atmosphere, will balance each other in the former case than in the latter.

My own experience, therefore, leads to the conclusion, that no levelling can be expected to give a correct result, unless it be performed in opposite directions, and the mean of both results be taken, instead of depending, as Captain Lloyd appears to have done*, on the consistency of separate sets of successive readings. I have myself invariably found, as that gentleman also did, the agreement of these to be almost identical, both in the going and returning series, notwithstanding the great progressive difference of these two series of levels from each other; of which progression not the smallest trace is discoverable in the separate columns of the same series.

I have entered the more minutely into this subject, because I am not aware that any one has described, or even noticed, the existence of such differences before; and should feel much interest in reading the statements of any experienced person, who had been engaged in a similar undertaking, and had conducted it with sufficent care to render the *law* of the errors in any degree discernible. —*T. G. B.*

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* See an account of levelling operations from Sheerness to London, in the Phil. Trans. for 1831.

LOGARITHMS

OF

NUMBERS

FROM

1 TO 10,000.

No.	Log.	No.	Log.	No.	Log.	No.	Log.
1	0·000000	26	1·414973	51	1·707570	76	1·880814
2	0·301030	27	1·431364	52	1·716003	77	1·886491
3	0·477121	28	1·447158	53	1·724276	78	1·892095
4	0·602060	29	1·462398	54	1·732394	79	1·897627
5	0·698970	30	1·477121	55	1·740363	80	1·903090
6	0·778151	31	1·491362	56	1·748188	81	1·908485
7	0·845098	32	1·505150	57	1·755875	82	1·913814
8	0·903090	33	1·518514	58	1·763428	83	1·919078
9	0·954243	34	1·531479	59	1·770852	84	1·924279
10	1·000000	35	1·544068	60	1·778151	85	1·929419
11	1·041393	36	1·556303	61	1·785330	86	1·934498
12	1·079181	37	1·568202	62	1·792392	87	1·939519
13	1·113943	38	1·579784	63	1·799341	88	1·944483
14	1·146128	39	1·591065	64	1·806180	89	1·949390
15	1·176091	40	1·602060	65	1·812913	90	1·954243
16	1·204120	41	1·612784	66	1·819544	91	1·959041
17	1·230449	42	1·623249	67	1·826075	92	1·963788
18	1·255273	43	1·633468	68	1·832509	93	1·968483
19	1·278754	44	1·643453	69	1·838849	94	1·973128
20	1·301030	45	1·653213	70	1·845098	95	1·977724
21	1·322219	46	1·662758	71	1·851258	96	1·982271
22	1·342423	47	1·672098	72	1·857332	97	1·986772
23	1·361728	48	1·681241	73	1·863323	98	1·991226
24	1·380211	49	1·690196	74	1·869232	99	1·995635
25	1·397940	50	1·698970	75	1·875061	100	2·000000

N.	0	1	2	3	4	5	6	7	8	9	D.
100	000000	0434	0868	1301	1734	2166	2598	3029	3461	3891	432
101	4321	4751	5181	5609	6038	6466	6894	7321	7748	8174	428
102	8600	9026	9451	9876							
102	01				0300	0724	1147	1570	1993	2415	424
103	2837	3259	3680	4100	4521	4940	5360	5779	6197	6616	420
104	7033	7451	7868	8284	8700	9116	9532	9947			
104	02								0361	0775	416
105	1189	1603	2016	2428	2841	3252	3664	4075	4486	4896	412
106	5306	5715	6125	6533	6942	7350	7757	8164	8571	8978	408
107	9384	9789									
107	03		0195	0600	1004	1408	1812	2216	2619	3021	404
108	3424	3826	4227	4628	5029	5430	5830	6230	6629	7028	400
109	7426	7825	8223	8620	9017	9414	9811				
109	04							0207	0602	0998	397
110	1393	1787	2182	2576	2969	3362	3755	4148	4540	4932	393
111	5323	5714	6105	6495	6885	7275	7664	8053	8442	8830	389
112	9218	9606	9993								
N.	0	1	2	3	4	5	6	7	8	9	D.

N.	0	1	2	3	4	5	6	7	8	9	D.
112	05			0380	0766	1153	1538	1924	2309	2694	386
113	3078	3463	3846	4230	4613	4996	5378	5760	6142	6524	383
114	6905	7286	7666	8046	8426	8805	9185	9563	9942		
114	06									0320	379
115	0698	1075	1452	1829	2206	2582	2958	3333	3709	4083	376
116	4458	4832	5206	5580	5953	6326	6699	7071	7443	7815	373
117	8186	8557	8927	9298	9668						
117	07					0038	0407	0776	1145	1514	370
118	1882	2250	2617	2985	3352	3718	4085	4451	4816	5182	366
119	5547	5912	6276	6640	7004	7368	7731	8094	8457	8819	363
120	9181	9543	9904								
120	08			0266	0626	0987	1347	1707	2067	2426	360
121	2785	3144	3503	3861	4219	4576	4934	5291	5647	6004	357
122	6360	6716	7071	7426	7781	8136	8490	8845	9198	9552	355
123	9905										
123	09	0258	0611	0963	1315	1667	2018	2370	2721	3071	352
124	3422	3772	4122	4471	4820	5169	5518	5866	6215	6562	349
125	6910	7257	7604	7951	8298	8644	8990	9335	9681		
125	10									0026	346
126	0371	0715	1059	1403	1747	2091	2434	2777	3119	3462	343
127	3804	4146	4487	4828	5169	5510	5851	6191	6531	6871	341
128	7210	7549	7888	8227	8565	8903	9241	9579	9916		
128	11									0253	338
129	0590	0926	1263	1599	1934	2270	2605	2940	3275	3609	335
130	3943	4277	4611	4944	5278	5611	5943	6276	6608	6940	333
131	7271	7603	7934	8265	8595	8926	9256	9586	9915		
131	12									0245	330
132	0574	0903	1231	1560	1888	2216	2544	2871	3198	3525	328
133	3852	4178	4504	4830	5156	5481	5806	6131	6456	6781	325
134	7105	7429	7753	8076	8399	8722	9045	9368	9690		
134	13									0012	323
135	0334	0655	0977	1298	1619	1939	2260	2580	2900	3219	321
136	3539	3858	4177	4496	4814	5133	5451	5769	6086	6403	318
137	6721	7037	7354	7671	7987	8303	8618	8934	9249	9564	316
138	9879										
138	14	0194	0508	0822	1136	1450	1763	2076	2389	2702	314
139	3015	3327	3639	3951	4263	4574	4885	5196	5507	5818	311
140	6128	6438	6748	7058	7367	7676	7985	8294	8603	8911	309
141	9219	9527	9835								
141	15			0142	0449	0756	1063	1370	1676	1982	307
142	2288	2594	2900	3205	3510	3815	4120	4424	4728	5032	305
143	5336	5640	5943	6246	6549	6852	7154	7457	7759	8061	303
144	8362	8664	8965	9266	9567	9868					
144	16						0168	0469	0769	1068	301
145	1368	1667	1967	2266	2564	2863	3161	3460	3758	4055	299
146	4353	4650	4947	5244	5541	5838	6134	6430	6726	7022	297
N.	0	1	2	3	4	5	6	7	8	9	D.

N.	0	1	2	3	4	5	6	7	8	9	D.
147	167317	7613	7908	8203	8497	8792	9086	9380	9674	9968	295
148	170262	0555	0848	1141	1434	1726	2019	2311	2603	2895	293
149	3186	3478	3769	4060	4351	4641	4932	5222	5512	5802	291
150	6091	6381	6670	6959	7248	7536	7825	8113	8401	8689	289
151	8977	9264	9552	9839							
151	18				0126	0413	0699	0986	1272	1558	287
152	1844	2129	2415	2700	2985	3270	3555	3839	4123	4407	285
153	4691	4975	5259	5542	5825	6108	6391	6674	6956	7239	283
154	7521	7803	8084	8366	8647	8928	9209	9490	9771		
154	19									0051	281
155	0332	0612	0892	1171	1451	1730	2010	2289	2567	2846	279
156	3125	3403	3681	3959	4237	4514	4792	5069	5346	5623	278
157	5900	6176	6453	6729	7005	7281	7556	7832	8107	8382	276
158	8657	8932	9206	9481	9755						
158	20					0029	0303	0577	0850	1124	274
159	1397	1670	1943	2216	2488	2761	3033	3305	3577	3848	272
160	4120	4391	4663	4934	5204	5475	5746	6016	6286	6556	271
161	6826	7096	7365	7634	7904	8173	8441	8710	8979	9247	269
162	9515	9783									
162	21		0051	0319	0586	0853	1121	1388	1654	1921	267
163	2188	2454	2720	2986	3252	3518	3783	4049	4314	4579	266
164	4844	5109	5373	5638	5902	6166	6430	6694	6957	7221	264
165	7484	7747	8010	8273	8536	8798	9060	9323	9585	9846	262
166	220108	0370	0631	0892	1153	1414	1675	1936	2196	2456	261
167	2716	2976	3236	3496	3755	4015	4274	4533	4792	5051	259
168	5309	5568	5826	6084	6342	6600	6858	7115	7372	7630	258
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173	8046	8297	8548	8799	9049	9299	9550	9800			
173	24								0050	0300	250
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176	5513	5759	6006	6252	6499	6745	6991	7237	7482	7728	246
177	7973	8219	8464	8709	8954	9198	9443	9687	9932		
177	25									0176	245
178	0420	0664	0908	1151	1395	1638	1881	2125	2368	2610	243
179	2853	3096	3338	3580	3822	4064	4306	4548	4790	5031	242
180	5273	5514	5755	5996	6237	6477	6718	6958	7198	7439	241
181	7679	7918	8158	8398	8637	8877	9116	9355	9594	9833	239
182	260071	0310	0548	0787	1025	1263	1501	1739	1976	2214	238
183	2451	2688	2925	3162	3399	3636	3873	4109	4346	4582	237
184	4818	5054	5290	5525	5761	5996	6232	6467	6702	6937	235
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189	6462	6692	6921	7151	7380	7609	7838	8067	8296	8525	229
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190	28						0123	0351	0578	0806	228
191	1033	1261	1488	1715	1942	2169	2396	2622	2849	3075	227
192	3301	3527	3753	3979	4205	4431	4656	4882	5107	5332	226
193	5557	5782	6007	6232	6456	6681	6905	7130	7354	7578	225
194	7802	8026	8249	8473	8696	8920	9143	9366	9589	9812	223
195	290035	0257	0480	0702	0925	1147	1369	1591	1813	2034	222
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200	1030	1247	1464	1681	1898	2114	2331	2547	2764	2980	217
201	3196	3412	3628	3844	4059	4275	4491	4706	4921	5136	216
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208	8063	8272	8481	8689	8898	9106	9314	9522	9730	9938	208
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211	4282	4488	4694	4899	5105	5310	5516	5721	5926	6131	205
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213	8380	8583	8787	8991	9194	9398	9601	9805			
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215	2438	2640	2842	3044	3246	3447	3649	3850	4051	4253	202
216	4454	4655	4856	5057	5257	5458	5658	5859	6059	6260	201
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219	0444	0642	0841	1039	1237	1435	1632	1830	2028	2225	198
220	2423	2620	2817	3014	3212	3409	3606	3802	3999	4196	197
221	4392	4589	4785	4981	5178	5374	5570	5766	5962	6157	196
222	6353	6549	6744	6939	7135	7330	7525	7720	7915	8110	195
223	8305	8500	8694	8889	9083	9278	9472	9666	9860		
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227	6026	6217	6408	6599	6790	6981	7172	7363	7554	7744	191
228	7935	8125	8316	8506	8696	8886	9076	9266	9456	9646	190
229	9835										
229	36	0025	0215	0404	0593	0783	0972	1161	1350	1539	189
230	1728	1917	2105	2294	2482	2671	2859	3048	3236	3424	188
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232	5488	5675	5862	6049	6236	6423	6610	6796	6983	7169	187
233	7356	7542	7729	7915	8101	8287	8473	8659	8845	9030	186
234	9216	9401	9587	9772	9958						
234	37					0143	0328	0513	0698	0883	185
235	1068	1253	1437	1622	1806	1991	2175	2360	2544	2728	184
236	2912	3096	3280	3464	3647	3831	4015	4198	4382	4565	184
237	4748	4932	5115	5298	5481	5664	5846	6029	6212	6394	183
238	6577	6759	6942	7124	7306	7488	7670	7852	8034	8216	182
239	8398	8580	8761	8943	9124	9306	9487	9668	9849		
239	38									0030	181
240	0211	0392	0573	0754	0934	1115	1296	1476	1656	1837	181
241	2017	2197	2377	2557	2737	2917	3097	3277	3456	3636	180
242	3815	3995	4174	4353	4533	4712	4891	5070	5249	5428	179
243	5606	5785	5964	6142	6321	6499	6677	6856	7034	7212	178
244	7390	7568	7746	7923	8101	8279	8456	8634	8811	8989	178
245	9166	9343	9520	9698	9875						
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246	0935	1112	1288	1464	1641	1817	1993	2169	2345	2521	176
247	2697	2873	3048	3224	3400	3575	3751	3926	4101	4277	176
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249	6199	6374	6548	6722	6896	7071	7245	7419	7592	7766	174
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251	9674	9847									
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253	3121	3292	3464	3635	3807	3978	4149	4320	4492	4663	171
254	4834	5005	5176	5346	5517	5688	5858	6029	6199	6370	171
255	6540	6710	6881	7051	7221	7391	7561	7731	7901	8070	170
256	8240	8410	8579	8749	8918	9087	9257	9426	9595	9764	169
257	9933										
257	41	0102	0271	0440	0609	0777	0946	1114	1283	1451	169
258	1620	1788	1956	2124	2293	2461	2629	2796	2964	3132	168
259	3300	3467	3635	3803	3970	4137	4305	4472	4639	4806	167
260	4973	5140	5307	5474	5641	5808	5974	6141	6308	6474	167
261	6641	6807	6973	7139	7306	7472	7638	7804	7970	8135	166
262	8301	8467	8633	8798	8964	9129	9295	9460	9625	9791	165
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266	4882	5045	5208	5371	5534	5697	5860	6023	6186	6349	163
267	6511	6674	6836	6999	7161	7324	7486	7648	7811	7973	162
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269	9752	9914									
269	43		0075	0236	0398	0559	0720	0881	1042	1203	161
270	1364	1525	1685	1846	2007	2167	2328	2488	2649	2809	161
271	2969	3130	3290	3450	3610	3770	3930	4090	4249	4409	160
272	4569	4729	4888	5048	5207	5367	5526	5685	5844	6004	159
273	6163	6322	6481	6640	6799	6957	7116	7275	7433	7592	159
274	7751	7909	8067	8226	8384	8542	8701	8859	9017	9175	158
275	9333	9491	9648	9806	9964						
275	44					0122	0279	0437	0594	0752	158
276	0909	1066	1224	1381	1538	1695	1852	2009	2166	2323	157
277	2480	2637	2793	2950	3106	3263	3419	3576	3732	3889	157
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279	5604	5760	5915	6071	6226	6382	6537	6692	6848	7003	155
280	7158	7313	7468	7623	7778	7933	8088	8242	8397	8552	155
281	8706	8861	9015	9170	9324	9478	9633	9787	9941		
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283	1786	1940	2093	2247	2400	2553	2706	2859	3012	3165	153
284	3318	3471	3624	3777	3930	4082	4235	4387	4540	4692	153
285	4845	4997	5150	5302	5454	5606	5758	5910	6062	6214	152
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287	7882	8033	8184	8336	8487	8638	8789	8940	9091	9242	151
288	9392	9543	9694	9845	9995						
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289	0898	1048	1198	1348	1499	1649	1799	1948	2098	2248	150
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291	3893	4042	4191	4340	4490	4639	4788	4936	5085	5234	149
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293	6868	7016	7164	7312	7460	7608	7756	7904	8052	8200	148
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295	9822	9969									
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296	1292	1438	1585	1732	1878	2025	2171	2318	2464	2610	146
297	2756	2903	3049	3195	3341	3487	3633	3779	3925	4071	146
298	4216	4362	4508	4653	4799	4944	5090	5235	5381	5526	146
299	5671	5816	5962	6107	6252	6397	6542	6687	6832	6976	145
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303	1443	1586	1729	1872	2016	2159	2302	2445	2588	2731	143
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307	7138	7280	7421	7563	7704	7845	7986	8127	8269	8410	141
308	8551	8692	8833	8974	9114	9255	9396	9537	9677	9818	141
309	9958										
309	49	0099	0239	0380	0520	0661	0801	0941	1081	1222	140
310	1362	1502	1642	1782	1922	2062	2201	2341	2481	2621	140
311	2760	2900	3040	3179	3319	3458	3597	3737	3876	4015	139
312	4155	4294	4433	4572	4711	4850	4989	5128	5267	5406	139
313	5544	5683	5822	5960	6099	6238	6376	6515	6653	6791	139
314	6930	7068	7206	7344	7483	7621	7759	7897	8035	8173	138
315	8311	8448	8586	8724	8862	8999	9137	9275	9412	9550	138
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318	2427	2564	2700	2837	2973	3109	3246	3382	3518	3655	136
319	3791	3927	4063	4199	4335	4471	4607	4743	4878	5014	136
320	5150	5286	5421	5557	5693	5828	5964	6099	6234	6370	136
321	6505	6640	6776	6911	7046	7181	7316	7451	7586	7721	135
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323	9203	9337	9471	9606	9740	9874					
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324	0545	0679	0813	0947	1081	1215	1349	1482	1616	1750	134
325	1883	2017	2151	2284	2418	2551	2684	2818	2951	3084	133
326	3218	3351	3484	3617	3750	3883	4016	4149	4282	4415	133
327	4548	4681	4813	4946	5079	5211	5344	5476	5609	5741	133
328	5874	6006	6139	6271	6403	6535	6668	6800	6932	7064	132
329	7196	7328	7460	7592	7724	7855	7987	8119	8251	8382	132
330	8514	8646	8777	8909	9040	9171	9303	9434	9566	9697	131
331	9828	9959									
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332	1138	1269	1400	1530	1661	1792	1922	2053	2183	2314	131
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334	3746	3876	4006	4136	4266	4396	4526	4656	4785	4915	130
335	5045	5174	5304	5434	5563	5693	5822	5951	6081	6210	129
336	6339	6469	6598	6727	6856	6985	7114	7243	7372	7501	129
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338	8917	9045	9174	9302	9430	9559	9687	9815	9943		
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339	0200	0328	0456	0584	0712	0840	0968	1096	1223	1351	128
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349	2825	2950	3074	3199	3323	3447	3571	3696	3820	3944	124
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351	5307	5431	5555	5678	5802	5925	6049	6172	6296	6419	124
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353	7775	7898	8021	8144	8267	8389	8512	8635	8758	8881	123
354	9003	9126	9249	9371	9494	9616	9739	9861	9984		
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356	1450	1572	1694	1816	1938	2060	2181	2303	2425	2547	122
357	2668	2790	2911	3033	3155	3276	3398	3519	3640	3762	121
358	3883	4004	4126	4247	4368	4489	4610	4731	4852	4973	121
359	5094	5215	5336	5457	5578	5699	5820	5940	6061	6182	121
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361	7507	7627	7748	7868	7988	8108	8228	8349	8469	8589	120
362	8709	8829	8948	9068	9188	9308	9428	9548	9667	9787	120
363	9907										
363	56	0026	0146	0265	0385	0504	0624	0743	0863	0982	119
364	1101	1221	1340	1459	1578	1698	1817	1936	2055	2174	119
365	2293	2412	2531	2650	2769	2887	3006	3125	3244	3362	119
366	3481	3600	3718	3837	3955	4074	4192	4311	4429	4548	119
367	4666	4784	4903	5021	5139	5257	5376	5494	5612	5730	118
368	5848	5966	6084	6202	6320	6437	6555	6673	6791	6909	118
369	7026	7144	7262	7379	7497	7614	7732	7849	7967	8084	118
370	8202	8319	8436	8554	8671	8788	8905	9023	9140	9257	117
371	9374	9491	9608	9725	9842	9959					
371	57						0076	0193	0309	0426	117
372	0543	0660	0776	0893	1010	1126	1243	1359	1476	1592	117
373	1709	1825	1942	2058	2174	2291	2407	2523	2639	2755	116
374	2872	2988	3104	3220	3336	3452	3568	3684	3800	3915	116
375	4031	4147	4263	4379	4494	4610	4726	4841	4957	5072	116
376	5188	5303	5419	5534	5650	5765	5880	5996	6111	6226	115
377	6341	6457	6572	6687	6802	6917	7032	7147	7262	7377	115
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380	9784	9898									
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382	2063	2177	2291	2404	2518	2631	2745	2858	2972	3085	114
383	3199	3312	3426	3539	3652	3765	3879	3992	4105	4218	113
384	4331	4444	4557	4670	4783	4896	5009	5122	5235	5348	113
385	5461	5574	5686	5799	5912	6024	6137	6250	6362	6475	113
386	6587	6700	6812	6925	7037	7149	7262	7374	7486	7599	112
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389	9950										
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390	1065	1176	1287	1399	1510	1621	1732	1843	1955	2066	111
391	2177	2288	2399	2510	2621	2732	2843	2954	3064	3175	111
392	3286	3397	3508	3618	3729	3840	3950	4061	4171	4282	111
393	4393	4503	4614	4724	4834	4945	5055	5165	5276	5386	110
394	5496	5606	5717	5827	5937	6047	6157	6267	6377	6487	110
395	6597	6707	6817	6927	7037	7146	7256	7366	7476	7586	110
396	7695	7805	7914	8024	8134	8243	8353	8462	8572	8681	110
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398	9883	9992									
398	60		0101	0210	0319	0428	0537	0646	0755	0864	109
399	0973	1082	1191	1299	1408	1517	1625	1734	1843	1951	109
400	2060	2169	2277	2386	2494	2603	2711	2819	2928	3036	108
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402	4226	4334	4442	4550	4658	4766	4874	4982	5089	5197	108
403	5305	5413	5521	5628	5736	5844	5951	6059	6166	6274	108
404	6381	6489	6596	6704	6811	6919	7026	7133	7241	7348	107
405	7455	7562	7669	7777	7884	7991	8098	8205	8312	8419	107
406	8526	8633	8740	8847	8954	9061	9167	9274	9381	9488	107
407	9594	9701	9808	9914							
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408	0660	0767	0873	0979	1086	1192	1298	1405	1511	1617	106
409	1723	1829	1936	2042	2148	2254	2360	2466	2572	2678	106
410	2784	2890	2996	3102	3207	3313	3419	3525	3630	3736	106
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412	4897	5003	5108	5213	5319	5424	5529	5634	5740	5845	105
413	5950	6055	6160	6265	6370	6476	6581	6686	6790	6895	105
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416	9093	9198	9302	9406	9511	9615	9719	9824	9928		
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417	0136	0240	0344	0448	0552	0656	0760	0864	0968	1072	104
418	1176	1280	1384	1488	1592	1695	1799	1903	2007	2110	104
419	2214	2318	2421	2525	2628	2732	2835	2939	3042	3146	104
420	3249	3353	3456	3559	3663	3766	3869	3973	4076	4179	103
421	4282	4385	4488	4591	4695	4798	4901	5004	5107	5210	103
422	5312	5415	5518	5621	5724	5827	5929	6032	6135	6238	103
423	6340	6443	6546	6648	6751	6853	6956	7058	7161	7263	103
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425	8389	8491	8593	8695	8797	8900	9002	9104	9206	9308	102
426	9410	9512	9613	9715	9817	9919					
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441	4439	4537	4636	4734	4832	4931	5029	5127	5226	5324	98
442	5422	5521	5619	5717	5815	5913	6011	6110	6208	6306	98
443	6404	6502	6600	6698	6796	6894	6992	7089	7187	7285	98
444	7383	7481	7579	7676	7774	7872	7969	8067	8165	8262	98
445	8360	8458	8555	8653	8750	8848	8945	9043	9140	9237	97
446	9335	9432	9530	9627	9724	9821	9919				
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447	0308	0405	0502	0599	0696	0793	0890	0987	1084	1181	97
448	1278	1375	1472	1569	1666	1762	1859	1956	2053	2150	97
449	2246	2343	2440	2536	2633	2730	2826	2923	3019	3116	97
450	3213	3309	3405	3502	3598	3695	3791	3888	3984	4080	96
451	4177	4273	4369	4465	4562	4658	4754	4850	4946	5042	96
452	5138	5235	5331	5427	5523	5619	5715	5810	5906	6002	96
453	6098	6194	6290	6386	6482	6577	6673	6769	6864	6960	96
454	7056	7152	7247	7343	7438	7534	7629	7725	7820	7916	96
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456	8965	9060	9155	9250	9346	9441	9536	9631	9726	9821	95
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459	1813	1907	2002	2096	2191	2286	2380	2475	2569	2663	95
460	2758	2852	2947	3041	3135	3230	3324	3418	3512	3607	94
461	3701	3795	3889	3983	4078	4172	4266	4360	4454	4548	94
462	4642	4736	4830	4924	5018	5112	5206	5299	5393	5487	94
463	5581	5675	5769	5862	5956	6050	6143	6237	6331	6424	94
464	6518	6612	6705	6799	6892	6986	7079	7173	7266	7360	94
465	7453	7546	7640	7733	7826	7920	8013	8106	8199	8293	93
466	8386	8479	8572	8665	8759	8852	8945	9038	9131	9224	93
467	9317	9410	9503	9596	9689	9782	9875	9967			
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468	0246	0339	0431	0524	0617	0710	0802	0895	0988	1080	93
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474	5778	5870	5962	6053	6145	6236	6328	6419	6511	6602	92
475	6694	6785	6876	6968	7059	7151	7242	7333	7424	7516	91
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478	9428	9519	9610	9700	9791	9882	9973				
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479	0336	0426	0517	0607	0698	0789	0879	0970	1060	1151	91
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485	5742	5831	5921	6010	6100	6189	6279	6368	6458	6547	89
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488	8420	8509	8598	8687	8776	8865	8953	9042	9131	9220	89
489	9309	9398	9486	9575	9664	9753	9841	9930			
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490	0196	0285	0373	0462	0550	0639	0728	0816	0905	0993	89
491	1081	1170	1258	1347	1435	1524	1612	1700	1789	1877	88
492	1965	2053	2142	2230	2318	2406	2494	2583	2671	2759	88
493	2847	2935	3023	3111	3199	3287	3375	3463	3551	3639	88
494	3727	3815	3903	3991	4078	4166	4254	4342	4430	4517	88
495	4605	4693	4781	4868	4956	5044	5131	5219	5307	5394	88
496	5482	5569	5657	5744	5832	5919	6007	6094	6182	6269	87
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506	4151	4236	4322	4408	4494	4579	4665	4751	4837	4922	86
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508	5864	5949	6035	6120	6206	6291	6376	6462	6547	6632	85
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510	7570	7655	7740	7826	7911	7996	8081	8166	8251	8336	85
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516	2650	2734	2818	2902	2986	3070	3154	3238	3323	3407	84
517	3491	3575	3659	3742	3826	3910	3994	4078	4162	4246	84
518	4330	4414	4497	4581	4665	4749	4833	4916	5000	5084	84
519	5167	5251	5335	5418	5502	5586	5669	5753	5836	5920	84
520	6003	6087	6170	6254	6337	6421	6504	6588	6671	6754	83
521	6838	6921	7004	7088	7171	7254	7338	7421	7504	7587	83
522	7671	7754	7837	7920	8003	8086	8169	8253	8336	8419	83
523	8502	8585	8668	8751	8834	8917	9000	9083	9165	9248	83
524	9331	9414	9497	9580	9663	9745	9828	9911	9994		
524	72									0077	83
525	0159	0242	0325	0407	0490	0573	0655	0738	0821	0903	83
526	0986	1068	1151	1233	1316	1398	1481	1563	1646	1728	82
527	1811	1893	1975	2058	2140	2222	2305	2387	2469	2552	82
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529	3456	3538	3620	3702	3784	3866	3948	4030	4112	4194	82
530	4276	4358	4440	4522	4604	4685	4767	4849	4931	5013	82
531	5095	5176	5258	5340	5422	5503	5585	5667	5748	5830	82
532	5912	5993	6075	6156	6238	6320	6401	6483	6564	6646	82
533	6727	6809	6890	6972	7053	7134	7216	7297	7379	7460	81
534	7541	7623	7704	7785	7866	7948	8029	8110	8191	8273	81
535	8354	8435	8516	8597	8678	8759	8841	8922	9003	9084	81
536	9165	9246	9327	9408	9489	9570	9651	9732	9813	9893	81
537	9974										
537	73	0055	0136	0217	0298	0378	0459	0540	0621	0702	81
538	0782	0863	0944	1024	1105	1186	1266	1347	1428	1508	81
539	1589	1669	1750	1830	1911	1991	2072	2152	2233	2313	81
540	2394	2474	2555	2635	2715	2796	2876	2956	3037	3117	80
541	3197	3278	3358	3438	3518	3598	3679	3759	3839	3919	80
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543	4800	4880	4960	5040	5120	5200	5279	5359	5439	5519	80
544	5599	5679	5759	5838	5918	5998	6078	6157	6237	6317	80
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546	7193	7272	7352	7431	7511	7590	7670	7749	7829	7908	79
547	7987	8067	8146	8225	8305	8384	8463	8543	8622	8701	79
548	8781	8860	8939	9018	9097	9177	9256	9335	9414	9493	79
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550	0363	0442	0521	0600	0678	0757	0836	0915	0994	1073	79
551	1152	1230	1309	1388	1467	1546	1624	1703	1782	1860	79
552	1939	2018	2096	2175	2254	2332	2411	2489	2568	2647	79
553	2725	2804	2882	2961	3039	3118	3196	3275	3353	3431	78
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557	5855	5933	6011	6089	6167	6245	6323	6401	6479	6556	78
558	6634	6712	6790	6868	6945	7023	7101	7179	7256	7334	78
559	7412	7489	7567	7645	7722	7800	7878	7955	8033	8110	78
560	8188	8266	8343	8421	8498	8576	8653	8731	8808	8885	77
561	8963	9040	9118	9195	9272	9350	9427	9504	9582	9659	77
562	9736	9814	9891	9968							
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563	0508	0586	0663	0740	0817	0894	0971	1048	1125	1202	77
564	1279	1356	1433	1510	1587	1664	1741	1818	1895	1972	77
565	2048	2125	2202	2279	2356	2433	2509	2586	2663	2740	77
566	2816	2893	2970	3047	3123	3200	3277	3353	3430	3506	77
567	3583	3660	3736	3813	3889	3966	4042	4119	4195	4272	77
568	4348	4425	4501	4578	4654	4730	4807	4883	4960	5036	76
569	5112	5189	5265	5341	5417	5494	5570	5646	5722	5799	76
570	5875	5951	6027	6103	6180	6256	6332	6408	6484	6560	76
571	6636	6712	6788	6864	6940	7016	7092	7168	7244	7320	76
572	7396	7472	7548	7624	7700	7775	7851	7927	8003	8079	76
573	8155	8230	8306	8382	8458	8533	8609	8685	8761	8836	76
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575	9668	9743	9819	9894	9970						
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576	0422	0498	0573	0649	0724	0799	0875	0950	1025	1101	75
577	1176	1251	1326	1402	1477	1552	1627	1702	1778	1853	75
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583	5669	5743	5818	5892	5966	6041	6115	6190	6264	6338	74
584	6413	6487	6562	6636	6710	6785	6859	6933	7007	7082	74
585	7156	7230	7304	7379	7453	7527	7601	7675	7749	7823	74
586	7898	7972	8046	8120	8194	8268	8342	8416	8490	8564	74
587	8638	8712	8786	8860	8934	9008	9082	9156	9230	9303	74
588	9377	9451	9525	9599	9673	9746	9820	9894	9968		
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589	0115	0189	0263	0336	0410	0484	0557	0631	0705	0778	74
590	0852	0926	0999	1073	1146	1220	1293	1367	1440	1514	74
591	1587	1661	1734	1808	1881	1955	2028	2102	2175	2248	73
592	2322	2395	2468	2542	2615	2688	2762	2835	2908	2981	73
593	3055	3128	3201	3274	3348	3421	3494	3567	3640	3713	73
594	3786	3860	3933	4006	4079	4152	4225	4298	4371	4444	73
595	4517	4590	4663	4736	4809	4882	4955	5028	5100	5173	73
596	5246	5319	5392	5465	5538	5610	5683	5756	5829	5902	73
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599	7427	7499	7572	7644	7717	7789	7862	7934	8006	8079	72
600	8151	8224	8296	8368	8441	8513	8585	8658	8730	8802	72
601	8874	8947	9019	9091	9163	9236	9308	9380	9452	9524	72
602	9596	9669	9741	9813	9885	9957					
602	78						0029	0101	0173	0245	72
603	0317	0389	0461	0533	0605	0677	0749	0821	0893	0965	72
604	1037	1109	1181	1253	1324	1396	1468	1540	1612	1684	72
605	1755	1827	1899	1971	2042	2114	2186	2258	2329	2401	72
606	2473	2544	2616	2688	2759	2831	2902	2974	3046	3117	72
607	3189	3260	3332	3403	3475	3546	3618	3689	3761	3832	71
608	3904	3975	4046	4118	4189	4261	4332	4403	4475	4546	71
609	4617	4689	4760	4831	4902	4974	5045	5116	5187	5259	71
610	5330	5401	5472	5543	5615	5686	5757	5828	5899	5970	71
611	6041	6112	6183	6254	6325	6396	6467	6538	6609	6680	71
612	6751	6822	6893	6964	7035	7106	7177	7248	7319	7390	71
613	7460	7531	7602	7673	7744	7815	7885	7956	8027	8098	71
614	8168	8239	8310	8381	8451	8522	8593	8663	8734	8804	71
615	8875	8946	9016	9087	9157	9228	9299	9369	9440	9510	71
616	9581	9651	9722	9792	9863	9933					
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624	5185	5254	5324	5393	5463	5532	5602	5672	5741	5811	70
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626	6574	6644	6713	6782	6852	6921	6990	7060	7129	7198	69
627	7268	7337	7406	7475	7545	7614	7683	7752	7821	7890	69
628	7960	8029	8098	8167	8236	8305	8374	8443	8513	8582	69
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633	1404	1472	1541	1609	1678	1747	1815	1884	1952	2021	69
634	2089	2158	2226	2295	2363	2432	2500	2568	2637	2705	69
635	2774	2842	2910	2979	3047	3116	3184	3252	3321	3389	68
636	3457	3525	3594	3662	3730	3798	3867	3935	4003	4071	68
637	4139	4208	4276	4344	4412	4480	4548	4616	4685	4753	68
638	4821	4889	4957	5025	5093	5161	5229	5297	5365	5433	68
639	5501	5569	5637	5705	5773	5841	5908	5976	6044	6112	68
640	6180	6248	6316	6384	6451	6519	6587	6655	6723	6790	68
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644	8886	8953	9021	9088	9156	9223	9290	9358	9425	9492	67
645	9560	9627	9694	9762	9829	9896	9964				
645	81							0031	0098	0165	67
646	0233	0300	0367	0434	0501	0569	0636	0703	0770	0837	67
647	0904	0971	1039	1106	1173	1240	1307	1374	1441	1508	67
648	1575	1642	1709	1776	1843	1910	1977	2044	2111	2178	67
649	2245	2312	2379	2445	2512	2579	2646	2713	2780	2847	67
650	2913	2980	3047	3114	3181	3247	3314	3381	3448	3514	67
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653	4913	4980	5046	5113	5179	5246	5312	5378	5445	5511	66
654	5578	5644	5711	5777	5843	5910	5976	6042	6109	6175	66
655	6241	6308	6374	6440	6506	6573	6639	6705	6771	6838	66
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663	1514	1579	1645	1710	1775	1841	1906	1972	2037	2103	65
664	2168	2233	2299	2364	2430	2495	2560	2626	2691	2756	65
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667	4126	4191	4256	4321	4386	4451	4516	4581	4646	4711	65
668	4776	4841	4906	4971	5036	5101	5166	5231	5296	5361	65
669	5426	5491	5556	5621	5686	5751	5815	5880	5945	6010	65
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671	6723	6787	6852	6917	6981	7046	7111	7175	7240	7305	65
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677	0589	0653	0717	0781	0845	0909	0973	1037	1102	1166	64
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691	9478	9541	9604	9667	9729	9792	9855	9918	9981		
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693	0733	0796	0859	0921	0984	1046	1109	1172	1234	1297	63
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695	1985	2047	2110	2172	2235	2297	2360	2422	2484	2547	62
696	2609	2672	2734	2796	2859	2921	2983	3046	3108	3170	62
697	3233	3295	3357	3420	3482	3544	3606	3669	3731	3793	62
698	3855	3918	3980	4042	4104	4166	4229	4291	4353	4415	62
699	4477	4539	4601	4664	4726	4788	4850	4912	4974	5036	62
700	5098	5160	5222	5284	5346	5408	5470	5532	5594	5656	62
701	5718	5780	5842	5904	5966	6028	6090	6151	6213	6275	62
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703	6955	7017	7079	7141	7202	7264	7326	7388	7449	7511	62
704	7573	7634	7696	7758	7819	7881	7943	8004	8066	8128	62
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706	8805	8866	8928	8989	9051	9112	9174	9235	9297	9358	61
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717	5519	5580	5640	5701	5761	5822	5882	5943	6003	6064	61
718	6124	6185	6245	6306	6366	6427	6487	6548	6608	6668	60
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726	0937	0996	1056	1116	1176	1236	1295	1355	1415	1475	60
727	1534	1594	1654	1714	1773	1833	1893	1952	2012	2072	60
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734	5696	5755	5814	5874	5933	5992	6051	6110	6169	6228	59
735	6287	6346	6405	6465	6524	6583	6642	6701	6760	6819	59
736	6878	6937	6996	7055	7114	7173	7232	7291	7350	7409	59
737	7467	7526	7585	7644	7703	7762	7821	7880	7939	7998	59
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741	9818	9877	9935	9994							
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742	0404	0462	0521	0579	0638	0696	0755	0813	0872	0930	58
743	0989	1047	1106	1164	1223	1281	1339	1398	1456	1515	58
744	1573	1631	1690	1748	1806	1865	1923	1981	2040	2098	58
745	2156	2215	2273	2331	2389	2448	2506	2564	2622	2681	58
746	2739	2797	2855	2913	2972	3030	3088	3146	3204	3262	58
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748	3902	3960	4018	4076	4134	4192	4250	4308	4366	4424	58
749	4482	4540	4598	4656	4714	4772	4830	4888	4945	5003	58
750	5061	5119	5177	5235	5293	5351	5409	5466	5524	5582	58
751	5640	5698	5756	5813	5871	5929	5987	6045	6102	6160	58
752	6218	6276	6333	6391	6449	6507	6564	6622	6680	6737	58
753	6795	6853	6910	6968	7026	7083	7141	7199	7256	7314	58
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758	88						0013	0070	0127	0185	57
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765	3661	3718	3775	3832	3888	3945	4002	4059	4115	4172	57
766	4229	4285	4342	4399	4455	4512	4569	4625	4682	4739	57
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768	5361	5418	5474	5531	5587	5644	5700	5757	5813	5870	57
769	5926	5983	6039	6096	6152	6209	6265	6321	6378	6434	56
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771	7054	7111	7167	7223	7280	7336	7392	7449	7505	7561	56
772	7617	7674	7730	7786	7842	7898	7955	8011	8067	8123	56
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776	9862	9918	9974								
776	89			0030	0086	0141	0197	0253	0309	0365	56
777	0421	0477	0533	0589	0645	0700	0756	0812	0868	0924	56
778	0980	1035	1091	1147	1203	1259	1314	1370	1426	1482	56
779	1537	1593	1649	1705	1760	1816	1872	1928	1983	2039	56
780	2095	2150	2206	2262	2317	2373	2429	2484	2540	2595	56
781	2651	2707	2762	2818	2873	2929	2985	3040	3096	3151	56
782	3207	3262	3318	3373	3429	3484	3540	3595	3651	3706	56
783	3762	3817	3873	3928	3984	4039	4094	4150	4205	4261	55
784	4316	4371	4427	4482	4538	4593	4648	4704	4759	4814	55
785	4870	4925	4980	5036	5091	5146	5201	5257	5312	5367	55
786	5423	5478	5533	5588	5644	5699	5754	5809	5864	5920	55
787	5975	6030	6085	6140	6195	6251	6306	6361	6416	6471	55
788	6526	6581	6636	6692	6747	6802	6857	6912	6967	7022	55
789	7077	7132	7187	7242	7297	7352	7407	7462	7517	7572	55
790	7627	7682	7737	7792	7847	7902	7957	8012	8067	8122	55
791	8176	8231	8286	8341	8396	8451	8506	8561	8615	8670	55
792	8725	8780	8835	8890	8944	8999	9054	9109	9164	9218	55
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794	9821	9875	9930	9985							
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796	0913	0968	1022	1077	1131	1186	1240	1295	1349	1404	55
797	1458	1513	1567	1622	1676	1731	1785	1840	1894	1948	54
798	2003	2057	2112	2166	2221	2275	2329	2384	2438	2492	54
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802	4174	4229	4283	4337	4391	4445	4499	4553	4607	4661	54
803	4716	4770	4824	4878	4932	4986	5040	5094	5148	5202	54
804	5256	5310	5364	5418	5472	5526	5580	5634	5688	5742	54
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807	6874	6927	6981	7035	7089	7143	7196	7250	7304	7358	54
808	7411	7465	7519	7573	7626	7680	7734	7787	7841	7895	54
809	7949	8002	8056	8110	8163	8217	8270	8324	8378	8431	54
810	8485	8539	8592	8646	8699	8753	8807	8860	8914	8967	54
811	9021	9074	9128	9181	9235	9289	9342	9396	9449	9503	54
812	9556	9610	9663	9716	9770	9823	9877	9930	9984		
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813	0091	0144	0197	0251	0304	0358	0411	0464	0518	0571	53
814	0624	0678	0731	0784	0838	0891	0944	0998	1051	1104	53
815	1158	1211	1264	1317	1371	1424	1477	1530	1584	1637	53
816	1690	1743	1797	1850	1903	1956	2009	2063	2116	2169	53
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820	3814	3867	3920	3973	4026	4079	4132	4184	4237	4290	53
821	4343	4396	4449	4502	4555	4608	4660	4713	4766	4819	53
822	4872	4925	4977	5030	5083	5136	5189	5241	5294	5347	53
823	5400	5453	5505	5558	5611	5664	5716	5769	5822	5875	53
824	5927	5980	6033	6085	6138	6191	6243	6296	6349	6401	53
825	6454	6507	6559	6612	6664	6717	6770	6822	6875	6927	53
826	6980	7033	7085	7138	7190	7243	7295	7348	7400	7453	53
827	7506	7558	7611	7663	7716	7768	7820	7873	7925	7978	52
828	8030	8083	8135	8188	8240	8293	8345	8397	8450	8502	52
829	8555	8607	8659	8712	8764	8816	8869	8921	8973	9026	52
830	9078	9130	9183	9235	9287	9340	9392	9444	9496	9549	52
831	9601	9653	9706	9758	9810	9862	9914	9967			
831	92								0019	0071	52
832	0123	0176	0228	0280	0332	0384	0436	0489	0541	0593	52
833	0645	0697	0749	0801	0853	0906	0958	1010	1062	1114	52
834	1166	1218	1270	1322	1374	1426	1478	1530	1582	1634	52
835	1686	1738	1790	1842	1894	1946	1998	2050	2102	2154	52
836	2206	2258	2310	2362	2414	2466	2518	2570	2622	2674	52
837	2725	2777	2829	2881	2933	2985	3037	3089	3140	3192	52
838	3244	3296	3348	3399	3451	3503	3555	3607	3658	3710	52
839	3762	3814	3865	3917	3969	4021	4072	4124	4176	4228	52
840	4279	4331	4383	4434	4486	4538	4589	4641	4693	4744	52
841	4796	4848	4899	4951	5003	5054	5106	5157	5209	5261	52
842	5312	5364	5415	5467	5518	5570	5621	5673	5725	5776	52
843	5828	5879	5931	5982	6034	6085	6137	6188	6240	6291	51
844	6342	6394	6445	6497	6548	6600	6651	6702	6754	6805	51
845	6857	6908	6959	7011	7062	7114	7165	7216	7268	7319	51
846	7370	7422	7473	7524	7576	7627	7678	7730	7781	7832	51
847	7883	7935	7986	8037	8088	8140	8191	8242	8293	8345	51
848	8396	8447	8498	8549	8601	8652	8703	8754	8805	8857	51
849	8908	8959	9010	9061	9112	9163	9215	9266	9317	9368	51
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851	9930	9981									
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852	0440	0491	0542	0592	0643	0694	0745	0796	0847	0898	51
853	0949	1000	1051	1102	1153	1203	1254	1305	1356	1407	51
854	1458	1509	1560	1610	1661	1712	1763	1814	1865	1915	51
855	1966	2017	2068	2118	2169	2220	2271	2322	2372	2423	51
856	2474	2524	2575	2626	2677	2727	2778	2829	2879	2930	51
857	2981	3031	3082	3133	3183	3234	3285	3335	3386	3437	51
858	3487	3538	3589	3639	3690	3740	3791	3841	3892	3943	51
859	3993	4044	4094	4145	4195	4246	4296	4347	4397	4448	51
860	4498	4549	4599	4650	4700	4751	4801	4852	4902	4953	50
861	5003	5054	5104	5154	5205	5255	5306	5356	5406	5457	50
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866	7518	7568	7618	7668	7718	7769	7819	7869	7919	7969	50
867	8019	8069	8119	8169	8219	8269	8319	8370	8420	8470	50
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871	940018	0068	0118	0168	0218	0267	0317	0367	0417	0467	50
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875	2008	2058	2107	2157	2207	2256	2306	2355	2405	2455	50
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877	3000	3049	3099	3148	3198	3247	3297	3346	3396	3445	49
878	3495	3544	3593	3643	3692	3742	3791	3841	3890	3939	49
879	3989	4038	4088	4137	4186	4236	4285	4335	4384	4433	49
880	4483	4532	4581	4631	4680	4729	4779	4828	4877	4927	49
881	4976	5025	5074	5124	5173	5222	5272	5321	5370	5419	49
882	5469	5518	5567	5616	5665	5715	5764	5813	5862	5912	49
883	5961	6010	6059	6108	6157	6207	6256	6305	6354	6403	49
884	6452	6501	6551	6600	6649	6698	6747	6796	6845	6894	49
885	6943	6992	7041	7090	7140	7189	7238	7287	7336	7385	49
886	7434	7483	7532	7581	7630	7679	7728	7777	7826	7875	49
887	7924	7973	8022	8070	8119	8168	8217	8266	8315	8364	49
888	8413	8462	8511	8560	8609	8657	8706	8755	8804	8853	49
889	8902	8951	8999	9048	9097	9146	9195	9244	9292	9341	49
890	9390	9439	9488	9536	9585	9634	9683	9731	9780	9829	49
891	9878	9926	9975								
891	95			0024	0073	0121	0170	0219	0267	0316	49
892	0365	0414	0462	0511	0560	0608	0657	0706	0754	0803	49
893	0851	0900	0949	0997	1046	1095	1143	1192	1240	1289	49
894	1338	1386	1435	1483	1532	1580	1629	1677	1726	1775	49
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897	2792	2841	2889	2938	2986	3034	3083	3131	3180	3228	48
898	3276	3325	3373	3421	3470	3518	3566	3615	3663	3711	48
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900	4243	4291	4339	4387	4435	4484	4532	4580	4628	4677	48
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903	5688	5736	5784	5832	5880	5928	5976	6024	6072	6120	48
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906	7128	7176	7224	7272	7320	7368	7416	7464	7512	7559	48
907	7607	7655	7703	7751	7799	7847	7894	7942	7990	8038	48
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911	9518	9566	9614	9661	9709	9757	9804	9852	9900	9947	48
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915	1421	1469	1516	1563	1611	1658	1706	1753	1801	1848	47
916	1895	1943	1990	2038	2085	2132	2180	2227	2275	2322	47
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918	2843	2890	2937	2985	3032	3079	3126	3174	3221	3268	47
919	3316	3363	3410	3457	3504	3552	3599	3646	3693	3741	47
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921	4260	4307	4354	4401	4448	4495	4542	4590	4637	4684	47
922	4731	4778	4825	4872	4919	4966	5013	5061	5108	5155	47
923	5202	5249	5296	5343	5390	5437	5484	5531	5578	5625	47
924	5672	5719	5766	5813	5860	5907	5954	6001	6048	6095	47
925	6142	6189	6236	6283	6329	6376	6423	6470	6517	6564	47
926	6611	6658	6705	6752	6799	6845	6892	6939	6986	7033	47
927	7080	7127	7173	7220	7267	7314	7361	7408	7454	7501	47
928	7548	7595	7642	7688	7735	7782	7829	7875	7922	7969	47
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933	97			0021	0068	0114	0161	0207	0254	0300	47
934	0347	0393	0440	0486	0533	0579	0626	0672	0719	0765	46
935	0812	0858	0904	0951	0997	1044	1090	1137	1183	1229	46
936	1276	1322	1369	1415	1461	1508	1554	1601	1647	1693	46
937	1740	1786	1832	1879	1925	1971	2018	2064	2110	2157	46
938	2203	2249	2295	2342	2388	2434	2481	2527	2573	2619	46
939	2666	2712	2758	2804	2851	2897	2943	2989	3035	3082	46
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957	0912	0957	1003	1048	1093	1139	1184	1229	1275	1320	45
958	1366	1411	1456	1501	1547	1592	1637	1683	1728	1773	45
959	1819	1864	1909	1954	2000	2045	2090	2135	2181	2226	45
960	2271	2316	2362	2407	2452	2497	2543	2588	2633	2678	45
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981	1669	1713	1758	1802	1846	1890	1935	1979	2023	2067	44
982	2111	2156	2200	2244	2288	2333	2377	2421	2465	2509	44
983	2554	2598	2642	2686	2730	2774	2819	2863	2907	2951	44
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1008	·0034605	1058	·0244857	1108	·0445398	1158	·0637086
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1047	·0199467	1097	·0402066	1147	·0595634	1197	·0780942
1048	·0203613	1098	·0406023	1148	·0599419	1198	·0784568
1049	·0207755	1099	·0409977	1149	·0603200	1199	·0788192
1050	·0211893	1100	·0413927	1150	·0606978	1200	·0791813

LOGARITHMS

OF

SINES AND TANGENTS

FOR EVERY MINUTE OF THE QUADRANT,

AND OF

SINES

FOR EVERY SIX SECONDS OF THE FIRST TWO DEGREES.

0 degrees.

0 min. to 30 min.

′	Sine.	Diff. for 100″	Cosine.	Diff. for 100″	Tangent.	Diff. for 100″	Cotang.	′
0	— ∞		10·000000		— ∞		+ ∞	60
		+ ∞		0		∞		
1	6·463726		10·000000		6·463726		13·536274	59
		501717		0		501717		
2	6·764756		10·000000		6·764756		13·235244	58
		293485		0		293485		
3	6·940847		10·000000		6·940847		13·059153	57
		208232		0		208232		
4	7·065786		10·000000		7·065786		12·934214	56
		161517		0		161517		
5	7·162696		10·000000		7·162696		12·837304	55
		131968		2		131970		
6	7·241877		9·999999		7·241878		12·758122	54
		111578		0		111578		
7	7·308824		9·999999		7·308825		12·691175	53
		96653		0		96653		
8	7·366816		9·999999		7·366817		12·633183	52
		85253		0		85255		
9	7·417968		9·999999		7·417970		12·582030	51
		76263		2		76262		
10	7·463726		9·999998		7·463727		12·536273	50
		68987		0		68988		
11	7·505118		9·999998		7·505120		12·494880	49
		62980		2		62982		
12	7·542906		9·999997		7·542909		12·457091	48
		57937		0		57938		
13	7·577668		9·999997		7·577672		12·422328	47
		53642		2		53642		
14	7·609853		9·999996		7·609857		12·390143	46
		49938		0		49938		
15	7·639816		9·999996		7·639820		12·360180	45
		46715		2		46715		
16	7·667845		9·999995		7·667849		12·332151	44
		43880		0		43883		
17	7·694173		9·999995		7·694179		12·305821	43
		41373		2		41373		
18	7·718997		9·999994		7·719003		12·280997	42
		39135		2		39135		
19	7·742478		9·999993		7·742484		12·257516	41
		37127		0		37128		
20	7·764754		9·999993		7·764761		12·235239	40
		35315		2		35317		
21	7·785943		9·999992		7·785951		12·214049	39
		33672		2		33673		
22	7·806146		9·999991		7·806155		12·193845	38
		32175		2		32175		
23	7·825451		9·999990		7·825460		12·174540	37
		30805		2		30807		
24	7·843934		9·999989		7·843944		12·156056	36
		29547		0		29550		
25	7·861662		9·999989		7·861674		12·138326	35
		28388		2		28390		
26	7·878695		9·999988		7·878708		12·121292	34
		27317		2		27318		
27	7·895085		9·999987		7·895099		12·104901	33
		26323		2		26325		
28	7·910879		9·999986		7·910894		12·089106	32
		25400		2		25400		
29	7·926119		9·999985		7·926134		12·073866	31
		24538		3		24540		
30	7·940842		9·999983		7·940858		12·059142	30
′	Cosine.		Sine.		Cotang.		Tangent.	′

30 min. to 60 min.

89 degrees.

0 degrees.

30 min. to 60 min.

′	Sine.	Diff. for 100′	Cosine.	Diff. for 100′	Tangent.	Diff. for 100′	Cotang.	′
30	7·940842	23733	9·999983	2	7·940858	23737	12·059142	30
31	7·955082	22980	9·999982	2	7·955100	22982	12·044900	29
32	7·968870	22272	9·999981	2	7·968889	22273	12·031111	28
33	7·982233	21608	9·999980	2	7·982253	21610	12·017747	27
34	7·995198	20982	9·999979	3	7·995219	20983	12·004781	26
35	8·007787	20390	9·999977	2	8·007809	20392	11·992191	25
36	8·020021	19830	9·999976	2	8·020044	19835	11·979956	24
37	8·031919	19303	9·999975	3	8·031945	19303	11·968055	23
38	8·043501	18800	9·999973	2	8·043527	18803	11·956473	22
39	8·054781	18325	9·999972	2	8·054809	18328	11·945191	21
40	8·065776	17873	9·999971	3	8·065806	17875	11·934194	20
41	8·076500	17442	9·999969	2	8·076531	17443	11·923469	19
42	8·086965	17030	9·999968	3	8·086997	17033	11·913003	18
43	8·097183	16640	9·999966	3	8·097217	16643	11·902783	17
44	8·107167	16265	9·999964	2	8·107203	16267	11·892797	16
45	8·116926	15908	9·999963	3	8·116963	15912	11·883037	15
46	8·126471	15565	9·999961	3	8·126510	15568	11·873490	14
47	8·135810	15238	9·999959	2	8·135851	15242	11·864149	13
48	8·144953	14923	9·999958	3	8·144996	14927	11·855004	12
49	8·153907	14623	9·999956	3	8·153952	14625	11·846048	11
50	8·162681	14332	9·999954	3	8·162727	14335	11·837273	10
51	8·171280	14055	9·999952	3	8·171328	14058	11·828672	9
52	8·179713	13787	9·999950	3	8·179763	13788	11·820237	8
53	8·187985	13528	9·999948	3	8·188036	13533	11·811964	7
54	8·196102	13280	9·999946	3	8·196156	13283	11·803844	6
55	8·204070	13042	9·999944	3	8·204126	13045	11·795874	5
56	8·211895	12810	9·999942	3	8·211953	12813	11·788047	4
57	8·219581	12588	9·999940	3	8·219641	12590	11·780359	3
58	8·227134	12372	9·999938	3	8·227195	12377	11·772805	2
59	8·234557	12163	9·999936	3	8·234621	12167	11·765379	1
60	8·241855		9·999934		8·241921		11·758079	0
′	Cosine.		Sine.		Cotang.		Tangent.	′

0 min. to 30 min.

89 degrees.

1 degree.

0 min. to 30 min.

′	Sine.	Diff. for 100″	Cosine.	Diff. for 100″	Tangent.	Diff. for 100″	Cotang.	′
0	8·241855	11963	9·999934	3	8·241921	11968	11·758079	60
1	8·249033	11768	9·999932	5	8·249102	11772	11·750898	59
2	8·256094	11580	9·999929	3	8·256165	11583	11·743835	58
3	8·263042	11398	9·999927	3	8·263115	11402	11·736885	57
4	8·269881	11222	9·999925	5	8·269956	11225	11·730044	56
5	8·276614	11048	9·999922	3	8·276691	11053	11·723309	55
6	8·283243	10883	9·999920	3	8·283323	10888	11·716677	54
7	8·289773	10723	9·999918	5	8·289856	10727	11·710144	53
8	8·296207	10565	9·999915	3	8·296292	10570	11·703708	52
9	8·302546	10413	9·999913	5	8·302634	10417	11·697366	51
10	8·308794	10267	9·999910	5	8·308884	10270	11·691116	50
11	8·314954	10122	9·999907	3	8·315046	10127	11·684954	49
12	8·321027	9982	9·999905	5	8·321122	9987	11·678878	48
13	8·327016	9847	9·999902	5	8·327114	9852	11·672886	47
14	8·332924	9715	9·999899	3	8·333025	9718	11·666975	46
15	8·338753	9585	9·999897	5	8·338856	9590	11·661144	45
16	8·344504	9462	9·999894	5	8·344610	9465	11·655390	44
17	8·350181	9337	9·999891	5	8·350289	9343	11·649711	43
18	8·355783	9220	9·999888	5	8·355895	9225	11·644105	42
19	8·361315	9103	9·999885	5	8·361430	9108	11·638570	41
20	8·366777	8990	9·999882	5	8·366895	8995	11·633105	40
21	8·372171	8880	9·999879	5	8·372292	8883	11·627708	39
22	8·377499	8772	9·999876	5	8·377622	8778	11·622378	38
23	8·382762	8667	9·999873	5	8·382889	8672	11·617111	37
24	8·387962	8565	9·999870	5	8·388092	8570	11·611908	36
25	8·393101	8463	9·999867	5	8·393234	8468	11·606766	35
26	8·398179	8367	9·999864	5	8·398315	8372	11·601685	34
27	8·403199	8270	9·999861	5	8·403338	8277	11·596662	33
28	8·408161	8178	9·999858	7	8·408304	8182	11·591696	32
29	8·413068	8085	9·999854	5	8·413213	8092	11·586787	31
30	8·417919		9·999851		8·418068		11·581932	30
′	Cosine.		Sine.		Cotang.		Tangent.	′

30 min. to 60 min.

88 degrees.

1 degree.

30 min. to 60 min.

′	Sine.	Diff. for 100″	Cosine.	Diff. for 100″	Tangent.	Diff. for 100″	Cotang.	′
30	8·417919	7997	9·999851	5	8·418068	8002	11·581932	30
31	8·422717	7908	9·999848	7	8·422869	7915	11·577131	29
32	8·427462	7823	9·999844	5	8·427618	7828	11·572382	28
33	8·432156	7740	9·999841	5	8·432315	7745	11·567685	27
34	8·436800	7657	9·999838	7	8·436962	7663	11·563038	26
35	8·441394	7578	9·999834	5	8·441560	7583	11·558440	25
36	8·445941	7498	9·999831	7	8·446110	7505	11·553890	24
37	8·450440	7422	9·999827	5	8·450613	7428	11·549387	23
38	8·454893	7347	9·999824	7	8·455070	7352	11·544930	22
39	8·459301	7273	9·999820	7	8·459481	7280	11·540519	21
40	8·463665	7200	9·999816	5	8·463849	7205	11·536151	20
41	8·467985	7130	9·999813	7	8·468172	7137	11·531828	19
42	8·472263	7058	9·999809	7	8·472454	7065	11·527546	18
43	8·476498	6992	9·999805	7	8·476693	6998	11·523307	17
44	8·480693	6925	9·999801	7	8·480892	6930	11·519108	16
45	8·484848	6858	9·999797	5	8·485050	6867	11·514950	15
46	8·488963	6795	9·999794	7	8·489170	6800	11·510830	14
47	8·493040	6730	9·999790	7	8·493250	6738	11·506750	13
48	8·497078	6670	9·999786	7	8·497293	6675	11·502707	12
49	8·501080	6608	9·999782	7	8·501298	6615	11·498702	11
50	8·505045	6548	9·999778	7	8·505267	6555	11·494733	10
51	8·508974	6488	9·999774	8	8·509200	6497	11·490800	9
52	8·512867	6432	9·999769	7	8·513098	6438	11·486902	8
53	8·516726	6375	9·999765	7	8·516961	6382	11·483039	7
54	8·520551	6320	9·999761	7	8·520790	6327	11·479210	6
55	8·524343	6265	9·999757	7	8·524586	6272	11·475414	5
56	8·528102	6210	9·999753	8	8·528349	6218	11·471651	4
57	8·531828	6158	9·999748	7	8·532080	6165	11·467920	3
58	8·535523	6105	9·999744	7	8·535779	6113	11·464221	2
59	8·539186	6055	9·999740	8	8·539447	6062	11·460553	1
60	8·542819		9·999735		8·543084		11·456916	0
′	Cosine.		Sine.		Cotang.		Tangent.	′

0 min. to 30 min.

88 degrees.

2 degrees.

0 min. to 30 min.

′	Sine.	Diff. for 100″	Cosine.	Diff. for 100″	Tangent.	Diff. for 100″	Cotang.	′
0	8·542819	6005	9·999735	7	8·543084	6012	11·456916	60
1	8·546422	5955	9·999731	8	8·546691	5962	11·453309	59
2	8·549995	5907	9·999726	7	8·550268	5915	11·449732	58
3	8·553539	5858	9·999722	8	8·553817	5865	11·446183	57
4	8·557054	5810	9·999717	7	8·557336	5820	11·442664	56
5	8·560540	5765	9·999713	8	8·560828	5772	11·439172	55
6	8·563999	5720	9·999708	7	8·564291	5727	11·435709	54
7	8·567431	5675	9·999704	8	8·567727	5683	11·432273	53
8	8·570836	5630	9·999699	8	8·571137	5638	11·428863	52
9	8·574214	5587	9·999694	8	8·574520	5595	11·425480	51
10	8·577566	5543	9·999689	7	8·577877	5552	11·422123	50
11	8·580892	5502	9·999685	8	8·581208	5510	11·418792	49
12	8·584193	5460	9·999680	8	8·584514	5468	11·415486	48
13	8·587469	5420	9·999675	8	8·587795	5427	11·412205	47
14	8·590721	5378	9·999670	8	8·591051	5387	11·408949	46
15	8·593948	5340	9·999665	8	8·594283	5348	11·405717	45
16	8·597152	5300	9·999660	8	8·597492	5308	11·402508	44
17	8·600332	5262	9·999655	8	8·600677	5270	11·399323	43
18	8·603489	5223	9·999650	8	8·603839	5232	11·396161	42
19	8·606623	5185	9·999645	8	8·606978	5193	11·393022	41
20	8·609734	5148	9·999640	8	8·610094	5158	11·389906	40
21	8·612823	5113	9·999635	10	8·613189	5122	11·386811	39
22	8·615891	5077	9·999629	8	8·616262	5085	11·383738	38
23	8·618937	5042	9·999624	8	8·619313	5050	11·380687	37
24	8·621962	5005	9·999619	8	8·622343	5015	11·377657	36
25	8·624965	4972	9·999614	10	8·625352	4980	11·374648	35
26	8·627948	4938	9·999608	8	8·628340	4947	11·371660	34
27	8·630911	4905	9·999603	10	8·631308	4913	11·368692	33
28	8·633854	4870	9·999597	8	8·634256	4880	11·365744	32
29	8·636776	4840	9·999592	10	8·637184	4848	11·362816	31
30	8·639680		9·999586		8·640093		11·359907	30
′	Cosine.		Sine.		Cotang.		Tangent.	′

30 min. to 60 min.

87 degrees.

2 degrees.

30 min. to 60 min.

′	Sine.	Diff. for 100″	Cosine.	Diff. for 100″	Tangent.	Diff. for 100″	Cotang.	′
30	8·639680	4805	9·999586	8	8·640093	4815	11·359907	30
31	8·642563	4775	9·999581	10	8·642982	4785	11·357018	29
32	8·645428	4743	9·999575	8	8·645853	4752	11·354147	28
33	8·648274	4713	9·999570	10	8·648704	4722	11·351296	27
34	8·651102	4682	9·999564	10	8·651537	4692	11·348463	26
35	8·653911	4652	9·999558	8	8·654352	4662	11·345648	25
36	8·656702	4622	9·999553	10	8·657149	4632	11·342851	24
37	8·659475	4592	9·999547	10	8·659928	4602	11·340072	23
38	8·662230	4563	9·999541	10	8·662689	4573	11·337311	22
39	8·664968	4535	9·999535	10	8·665433	4545	11·334567	21
40	8·667689	4507	9·999529	8	8·668160	4517	11·331840	20
41	8·670393	4478	9·999524	10	8·670870	4488	11·329130	19
42	8·673080	4452	9·999518	10	8·673563	4460	11·326437	18
43	8·675751	4423	9·999512	10	8·676239	4435	11·323761	17
44	8·678405	4397	9·999506	10	8·678900	4407	11·321100	16
45	8·681043	4370	9·999500	12	8·681544	4380	11·318456	15
46	8·683665	4345	9·999493	10	8·684172	4353	11·315828	14
47	8·686272	4318	9·999487	10	8·686784	4328	11·313216	13
48	8·688863	4292	9·999481	10	8·689381	4303	11·310619	12
49	8·691438	4267	9·999475	10	8·691963	4277	11·308037	11
50	8·693998	4242	9·999469	10	8·694529	4253	11·305471	10
51	8·696543	4217	9·999463	12	8·697081	4227	11·302919	9
52	8·699073	4193	9·999456	10	8·699617	4203	11·300383	8
53	8·701589	4168	9·999450	12	8·702139	4178	11·297861	7
54	8·704090	4145	9·999443	10	8·704646	4157	11·295354	6
55	8·706577	4120	9·999437	10	8·707140	4130	11·292860	5
56	8·709049	4097	9·999431	12	8·709618	4108	11·290382	4
57	8·711507	4075	9·999424	10	8·712083	4085	11·287917	3
58	8·713952	4052	9·999418	12	8·714534	4063	11·285466	2
59	8·716383	4028	9·999411	12	8·716972	4040	11·283028	1
60	8·718800		9·999404		8·719396		11·280604	0
′	Cosine.		Sine.		Cotang.		Tangent.	′

0 min. to 30 min.

87 degrees.

3 degrees.

0 min. to 30 min.

′	Sine.	Diff. for 100″	Cosine.	Diff. for 100″	Tangent.	Diff. for 100″	Cotang.	′
0	8·718800	4007	9·999404	10	8·719396	4017	11·280604	60
1	8·721204	3985	9·999398	12	8·721806	3997	11·278194	59
2	8·723595	3962	9·999391	12	8·724204	3973	11·275796	58
3	8·725972	3942	9·999384	10	8·726588	3952	11·273412	57
4	8·728337	3918	9·999378	12	8·728959	3930	11·271041	56
5	8·730688	3898	9·999371	12	8·731317	3910	11·268683	55
6	8·733027	3878	9·999364	12	8·733663	3888	11·266337	54
7	8·735354	3855	9·999357	12	8·735996	3868	11·264004	53
8	8·737667	3837	9·999350	12	8·738317	3848	11·261683	52
9	8·739969	3817	9·999343	12	8·740626	3827	11·259374	51
10	8·742259	3795	9·999336	12	8·742922	3808	11·257078	50
11	8·744536	3777	9·999329	12	8·745207	3787	11·254793	49
12	8·746802	3755	9·999322	12	8·747479	3768	11·252521	48
13	8·749055	3737	9·999315	12	8·749740	3748	11·250260	47
14	8·751297	3718	9·999308	12	8·751989	3730	11·248011	46
15	8·753528	3698	9·999301	12	8·754227	3710	11·245773	45
16	8·755747	3680	9·999294	12	8·756453	3692	11·243547	44
17	8·757955	3660	9·999287	13	8·758668	3673	11·241332	43
18	8·760151	3643	9·999279	12	8·760872	3655	11·239128	42
19	8·762337	3623	9·999272	12	8·763065	3635	11·236935	41
20	8·764511	3607	9·999265	13	8·765246	3618	11·234754	40
21	8·766675	3588	9·999257	12	8·767417	3602	11·232583	39
22	8·768828	3570	9·999250	13	8·769578	3582	11·230422	38
23	8·770970	3552	9·999242	12	8·771727	3565	11·228273	37
24	8·773101	3537	9·999235	13	8·773866	3548	11·226134	36
25	8·775223	3517	9·999227	12	8·775995	3532	11·224005	35
26	8·777333	3502	9·999220	13	8·778114	3513	11·221886	34
27	8·779434	3483	9·999212	12	8·780222	3497	11·219778	33
28	8·781524	3468	9·999205	13	8·782320	3480	11·217680	32
29	8·783605	3450	9·999197	13	8·784408	3463	11·215592	31
30	8·785675		9·999189		8·786486		11·213514	30
′	Cosine.		Sine.		Cotang.		Tangent.	′

30 min. to 60 min.

86 degrees.

3 degrees.

30 min. to 60 min.

′	Sine.	Diff. for 100″	Cosine.	Diff. for 100″	Tangent.	Diff. for 100″	Cotang.	′
30	8·785675	3435	9·999189	13	8·786486	3447	11·213514	30
31	8·787736	3418	9·999181	12	8·788554	3432	11·211446	29
32	8·789787	3402	9·999174	13	8·790613	3415	11·209387	28
33	8·791828	3385	9·999166	13	8·792662	3398	11·207338	27
34	8·793859	3370	9·999158	13	8·794701	3383	11·205299	26
35	8·795881	3355	9·999150	13	8·796731	3368	11·203269	25
36	8·797894	3338	9·999142	13	8·798752	3352	11·201248	24
37	8·799897	3325	9·999134	13	8·800763	3337	11·199237	23
38	8·801892	3307	9·999126	13	8·802765	3322	11·197235	22
39	8·803876	3293	9·999118	13	8·804758	3307	11·195242	21
40	8·805852	3278	9·999110	13	8·806742	3292	11·193258	20
41	8·807819	3263	9·999102	13	8·808717	3277	11·191283	19
42	8·809777	3248	9·999094	13	8·810683	3263	11·189317	18
43	8·811726	3235	9·999086	15	8·812641	3247	11·187359	17
44	8·813667	3220	9·999077	13	8·814589	3233	11·185411	16
45	8·815599	3205	9·999069	13	8·816529	3220	11·183471	15
46	8·817522	3190	9·999061	13	8·818461	3205	11·181539	14
47	8·819436	3178	9·999053	15	8·820384	3190	11·179616	13
48	8·821343	3162	9·999044	13	8·822298	3178	11·177702	12
49	8·823240	3150	9·999036	15	8·824205	3163	11·175795	11
50	8·825130	3135	9·999027	13	8·826103	3148	11·173897	10
51	8·827011	3122	9·999019	15	8·827992	3137	11·172008	9
52	8·828884	3108	9·999010	13	8·829874	3123	11·170126	8
53	8·830749	3097	9·999002	15	8·831748	3108	11·168252	7
54	8·832607	3082	9·998993	15	8·833613	3097	11·166387	6
55	8·834456	3068	9·998984	13	8·835471	3083	11·164529	5
56	8·836297	3055	9·998976	15	8·837321	3070	11·162679	4
57	8·838130	3043	9·998967	15	8·839163	3058	11·160837	3
58	8·839956	3030	9·998958	13	8·840998	3045	11·159002	2
59	8·841774	3018	9·998950	15	8·842825	3032	11·157175	1
60	8·843585		9·998941		8·844644		11·155356	0
′	Cosine.		Sine.		Cotang.		Tangent.	′

0 min. to 30 min.

86 degrees.

4 degrees.

0 min. to 30 min.

′	Sine.	Diff. for 100″	Cosine.	Diff. for 100″	Tangent	Diff. for 100″	Cotang.	′
0	8·843585	3003	9·998941	15	8·844644	3018	11·155356	60
1	8·845387	2993	9·998932	15	8·846455	3008	11·153545	59
2	8·847183	2980	9·998923	15	8·848260	2995	11·151740	58
3	8·848971	2967	9·998914	15	8·850057	2982	11·149943	57
4	8·850751	2957	9·998905	15	8·851846	2970	11·148154	56
5	8·852525	2943	9·998896	15	8·853628	2958	11·146372	55
6	8·854291	2930	9·998887	15	8·855403	2947	11·144597	54
7	8·856049	2920	9·998878	15	8·857171	2935	11·142829	53
8	8·857801	2908	9·998869	15	8·858932	2923	11·141068	52
9	8·859546	2895	9·998860	15	8·860686	2912	11·139314	51
10	8·861283	2885	9·998851	17	8·862433	2900	11·137567	50
11	8·863014	2873	9·998841	15	8·864173	2888	11·135827	49
12	8·864738	2862	9·998832	15	8·865906	2877	11·134094	48
13	8·866455	2850	9·998823	17	8·867632	2865	11·132368	47
14	8·868165	2838	9·998813	15	8·869351	2855	11·130649	46
15	8·869868	2828	9·998804	15	8·871064	2843	11·128936	45
16	8·871565	2817	9·998795	17	8·872770	2832	11·127230	44
17	8·873255	2805	9·998785	15	8·874469	2822	11·125531	43
18	8·874938	2795	9·998776	17	8·876162	2812	11·123838	42
19	8·876615	2783	9·998766	15	8·877849	2800	11·122151	41
20	8·878285	2773	9·998757	17	8·879529	2788	11·120471	40
21	8·879949	2763	9·998747	15	8·881202	2778	11·118798	39
22	8·881607	2752	9·998738	17	8·882869	2768	11·117131	38
23	8·883258	2742	9·998728	17	8·884530	2758	11·115470	37
24	8·884903	2732	9·998718	17	8·886185	2747	11·113815	36
25	8·886542	2720	9·998708	15	8·887833	2738	11·112167	35
26	8·888174	2712	9·998699	17	8·889476	2727	11·110524	34
27	8·889801	2700	9·998689	17	8·891112	2717	11·108888	33
28	8·891421	2690	9·998679	17	8·892742	2707	11·107258	32
29	8·893035	2680	9·998669	17	8·894366	2697	11·105634	31
30	8·894643		9·998659		8·895984		11·104016	30
′	Cosine.		Sine.		Cotang.		Tangent.	′

30 min. to 60 min.

85 degrees.

4 degrees.

30 min. to 60 min.

′	Sine.	Diff. for 100″	Ccsine.	Diff. for 100″	Tangent.	Diff. for 100″	Cotang.	′
30	8·894643	2672	9·998659	17	8·895984	2687	11·104016	30
31	8·896246	2660	9·998649	17	8·897596	2678	11·102404	29
32	8·897842	2650	9·998639	17	8·899203	2667	11·100797	28
33	8·899432	2642	9·998629	17	8·900803	2658	11·099197	27
34	8·901017	2632	9·998619	17	8·902398	2648	11·097602	26
35	8·902596	2622	9·998609	17	8·903987	2638	11·096013	25
36	8·904169	2612	9·998599	17	8·905570	2628	11·094430	24
37	8·905736	2602	9·998589	18	8·907147	2620	11·092853	23
38	8·907297	2593	9·998578	17	8·908719	2610	11·091281	22
39	8·908853	2585	9·998568	17	8·910285	2602	11·089715	21
40	8·910404	2575	9·998558	17	8·911846	2592	11·088154	20
41	8·911949	2565	9·998548	18	8·913401	2583	11·086599	19
42	8·913488	2557	9·998537	17	8·914951	2573	11·085049	18
43	8·915022	2547	9·998527	18	8·916495	2565	11·083505	17
44	8·916550	2538	9·998516	17	8·918034	2557	11·081966	16
45	8·918073	2530	9·998506	18	8·919568	2547	11·080432	15
46	8·919591	2520	9·998495	17	8·921096	2538	11·078904	14
47	8·921103	2512	9·998485	18	8·922619	2528	11·077381	13
48	8·922610	2503	9·998474	17	8·924136	2522	11·075864	12
49	8·924112	2495	9·998464	18	8·925649	2512	11·074351	11
50	8·925609	2485	9·998453	18	8·927156	2503	11·072844	10
51	8·927100	2478	9·998442	18	8·928658	2495	11·071342	9
52	8·928587	2468	9·998431	17	8·930155	2487	11·069845	8
53	8·930068	2460	9·998421	18	8·931647	2478	11·068353	7
54	8·931544	2452	9·998410	18	8·933134	2470	11·066866	6
55	8·933015	2443	9·998399	18	8·934616	2462	11·065384	5
56	8·934481	2435	9·998388	18	8·936093	2453	11·063907	4
57	8·935942	2427	9·998377	18	8·937565	2445	11·062435	3
58	8·937398	2420	9·998366	18	8·939032	2437	11·060968	2
59	8·938850	2410	9·998355	18	8·940494	2430	11·059506	1
60	8·940296		9·998344		8·941952		11·058048	0
′	Cosine.		Sine.		Cotang.		Tangent.	′

0 min. to 30 min.

85 degrees.

5 degrees.

0 min. to 30 min.

′	Sine.	Diff. for 100″	Cosine.	Diff. for 100″	Tangent.	Diff. for 100″	Cotang.	′
0	8·940296	2403	9·998344	18	8·941952	2420	11·058048	60
1	8·941738	2393	9·998333	18	8·943404	2413	11·056596	59
2	8·943174	2387	9·998322	18	8·944852	2405	11·055148	58
3	8·944606	2380	9·998311	18	8·946295	2398	11·053705	57
4	8·946034	2370	9·998300	18	8·947734	2390	11·052266	56
5	8·947456	2363	9·998289	20	8·949168	2382	11·050832	55
6	8·948874	2355	9·998277	18	8·950597	2373	11·049403	54
7	8·950287	2348	9·998266	18	8·952021	2367	11·047979	53
8	8·951696	2340	9·998255	20	8·953441	2358	11·046559	52
9	8·953100	2332	9·998243	18	8·954856	2352	11·045144	51
10	8·954499	2325	9·998232	20	8·956267	2345	11·043733	50
11	8·955894	2317	9·998220	18	8·957674	2335	11·042326	49
12	8·957284	2310	9·998209	20	8·959075	2330	11·040925	48
13	8·958670	2303	9·998197	18	8·960473	2322	11·039527	47
14	8·960052	2295	9·998186	20	8·961866	2315	11·038134	46
15	8·961429	2287	9·998174	18	8·963255	2307	11·036745	45
16	8·962801	2282	9·998163	20	8·964639	2300	11·035361	44
17	8·964170	2273	9·998151	20	8·966019	2292	11·033981	43
18	8·965534	2265	9·998139	18	8·967394	2287	11·032606	42
19	8·966893	2260	9·998128	20	8·968766	2278	11·031234	41
20	8·968249	2252	9·998116	20	8·970133	2272	11·029867	40
21	8·969600	2245	9·998104	20	8·971496	2265	11·028504	39
22	8·970947	2237	9·998092	20	8·972855	2257	11·027145	38
23	8·972289	2232	9·998080	20	8·974209	2252	11·025791	37
24	8·973628	2223	9·998068	20	8·975560	2243	11·024440	36
25	8·974962	2218	9·998056	20	8·976906	2237	11·023094	35
26	8·976293	2210	9·998044	20	8·978248	2230	11·021752	34
27	8·977619	2203	9·998032	20	8·979586	2225	11·020414	33
28	8·978941	2197	9·998020	20	8·980921	2217	11·019079	32
29	8·980259	2190	9·998008	20	8·982251	2210	11·017749	31
30	8·981573		9·997996		8·983577		11·016423	30
′	Cosine.		Sine.		Cotang.		Tangent.	′

30 min. to 60 min.

84 degrees.

5 degrees.

30 min. to 60 min.

′	Sine.	Diff. for 100″	Cosine.	Diff. for 100″	Tangent.	Diff. for 100″	Cotang.	′
30	8·981573	2183	9·997996	20	8·983577	2203	11·016423	30
31	8·982883	2177	9·997984	20	8·984899	2197	11·015101	29
32	8·984189	2170	9·997972	22	8·986217	2192	11·013783	28
33	8·985491	2163	9·997959	20	8·987532	2183	11·012468	27
34	8·986789	2157	9·997947	20	8·988842	2178	11·011158	26
35	8·988083	2152	9·997935	22	8·990149	2170	11·009851	25
36	8·989374	2143	9·997922	20	8·991451	2165	11·008549	24
37	8·990660	2138	9·997910	22	8·992750	2158	11·007250	23
38	8·991943	2132	9·997897	20	8·994045	2153	11·005955	22
39	8·993222	2125	9·997885	22	8·995337	2145	11·004663	21
40	8·994497	2118	9·997872	20	8·996624	2140	11·003376	20
41	8·995768	2113	9·997860	22	8·997908	2133	11·002092	19
42	8·997036	2105	9·997847	20	8·999188	2128	11·000812	18
43	8·998299	2102	9·997835	22	9·000465	2122	10·999535	17
44	8·999560	2093	9·997822	22	9·001738	2115	10·998262	16
45	9·000816	2088	9·997809	20	9·003007	2108	10·996993	15
46	9·002069	2082	9·997797	22	9·004272	2103	10·995728	14
47	9·003318	2075	9·997784	22	9·005534	2097	10·994466	13
48	9·004563	2070	9·997771	22	9·006792	2092	10·993208	12
49	9·005805	2065	9·997758	22	9·008047	2085	10·991953	11
50	9·007044	2057	9·997745	22	9·009298	2080	10·990702	10
51	9·008278	2053	9·997732	22	9·010546	2073	10·989454	9
52	9·009510	2045	9·997719	22	9·011790	2068	10·988210	8
53	9·010737	2042	9·997706	22	9·013031	2062	10·986969	7
54	9·011962	2033	9·997693	22	9·014268	2057	10·985732	6
55	9·013182	2030	9·997680	22	9·015502	2050	10·984498	5
56	9·014400	2022	9·997667	22	9·016732	2045	10·983268	4
57	9·015613	2018	9·997654	22	9·017959	2040	10·982041	3
58	9·016824	2012	9·997641	22	9·019183	2033	10·980817	2
59	9·018031	2007	9·997628	23	9·020403	2028	10·979597	1
60	9·019235		9·997614		9·021620		10·978380	0
′	Cosine.		Sine.		Cotang.		Tangent.	′

0 min. to 30 min.

84 degrees.

6 degrees.

0 min. to 30 min.

′	Sine.	Diff. for 100″	Cosine.	Diff. for 100″	Tangent.	Diff. for 100″	Cotang.	′
0	9·019235	2000	9·997614	22	9·021620	2023	10·978380	60
1	9·020435	1995	9·997601	22	9·022834	2017	10·977166	59
2	9·021632	1988	9·997588	23	9·024044	2012	10·975956	58
3	9·022825	1985	9·997574	22	9·025251	2007	10·974749	57
4	9·024016	1978	9·997561	23	9·026455	2000	10·973545	56
5	9·025203	1972	9·997547	22	9·027655	1995	10·972345	55
6	9·026386	1968	9·997534	23	9·028852	1990	10·971148	54
7	9·027567	1962	9·997520	22	9·030046	1985	10·969954	53
8	9·028744	1957	9·997507	23	9·031237	1980	10·968763	52
9	9·029918	1952	9·997493	22	9·032425	1973	10·967575	51
10	9·031089	1947	9·997480	23	9·033609	1970	10·966391	50
11	9·032257	1940	9·997466	23	9·034791	1963	10·965209	49
12	9·033421	1935	9·997452	22	9·035969	1958	10·964031	48
13	9·034582	1932	9·997439	23	9·037144	1953	10·962856	47
14	9·035741	1925	9·997425	23	9·038316	1948	10·961684	46
15	9·036896	1920	9·997411	23	9·039485	1943	10·960515	45
16	9·038048	1915	9·997397	23	9·040651	1937	10·959349	44
17	9·039197	1908	9·997383	23	9·041813	1933	10·958187	43
18	9·040342	1905	9·997369	23	9·042973	1928	10·957027	42
19	9·041485	1900	9·997355	23	9·044130	1923	10·955870	41
20	9·042625	1895	9·997341	23	9·045284	1917	10·954716	40
21	9·043762	1888	9·997327	23	9·046434	1913	10·953566	39
22	9·044895	1885	9·997313	23	9·047582	1908	10·952418	38
23	9·046026	1880	9·997299	23	9·048727	1903	10·951273	37
24	9·047154	1875	9·997285	23	9·049869	1898	10·950131	36
25	9·048279	1868	9·997271	23	9·051008	1893	10·948992	35
26	9·049400	1865	9·997257	25	9·052144	1888	10·947856	34
27	9·050519	1860	9·997242	23	9·053277	1883	10·946723	33
28	9·051635	1857	9·997228	23	9·054407	1880	10·945593	32
29	9·052749	1850	9·997214	25	9·055535	1873	10·944465	31
30	9·053859		9·997199		9·056659		10·943341	30
′	Cosine.		Sine.		Cotang.		Tangent.	′

30 min. to 60 min.

83 degrees.

6 degrees.

30 min. to 60 min.

′	Sine.	Diff. for 100″	Cosine.	Diff. for 100″	Tangent.	Diff. for 100″	Cotang.	′
30	9·053859	1845	9·997199	23	9·056659	1870	10·943341	30
31	9·054966	1842	9·997185	25	9·057781	1865	10·942219	29
32	9·056071	1835	9·997170	23	9·058900	1860	10·941100	28
33	9·057172	1832	9·997156	25	9·060016	1857	10·939984	27
34	9·058271	1827	9·997141	23	9·061130	1850	10·938870	26
35	9·059367	1822	9·997127	25	9·062240	1847	10·937760	25
36	9·060460	1818	9·997112	23	9·063348	1842	10·936652	24
37	9·061551	1813	9·997098	25	9·064453	1838	10·935547	23
38	9·062639	1808	9·997083	25	9·065556	1832	10·934444	22
39	9·063724	1803	9·997068	25	9·066655	1828	10·933345	21
40	9·064806	1798	9·997053	23	9·067752	1823	10·932248	20
41	9·065885	1795	9·997039	25	9·068846	1820	10·931154	19
42	9·066962	1790	9·997024	25	9·069938	1815	10·930062	18
43	9·068036	1785	9·997009	25	9·071027	1810	10·928973	17
44	9·069107	1782	9·996994	25	9·072113	1807	10·927887	16
45	9·070176	1777	9·996979	25	9·073197	1802	10·926803	15
46	9·071242	1773	9·996964	25	9·074278	1797	10·925722	14
47	9·072306	1767	9·996949	25	9·075356	1793	10·924644	13
48	9·073366	1763	9·996934	25	9·076432	1788	10·923568	12
49	9·074424	1760	9·996919	25	9·077505	1785	10·922495	11
50	9·075480	1755	9·996904	25	9·078576	1780	10·921424	10
51	9·076533	1750	9·996889	25	9·079644	1777	10·920356	9
52	9·077583	1747	9·996874	27	9·080710	1772	10·919290	8
53	9·078631	1742	9·996858	25	9·081773	1767	10·918227	7
54	9·079676	1738	9·996843	25	9·082833	1763	10·917167	6
55	9·080719	1733	9·996828	27	9·083891	1760	10·916109	5
56	9·081759	1730	9·996812	25	9·084947	1755	10·915053	4
57	9·082797	1725	9·996797	25	9·086000	1750	10·914000	3
58	9·083832	1720	9·996782	27	9·087050	1747	10·912950	2
59	9·084864	1717	9·996766	25	9·088098	1743	10·911902	1
60	9·085894		9·996751		9·089144		10·910856	0
′	Cosine.		Sine.		Cotang.		Tangent.	′

0 min. to 30 min.

83 degrees.

7 degrees.

0 min. to 30 min.

′	Sine.	Diff. for 100″	Cosine.	Diff for 100″	Tangent.	Diff. for 100″	Cotang.	′
0	9·085894	1713	9·996751	27	9·089144	1738	10·910856	60
1	9·086922	1708	9·996735	25	9·090187	1735	10·909813	59
2	9·087947	1705	9·996720	27	9·091228	1730	10·908772	58
3	9·088970	1700	9·996704	27	9·092266	1727	10·907734	57
4	9·089990	1697	9·996688	25	9·093302	1723	10·906698	56
5	9·091008	1693	9·996673	27	9·094336	1718	10·905664	55
6	9·092024	1688	9·996657	27	9·095367	1713	10·904633	54
7	9·093037	1683	9·996641	27	9·096395	1712	10·903605	53
8	9·094047	1682	9·996625	25	9·097422	1707	10·902578	52
9	9·095056	1677	9·996610	27	9·098446	1703	10·901554	51
10	9·096062	1672	9·996594	27	9·099468	1698	10·900532	50
11	9·097065	1668	9·996578	27	9·100487	1695	10·899513	49
12	9·098066	1665	9·996562	27	9·101504	1692	10·898496	48
13	9·099065	1662	9·996546	27	9·102519	1688	10·897481	47
14	9·100062	1657	9·996530	27	9·103532	1683	10·896468	46
15	9·101056	1653	9·996514	27	9·104542	1680	10·895458	45
16	9·102048	1648	9·996498	27	9·105550	1677	10·894450	44
17	9·103037	1647	9·996482	28	9·106556	1672	10·893444	43
18	9·104025	1642	9·996465	27	9·107559	1668	10·892441	42
19	9·105010	1637	9·996449	27	9·108560	1665	10·891440	41
20	9·105992	1635	9·996433	27	9·109559	1662	10·890441	40
21	9·106973	1630	9·996417	28	9·110556	1658	10·889444	39
22	9·107951	1627	9·996400	27	9·111551	1653	10·888449	38
23	9·108927	1623	9·996384	27	9·112543	1650	10·887457	37
24	9·109901	1620	9·996368	28	9·113533	1647	10·886467	36
25	9·110873	1615	9·996351	27	9·114521	1643	10·885479	35
26	9·111842	1612	9·996335	28	9·115507	1640	10·884493	34
27	9·112809	1608	9·996318	27	9·116491	1635	10·883509	33
28	9·113774	1605	9·996302	28	9·117472	1633	10·882528	32
29	9·114737	1602	9·996285	27	9·118452	1628	10·881548	31
30	9·115698		9·996269		9·119429		10·880571	30
′	Cosine.		Sine.		Cotang.		Tangent.	′

30 min. to 60 min.

82 degrees.

7 degrees.

30 min. to 60 min.

′	Sine.	Diff. for 100″	Cosine.	Diff. for 100″	Tangent.	Diff. for 100″	Cotang.	′
30	9·115698	1597	9·996269	28	9·119429	1625	10·880571	30
31	9·116656	1595	9·996252	28	9·120404	1622	10·879596	29
32	9·117613	1590	9·996235	27	9·121377	1618	10·878623	28
33	9·118567	1587	9·996219	28	9·122348	1615	10·877652	27
34	9·119519	1583	9·996202	28	9·123317	1612	10·876683	26
35	9·120469	1580	9·996185	28	9·124284	1608	10·875716	25
36	9·121417	1575	9·996168	28	9·125249	1603	10·874751	24
37	9·122362	1573	9·996151	28	9·126211	1602	10·873789	23
38	9·123306	1570	9·996134	28	9·127172	1597	10·872828	22
39	9·124248	1565	9·996117	28	9·128130	1595	10·871870	21
40	9·125187	1563	9·996100	28	9·129087	1590	10·870913	20
41	9·126125	1558	9·996083	28	9·130041	1588	10·869959	19
42	9·127060	1555	9·996066	28	9·130994	1583	10·869006	18
43	9·127993	1553	9·996049	28	9·131944	1582	10·868056	17
44	9·128925	1548	9·996032	28	9·132893	1577	10·867107	16
45	9·129854	1545	9·996015	28	9·133839	1575	10·866161	15
46	9·130781	1542	9·995998	30	9·134784	1570	10·865216	14
47	9·131706	1540	9·995980	28	9·135726	1568	10·864274	13
48	9·132630	1535	9·995963	28	9·136667	1563	10·863333	12
49	9·133551	1532	9·995946	30	9·137605	1562	10·862395	11
50	9·134470	1528	9·995928	28	9·138542	1557	10·861458	10
51	9·135387	1527	9·995911	28	9·139476	1555	10·860524	9
52	9·136303	1522	9·995894	30	9·140409	1552	10·859591	8
53	9·137216	1520	9·995876	28	9·141340	1548	10·858660	7
54	9·138128	1515	9·995859	30	9·142269	1545	10·857731	6
55	9·139037	1512	9·995841	30	9·143196	1542	10·856804	5
56	9·139944	1510	9·995823	28	9·144121	1538	10·855879	4
57	9·140850	1507	9·995806	30	9·145044	1537	10·854956	3
58	9·141754	1502	9·995788	28	9·145966	1532	10·854034	2
59	9·142655	1500	9·995771	30	9·146885	1530	10·853115	1
60	9·143555		9·995753		9·147803		10·852197	0
′	Cosine.		Sine.		Cotang.		Tangent.	′

0 min. to 30 min.

82 degrees.

8 degrees.

0 min. to 30 min.

′	Sine.	Diff. for 100″	Cosine.	Diff. for 100″	Tangent.	Diff. for 100″	Cotang.	′
0	9·143555	1497	9·995753	30	9·147803	1525	10·852197	60
1	9·144453	1493	9·995735	30	9·148718	1523	10·851282	59
2	9·145349	1490	9·995717	30	9·149632	1520	10·850368	58
3	9·146243	1488	9·995699	30	9·150544	1517	10·849456	57
4	9·147136	1483	9·995681	28	9·151454	1515	10·848546	56
5	9·148026	1482	9·995664	30	9·152363	1510	10·847637	55
6	9·148915	1478	9·995646	30	9·153269	1508	10·846731	54
7	9·149802	1473	9·995628	30	9·154174	1505	10·845826	53
8	9·150686	1472	9·995610	32	9·155077	1502	10·844923	52
9	9·151569	1470	9·995591	30	9·155978	1498	10·844022	51
10	9·152451	1465	9·995573	30	9·156877	1497	10·843123	50
11	9·153330	1463	9·995555	30	9·157775	1493	10·842225	49
12	9·154208	1458	9·995537	30	9·158671	1490	10·841329	48
13	9·155083	1457	9·995519	30	9·159565	1487	10·840435	47
14	9·155957	1455	9·995501	32	9·160457	1483	10·839543	46
15	9·156830	1450	9·995482	30	9·161347	1482	10·838653	45
16	9·157700	1448	9·995464	30	9·162236	1478	10·837764	44
17	9·158569	1443	9·995446	32	9·163123	1475	10·836877	43
18	9·159435	1443	9·995427	30	9·164008	1473	10·835992	42
19	9·160301	1438	9·995409	32	9·164892	1470	10·835108	41
20	9·161164	1435	9·995390	30	9·165774	1467	10·834226	40
21	9·162025	1433	9·995372	32	9·166654	1463	10·833346	39
22	9·162885	1430	9·995353	32	9·167532	1462	10·832468	38
23	9·163743	1428	9·995334	30	9·168409	1458	10·831591	37
24	9·164600	1423	9·995316	32	9·169284	1455	10·830716	36
25	9·165454	1422	9·995297	32	9·170157	1453	10·829843	35
26	9·166307	1420	9·995278	30	9·171029	1450	10·828971	34
27	9·167159	1415	9·995260	32	9·171899	1447	10·828101	33
28	9·168008	1413	9·995241	32	9·172767	1445	10·827233	32
29	9·168856	1410	9·995222	32	9·173634	1442	10·826366	31
30	9·169702		9·995203		9·174499		10·825501	30
′	Cosine.		Sine.		Cotang.		Tangent.	′

30 min. to 60 min.

81 degrees.

8 degrees.

30 min. to 60 min.

′	Sine.	Diff. for 100″	Cosine.	Diff. for 100″	Tangent.	Diff. for 100″	Cotang.	′
30	9·169702	1408	9·995203	32	9·174499	1438	10·825501	30
31	9·170547	1403	9·995184	32	9·175362	1437	10·824638	29
32	9·171389	1402	9·995165	32	9·176224	1433	10·823776	28
33	9·172230	1400	9·995146	32	9·177084	1430	10·822916	27
34	9·173070	1397	9·995127	32	9·177942	1428	10·822058	26
35	9·173908	1393	9·995108	32	9·178799	1427	10·821201	25
36	9·174744	1390	9·995089	32	9·179655	1422	10·820345	24
37	9·175578	1388	9·995070	32	9·180508	1420	10·819492	23
38	9·176411	1385	9·995051	32	9·181360	1418	10·818640	22
39	9·177242	1383	9·995032	32	9·182211	1413	10·817789	21
40	9·178072	1380	9·995013	33	9·183059	1413	10·816941	20
41	9·178900	1377	9·994993	32	9·183907	1408	10·816093	19
42	9·179726	1375	9·994974	32	9·184752	1408	10·815248	18
43	9·180551	1372	9·994955	33	9·185597	1403	10·814403	17
44	9·181374	1370	9·994935	32	9·186439	1402	10·813561	16
45	9·182196	1367	9·994916	33	9·187280	1400	10·812720	15
46	9·183016	1363	9·994896	32	9·188120	1397	10·811880	14
47	9·183834	1362	9·994877	33	9·188958	1393	10·811042	13
48	9·184651	1358	9·994857	32	9·189794	1392	10·810206	12
49	9·185466	1357	9·994838	33	9·190629	1388	10·809371	11
50	9·186280	1353	9·994818	33	9·191462	1387	10·808538	10
51	9·187092	1352	9·994798	32	9·192294	1383	10·807706	9
52	9·187903	1348	9·994779	33	9·193124	1382	10·806876	8
53	9·188712	1345	9·994759	33	9·193953	1378	10·806047	7
54	9·189519	1343	9·994739	32	9·194780	1377	10·805220	6
55	9·190325	1342	9·994720	33	9·195606	1373	10·804394	5
56	9·191130	1338	9·994700	33	9·196430	1372	10·803570	4
57	9·191933	1335	9·994680	33	9·197253	1368	10·802747	3
58	9·192734	1333	9·994660	33	9·198074	1367	10·801926	2
59	9·193534	1330	9·994640	33	9·198894	1365	10·801106	1
60	9·194332		9·994620		9·199713		10·800287	0
′	Cosine.		Sine.		Cotang.		Tangent.	′

0 min. to 30 min.

81 degrees.

9 degrees.

0 min. to 30 min.

′	Sine.	Diff. for 100″	Cosine.	Diff. for 100″	Tangent.	Diff. for 100″	Cotang.	′
0	9·194332	1328	9·994620	33	9·199713	1360	10·800287	60
1	9·195129	1327	9·994600	33	9·200529	1360	10·799471	59
2	9·195925	1323	9·994580	33	9·201345	1357	10·798655	58
3	9·196719	1320	9·994560	33	9·202159	1353	10·797841	57
4	9·197511	1318	9·994540	35	9·202971	1352	10·797029	56
5	9·198302	1315	9·994519	33	9·203782	1350	10·796218	55
6	9·199091	1313	9·994499	33	9·204592	1347	10·795408	54
7	9·199879	1312	9·994479	33	9·205400	1345	10·794600	53
8	9·200666	1308	9·994459	35	9·206207	1343	10·793793	52
9	9·201451	1305	9·994438	33	9·207013	1340	10·792987	51
10	9·202234	1305	9·994418	33	9·207817	1337	10·792183	50
11	9·203017	1300	9·994398	35	9·208619	1335	10·791381	49
12	9·203797	1300	9·994377	33	9·209420	1333	10·790580	48
13	9·204577	1295	9·994357	35	9·210220	1330	10·789780	47
14	9·205354	1295	9·994336	33	9·211018	1328	10·788982	46
15	9·206131	1292	9·994316	35	9·211815	1327	10·788185	45
16	9·206906	1288	9·994295	35	9·212611	1323	10·787389	44
17	9·207679	1288	9·994274	33	9·213405	1322	10·786595	43
18	9·208452	1283	9·994254	35	9·214198	1318	10·785802	42
19	9·209222	1283	9·994233	35	9·214989	1318	10·785011	41
20	9·209992	1280	9·994212	35	9·215780	1313	10·784220	40
21	9·210760	1277	9·994191	33	9·216568	1313	10·783432	39
22	9·211526	1275	9·994171	35	9·217356	1310	10·782644	38
23	9·212291	1273	9·994150	35	9·218142	1307	10·781858	37
24	9·213055	1272	9·994129	35	9·218926	1307	10·781074	36
25	9·213818	1268	9·994108	35	9·219710	1303	10·780290	35
26	9·214579	1265	9·994087	35	9 220492	1300	10·779508	34
27	9·215338	1265	9·994066	35	9·221272	1300	10·778728	33
28	9·216097	1262	9·994045	35	9·222052	1297	10·777948	32
29	9·216854	1258	9·994024	35	9·222830	1295	10·777170	31
30	9·217609		9·994003		9·223607		10·776393	30
′	Cosine.		Sine.		Cotang.		Tangent.	′

30 min. to 60 min.

80 degrees.

9 degrees.

30 min. to 60 min.

′	Sine.	Diff. for 100″	Cosine.	Diff. for 100″	Tangent.	Diff. for 100″	Cotang.	′
30	9·217609	1257	9·994003	35	9·223607	1292	10·776393	30
31	9·218363	1255	9·993982	37	9·224382	1290	10·775618	29
32	9·219116	1253	9·993960	35	9·225156	1288	10·774844	28
33	9·219868	1250	9·993939	35	9·225929	1285	10·774071	27
34	9·220618	1248	9·993918	35	9·226700	1285	10·773300	26
35	9·221367	1247	9·993897	37	9·227471	1280	10·772529	25
36	9·222115	1243	9·993875	35	9·228239	1280	10·771761	24
37	9·222861	1242	9·993854	37	9·229007	1277	10·770993	23
38	9·223606	1238	9·993832	35	9·229773	1277	10·770227	22
39	9·224349	1238	9·993811	37	9·230539	1272	10·769461	21
40	9·225092	1235	9·993789	35	9·231302	1272	10·768698	20
41	9·225833	1233	9·993768	37	9·232065	1268	10·767935	19
42	9·226573	1230	9·993746	35	9·232826	1267	10·767174	18
43	9·227311	1228	9·993725	37	9·233586	1265	10·766414	17
44	9·228048	1227	9·993703	37	9·234345	1263	10·765655	16
45	9·228784	1223	9·993681	35	9·235103	1260	10·764897	15
46	9·229518	1223	9·993660	37	9·235859	1258	10·764141	14
47	9·230252	1220	9·993638	37	9·236614	1257	10·763386	13
48	9·230984	1218	9·993616	37	9·237368	1253	10·762632	12
49	9·231715	1215	9·993594	37	9·238120	1253	10·761880	11
50	9·232444	1213	9·993572	37	9·238872	1250	10·761128	10
51	9·233172	1212	9·993550	37	9·239622	1248	10·760378	9
52	9·233899	1210	9·993528	37	9·240371	1245	10·759629	8
53	9·234625	1207	9·993506	37	9·241118	1245	10·758882	7
54	9·235349	1207	9·993484	37	9·241865	1242	10·758135	6
55	9·236073	1203	9·993462	37	9·242610	1240	10·757390	5
56	9·236795	1200	9·993440	37	9·243354	1238	10·756646	4
57	9·237515	1200	9·993418	37	9·244097	1237	10·755903	3
58	9·238235	1197	9·993396	37	9·244839	1233	10·755161	2
59	9·238953	1195	9·993374	38	9·245579	1233	10·754421	1
60	9·239670		9·993351		9·246319		10·753681	0
′	Cosine.		Sine.		Cotang.		Tangent.	′

0 min. to 30 min.

80 degrees.

10 degrees.

0 min. to 30 min.

′	Sine.	Diff. for 100″	Cosine.	Diff. for 100″	Tangent.	Diff. for 100″	Cotang.	′
0	9·239670	1193	9·993351	37	9·246319	1230	10·753681	60
1	9·240386	1192	9·993329	37	9·247057	1228	10·752943	59
2	9·241101	1188	9·993307	38	9·247794	1227	10·752206	58
3	9·241814	1187	9·993284	37	9·248530	1223	10·751470	57
4	9·242526	1185	9·993262	37	9·249264	1223	10·750736	56
5	9·243237	1183	9·993240	38	9·249998	1220	10·750002	55
6	9·243947	1182	9·993217	37	9·250730	1218	10·749270	54
7	9·244656	1178	9·993195	38	9·251461	1217	10·748539	53
8	9·245363	1177	9·993172	38	9·252191	1215	10·747809	52
9	9·246069	1177	9·993149	37	9·252920	1213	10·747080	51
10	9·246775	1172	9·993127	38	9·253648	1210	10·746352	50
11	9·247478	1172	9·993104	38	9·254374	1210	10·745626	49
12	9·248181	1170	9·993081	37	9·255100	1207	10·744900	48
13	9·248883	1167	9·993059	38	9·255824	1205	10·744176	47
14	9·249583	1165	9·993036	38	9·256547	1203	10·743453	46
15	9·250282	1163	9·993013	38	9·257269	1202	10·742731	45
16	9·250980	1162	9·992990	38	9·257990	1200	10·742010	44
17	9·251677	1160	9·992967	38	9·258710	1198	10·741290	43
18	9·252373	1157	9·992944	38	9·259429	1195	10·740571	42
19	9·253067	1157	9·992921	38	9·260146	1195	10·739854	41
20	9·253761	1153	9·992898	38	9·260863	1192	10·739137	40
21	9·254453	1152	9·992875	38	9·261578	1190	10·738422	39
22	9·255144	1150	9·992852	38	9·262292	1188	10·737708	38
23	9·255834	1148	9·992829	38	9·263005	1187	10·736995	37
24	9·256523	1147	9·992806	38	9·263717	1185	10·736283	36
25	9·257211	1145	9·992783	40	9·264428	1183	10·735572	35
26	9·257898	1142	9·992759	38	9·265138	1182	10·734862	34
27	9·258583	1142	9·992736	38	9·265847	1180	10·734153	33
28	9·259268	1138	9·992713	38	9·266555	1177	10·733445	32
29	9·259951	1137	9·992690	40	9·267261	1177	10·732739	31
30	9·260633		9·992666		9·267967		10·732033	30
′	Cosine.		Sine.		Cotang.		Tangent.	′

30 min. to 60 min.

79 degrees.

10 degrees.

30 min. to 60 min.

′	Sine.	Diff. for 100″	Cosine.	Diff. for 100″	Tangent.	Diff. for 100″	Cotang.	′
30	9·260633	1135	9·992666	38	9·267967	1173	10·732033	30
31	9·261314	1133	9·992643	40	9·268671	1173	10·731329	29
32	9·261994	1132	9·992619	38	9·269375	1170	10·730625	28
33	9·262673	1130	9·992596	40	9·270077	1170	10·729923	27
34	9·263351	1127	9·992572	38	9·270779	1167	10·729221	26
35	9·264027	1127	9·992549	40	9·271479	1165	10·728521	25
36	9·264703	1123	9·992525	40	9·272178	1163	10·727822	24
37	9·265377	1123	9·992501	38	9·272876	1162	10·727124	23
38	9·266051	1120	9·992478	40	9·273573	1160	10·726427	22
39	9·266723	1120	9·992454	40	9·274269	1158	10·725731	21
40	9·267395	1117	9·992430	40	9·274964	1157	10·725036	20
41	9·268065	1115	9·992406	40	9·275658	1155	10·724342	19
42	9·268734	1113	9·992382	38	9·276351	1153	10·723649	18
43	9·269402	1112	9·992359	40	9·277043	1152	10·722957	17
44	9·270069	1110	9·992335	40	9·277734	1150	10·722266	16
45	9·270735	1108	9·992311	40	9·278424	1148	10·721576	15
46	9·271400	1107	9·992287	40	9·279113	1147	10·720887	14
47	9·272064	1103	9·992263	40	9·279801	1145	10·720199	13
48	9·272726	1103	9·992239	42	9·280488	1143	10·719512	12
49	9·273388	1102	9·992214	40	9·281174	1140	10·718826	11
50	9·274049	1098	9·992190	40	9·281858	1140	10·718142	10
51	9·274708	1098	9·992166	40	9·282542	1138	10·717458	9
52	9·275367	1097	9·992142	40	9·283225	1137	10·716775	8
53	9·276025	1093	9·992118	42	9·283907	1135	10·716093	7
54	9·276681	1093	9·992093	40	9·284588	1133	10·715412	6
55	9·277337	1090	9·992069	42	9·285268	1132	10·714732	5
56	9·277991	1090	9·992044	40	9·285947	1128	10·714053	4
57	9·278645	1087	9·992020	40	9·286624	1128	10·713376	3
58	9·279297	1085	9·991996	42	9·287301	1127	10·712699	2
59	9·279948	1085	9·991971	40	9·287977	1125	10·712023	1
60	9·280599		9·991947		9·288652		10·711348	0
′	Cosine.		Sine.		Cotang.		Tangent.	′

0 min. to 30 min.

79 degrees.

11 degrees.

0 min. to 30 min.

′	Sine.	Diff. for 100″	Cosine.	Diff. for 100″	Tangent.	Diff. for 100″	Cotang.	′
0	9·280599	1082	9·991947	42	9·288652	1123	10·711348	60
1	9·281248	1082	9·991922	42	9·289326	1122	10·710674	59
2	9·281897	1078	9·991897	40	9·289999	1120	10·710001	58
3	9·282544	1077	9·991873	42	9·290671	1118	10·709329	57
4	9·283190	1077	9·991848	42	9·291342	1118	10·708658	56
5	9·283836	1073	9·991823	40	9·292013	1115	10·707987	55
6	9·284480	1073	9·991799	42	9·292682	1113	10·707318	54
7	9·285124	1070	9·991774	42	9·293350	1112	10·706650	53
8	9·285766	1070	9·991749	42	9·294017	1112	10·705983	52
9	9·286408	1067	9·991724	42	9·294684	1108	10·705316	51
10	9·287048	1067	9·991699	42	9·295349	1107	10·704651	50
11	9·287688	1063	9·991674	42	9·296013	1107	10·703987	49
12	9·288326	1063	9·991649	42	9·296677	1103	10·703323	48
13	9·288964	1060	9·991624	42	9·297339	1103	10·702661	47
14	9·289600	1060	9·991599	42	9·298001	1102	10·701999	46
15	9·290236	1057	9·991574	42	9·298662	1100	10·701338	45
16	9·290870	1057	9·991549	42	9·299322	1097	10·700678	44
17	9·291504	1055	9·991524	43	9·299980	1097	10·700020	43
18	9·292137	1052	9·991498	42	9·300638	1095	10·699362	42
19	9·292768	1052	9·991473	42	9·301295	1093	10·698705	41
20	9·293399	1050	9·991448	43	9·301951	1093	10·698049	40
21	9·294029	1048	9·991422	42	9·302607	1090	10·697393	39
22	9·294658	1047	9·991397	42	9·303261	1088	10·696739	38
23	9·295286	1045	9·991372	43	9·303914	1088	10·696086	37
24	9·295913	1043	9·991346	42	9·304567	1085	10·695433	36
25	9·296539	1042	9·991321	43	9·305218	1085	10·694782	35
26	9·297164	1040	9·991295	42	9·305869	1083	10·694131	34
27	9·297788	1040	9·991270	43	9·306519	1082	10·693481	33
28	9·298412	1037	9·991244	43	9·307168	1080	10·692832	32
29	9·299034	1035	9·991218	42	9·307816	1078	10·692184	31
30	9·299655		9·991193		9·308463		10·691537	30
′	Cosine.		Sine.		Cotang.		Tangent.	′

30 min. to 60 min.

78 degrees.

11 degrees.

30 min. to 60 min.

′	Sine.	Diff. for 100″	Cosine.	Diff. for 100″	Tangent.	Diff. for 100″	Cotang.	′
30	9·299655	1035	9·991193	43	9·308463	1077	10·691537	30
31	9·300276	1032	9·991167	43	9·309109	1075	10·690891	29
32	9·300895	1032	9·991141	43	9·309754	1075	10·690246	28
33	9·301514	1030	9·991115	42	9·310399	1072	10·689601	27
34	9·302132	1027	9·991090	43	9·311042	1072	10·688958	26
35	9·302748	1027	9·991064	43	9·311685	1070	10·688315	25
36	9·303364	1025	9·991038	43	9·312327	1068	10·687673	24
37	9·303979	1023	9·991012	43	9·312968	1067	10·687032	23
38	9·304593	1023	9·990986	43	9·313608	1065	10·686392	22
39	9·305207	1020	9·990960	43	9·314247	1063	10·685753	21
40	9·305819	1018	9·990934	43	9·314885	1063	10·685115	20
41	9·306430	1018	9·990908	43	9·315523	1060	10·684477	19
42	9·307041	1015	9·990882	45	9·316159	1060	10·683841	18
43	9·307650	1015	9·990855	43	9·316795	1058	10·683205	17
44	9·308259	1013	9·990829	43	9·317430	1057	10·682570	16
45	9·308867	1012	9·990803	43	9·318064	1055	10·681936	15
46	9·309474	1010	9·990777	45	9·318697	1055	10·681303	14
47	9·310080	1008	9·990750	43	9·319330	1052	10·680670	13
48	9·310685	1007	9·990724	45	9·319961	1052	10·680039	12
49	9·311289	1007	9·990697	43	9·320592	1050	10·679408	11
50	9·311893	1003	9·990671	43	9·321222	1048	10·678778	10
51	9·312495	1003	9·990645	45	9·321851	1047	10·678149	9
52	9·313097	1002	9·990618	45	9·322479	1045	10·677521	8
53	9·313698	998	9·990591	43	9·323106	1045	10·676894	7
54	9·314297	1000	9·990565	45	9·323733	1042	10·676267	6
55	9·314897	997	9·990538	45	9·324358	1042	10·675642	5
56	9·315495	995	9·990511	43	9·324983	1040	10·675017	4
57	9·316092	995	9·990485	45	9·325607	1040	10·674393	3
58	9·316689	992	9·990458	45	9·326231	1037	10·673769	2
59	9·317284	992	9·990431	45	9·326853	1037	10·673147	1
60	9·317879		9·990404		9·327475		10·672525	0
′	Cosine.		Sine.		Cotang.		Tangent.	′

0 min. to 30 min.

78 degrees.

12 degrees.

0 min. to 30 min.

′	Sine.	Diff. for 100″	Cosine.	Diff. for 100″	Tangent.	Diff. for 100″	Cotang.	′
0	9·317879	990	9·990404	43	9·327475	1033	10·672525	60
1	9·318473	988	9·990378	45	9·328095	1033	10·671905	59
2	9·319066	987	9·990351	45	9·328715	1032	10·671285	58
3	9·319658	985	9·990324	45	9·329334	1032	10·670666	57
4	9·320249	985	9·990297	45	9·329953	1028	10·670047	56
5	9·320840	983	9·990270	45	9·330570	1028	10·669430	55
6	9·321430	982	9·990243	47	9·331187	1027	10·668813	54
7	9·322019	980	9·990215	45	9·331803	1025	10·668197	53
8	9·322607	978	9·990188	45	9·332418	1025	10·667582	52
9	9·323194	977	9·990161	45	9·333033	1022	10·666967	51
10	9·323780	977	9·990134	45	9·333646	1022	10·666354	50
11	9·324366	973	9·990107	47	9·334259	1020	10·665741	49
12	9·324950	973	9·990079	45	9·334871	1018	10·665129	48
13	9·325534	972	9·990052	45	9·335482	1018	10·664518	47
14	9·326117	972	9·990025	47	9·336093	1015	10·663907	46
15	9·326700	968	9·989997	45	9·336702	1015	10·663298	45
16	9·327281	968	9·989970	47	9·337311	1013	10·662689	44
17	9·327862	967	9·989942	45	9·337919	1013	10·662081	43
18	9·328442	965	9·989915	47	9·338527	1010	10·661473	42
19	9·329021	963	9·989887	45	9·339133	1010	10·660867	41
20	9·329599	962	9·989860	47	9·339739	1008	10·660261	40
21	9·330176	962	9·989832	47	9·340344	1007	10·659656	39
22	9·330753	960	9·989804	45	9·340948	1007	10·659052	38
23	9·331329	957	9·989777	47	9·341552	1005	10·658448	37
24	9·331903	958	9·989749	47	9·342155	1003	10·657845	36
25	9·332478	955	9·989721	47	9·342757	1002	10·657243	35
26	9·333051	955	9·989693	47	9·343358	1000	10·656642	34
27	9·333624	952	9·989665	47	9·343958	1000	10·656042	33
28	9·334195	953	9·989637	45	9·344558	998	10·655442	32
29	9·334767	950	9·989610	47	9·345157	997	10·654843	31
30	9·335337		9·989582		9·345755		10·654245	30
′	Cosine.		Sine.		Cotang.		Tangent.	′

30 min. to 60 min.

77 degrees.

12 degrees.

30 min. to 60 min.

′	Sine.	Diff. for 100″	Cosine.	Diff. for 100″	Tangent.	Diff. for 100″	Cotang.	′
30	9·335337	948	9·989582	48	9·345755	997	10·654245	30
31	9·335906	948	9·989553	47	9·346353	993	10·653647	29
32	9·336475	947	9·989525	47	9·346949	993	10·653051	28
33	9·337043	945	9·989497	47	9·347545	993	10·652455	27
34	9·337610	943	9·989469	47	9·348141	990	10·651859	26
35	9·338176	943	9·989441	47	9·348735	990	10·651265	25
36	9·338742	942	9·989413	47	9·349329	988	10·650671	24
37	9·339307	940	9·989385	48	9·349922	987	10·650078	23
38	9·339871	938	9·989356	47	9·350514	987	10·649486	22
39	9·340434	937	9·989328	47	9·351106	985	10·648894	21
40	9·340996	937	9·989300	48	9·351697	983	10·648303	20
41	9·341558	935	9·989271	47	9·352287	982	10·647713	19
42	9·342119	933	9·989243	48	9·352876	982	10·647124	18
43	9·342679	933	9·989214	47	9·353465	980	10·646535	17
44	9·343239	930	9·989186	48	9·354053	978	10·645947	16
45	9·343797	930	9·989157	48	9·354640	978	10·645360	15
46	9·344355	928	9·989128	47	9·355227	977	10·644773	14
47	9·344912	928	9·989100	48	9·355813	975	10·644187	13
48	9·345469	925	9·989071	48	9·356398	973	10·643602	12
49	9·346024	925	9·989042	47	9·356982	973	10·643018	11
50	9·346579	925	9·989014	48	9·357566	972	10·642434	10
51	9·347134	922	9·988985	48	9·358149	970	10·641851	9
52	9·347687	922	9·988956	48	9·358731	970	10·641269	8
53	9·348240	920	9·988927	48	9·359313	967	10·640687	7
54	9·348792	918	9·988898	48	9·359893	968	10·640107	6
55	9·349343	917	9·988869	48	9·360474	965	10·639526	5
56	9·349893	917	9·988840	48	9·361053	965	10·638947	4
57	9·350443	915	9·988811	48	9·361632	963	10·638368	3
58	9·350992	913	9·988782	48	9·362210	962	10·637790	2
59	9·351540	913	9·988753	48	9·362787	962	10·637213	1
60	9·352088		9·988724		9·363364		10·636636	0
′	Cosine.		Sine.		Cotang.		Tangent.	′

0 min. to 30 min.

77 degrees.

13 degrees.

0 min. to 30 min.

′	Sine.	Diff. for 100″	Cosine.	Diff. for 100″	Tangent.	Diff. for 100″	Cotang.	′
0	9·352088	912	9·988724	48	9·363364	960	10·636636	60
1	9·352635	910	9·988695	48	9·363940	958	10·636060	59
2	9·353181	908	9·988666	50	9·364515	958	10·635485	58
3	9·353726	908	9·988636	48	9·365090	957	10·634910	57
4	9·354271	907	9·988607	48	9·365664	955	10·634336	56
5	9·354815	905	9·988578	50	9·366237	955	10·633763	55
6	9·355358	905	9·988548	48	9·366810	953	10·633190	54
7	9·355901	903	9·988519	50	9·367382	952	10·632618	53
8	9·356443	902	9·988489	48	9·367953	952	10·632047	52
9	9·356984	900	9·988460	50	9·368524	950	10·631476	51
10	9·357524	900	9·988430	48	9·369094	948	10·630906	50
11	9·358064	898	9·988401	50	9·369663	948	10·630337	49
12	9·358603	897	9·988371	48	9·370232	945	10·629768	48
13	9·359141	895	9·988342	50	9·370799	947	10·629201	47
14	9·359678	895	9·988312	50	9·371367	943	10·628633	46
15	9·360215	895	9·988282	50	9·371933	943	10·628067	45
16	9·360752	892	9·988252	48	9·372499	942	10·627501	44
17	9·361287	892	9·988223	50	9·373064	942	10·626936	43
18	9·361822	890	9·988193	50	9·373629	940	10·626371	42
19	9·362356	888	9·988163	50	9·374193	938	10·625807	41
20	9·362889	888	9·988133	50	9·374756	938	10·625244	40
21	9·363422	887	9·988103	50	9·375319	937	10·624681	39
22	9·363954	885	9·988073	50	9·375881	935	10·624119	38
23	9·364485	885	9·988043	50	9·376442	935	10·623558	37
24	9·365016	883	9·988013	50	9·377003	933	10·622997	36
25	9·365546	882	9·987983	50	9·377563	932	10·622437	35
26	9·366075	882	9·987953	52	9·378122	932	10·621878	34
27	9·366604	878	9·987922	50	9·378681	930	10·621319	33
28	9·367131	880	9·987892	50	9·379239	930	10·620761	32
29	9·367659	877	9·987862	50	9·379797	928	10·620203	31
30	9·368185		9·987832		9·380354		10·619646	30
′	Cosine.		Sine.		Cotang.		Tangent.	′

30 min. to 60 min.

76 degrees.

13 degrees.

30 min. to 60 min.

′	Sine.	Diff. for 100″	Cosine.	Diff. for 100″	Tangent.	Diff. for 100″	Cotang.	′
30	9·368185	877	9·987832	52	9·380354	927	10·619646	30
31	9·368711	875	9·987801	50	9·380910	927	10·619090	29
32	9·369236	875	9·987771	52	9·381466	923	10·618534	28
33	9·369761	873	9·987740	50	9·382020	925	10·617980	27
34	9·370285	872	9·987710	52	9·382575	923	10·617425	26
35	9·370808	870	9·987679	50	9·383129	922	10·616871	25
36	9·371330	870	9·987649	52	9·383682	920	10·616318	24
37	9·371852	868	9·987618	50	9·384234	920	10·615766	23
38	9·372373	868	9·987588	52	9·384786	918	10·615214	22
39	9·372894	867	9·987557	52	9·385337	918	10·614663	21
40	9·373414	865	9·987526	50	9·385888	917	10·614112	20
41	9·373933	865	9·987496	52	9·386438	915	10·613562	19
42	9·374452	863	9·987465	52	9·386987	915	10·613013	18
43	9·374970	862	9·987434	52	9·387536	913	10·612464	17
44	9·375487	860	9·987403	52	9·388084	912	10·611916	16
45	9·376003	860	9·987372	52	9·388631	912	10·611369	15
46	9·376519	860	9·987341	52	9·389178	910	10·610822	14
47	9·377035	857	9·987310	52	9·389724	910	10·610276	13
48	9·377549	857	9·987279	52	9·390270	908	10·609730	12
49	9·378063	857	9·987248	52	9·390815	908	10·609185	11
50	9·378577	853	9·987217	52	9·391360	905	10·608640	10
51	9·379089	853	9·987186	52	9·391903	907	10·608097	9
52	9·379601	853	9·987155	52	9·392447	903	10·607553	8
53	9·380113	852	9·987124	53	9·392989	903	10·607011	7
54	9·380624	850	9·987092	52	9·393531	903	10·606469	6
55	9·381134	848	9·987061	52	9·394073	902	10·605927	5
56	9·381643	848	9·987030	53	9·394614	900	10·605386	4
57	9·382152	848	9·986998	52	9·395154	900	10·604846	3
58	9·382661	845	9·986967	52	9·395694	898	10·604306	2
59	9·383168	845	9·986936	53	9·396233	897	10·603767	1
60	9·383675		9·986904		9·396771		10·603229	0
′	Cosine.		Sine.		Cotang.		Tangent.	′

0 min. to 30 min.

76 degrees.

14 degrees.

0 min. to 30 min.

′	Sine.	Diff. for 100″	Cosine.	Diff. for 100″	Tangent.	Diff. for 100″	Cotang.	′
0	9·383675	845	9·986904	52	9·396771	897	10·603229	60
1	9·384182	842	9·986873	53	9·397309	895	10·602691	59
2	9·384687	842	9·986841	53	9·397846	895	10·602154	58
3	9·385192	842	9·986809	52	9·398383	893	10·601617	57
4	9·385697	840	9·986778	53	9·398919	893	10·601081	56
5	9·386201	838	9·986746	53	9·399455	892	10·600545	55
6	9·386704	838	9·986714	52	9·399990	890	10·600010	54
7	9·387207	837	9·986683	53	9·400524	890	10·599476	53
8	9·387709	835	9·986651	53	9·401058	888	10·598942	52
9	9·388210	835	9·986619	53	9·401591	888	10·598409	51
10	9·388711	833	9·986587	53	9·402124	887	10·597876	50
11	9·389211	833	9·986555	53	9·402656	885	10·597344	49
12	9·389711	832	9·986523	53	9·403187	885	10·596813	48
13	9·390210	830	9·986491	53	9·403718	885	10·596282	47
14	9·390708	830	9·986459	53	9·404249	882	10·595751	46
15	9·391206	828	9·986427	53	9·404778	883	10·595222	45
16	9·391703	827	9·986395	53	9·405308	880	10·594692	44
17	9·392199	827	9·986363	53	9·405836	880	10·594164	43
18	9·392695	827	9·986331	53	9·406364	880	10·593636	42
19	9·393191	823	9·986299	55	9·406892	878	10·593108	41
20	9·393685	823	9·986266	53	9·407419	877	10·592581	40
21	9·394179	823	9·986234	53	9·407945	877	10·592055	39
22	9·394673	822	9·986202	55	9·408471	875	10·591529	38
23	9·395166	820	9·986169	53	9·408996	875	10·591004	37
24	9·395658	820	9·986137	55	9·409521	873	10·590479	36
25	9·396150	818	9·986104	53	9·410045	873	10·589955	35
26	9·396641	818	9·986072	55	9·410569	872	10·589431	34
27	9·397132	815	9·986039	53	9·411092	872	10·588908	33
28	9·397621	817	9·986007	55	9·411615	870	10·588385	32
29	9·398111	815	9·985974	53	9·412137	868	10·587863	31
30	9·398600		9·985942		9·412658		10·587342	30
′	Cosine.		Sine.		Cotang.		Tangent.	′

30 min. to 60 min.

75 degrees.

14 degrees.

30 min. to 60 min.

′	Sine.	Diff. for 100″	Cosine.	Diff. for 100″	Tangent.	Diff. for 100″	Cotang.	′
30	9·398600	813	9·985942	55	9·412658	868	10·587342	30
31	9·399088	812	9·985909	55	9·413179	867	10·586821	29
32	9·399575	812	9·985876	55	9·413699	867	10·586301	28
33	9·400062	812	9·985843	53	9·414219	865	10·585781	27
34	9·400549	810	9·985811	55	9·414738	865	10·585262	26
35	9·401035	808	9·985778	55	9·415257	863	10·584743	25
36	9·401520	808	9·985745	55	9·415775	863	10·584225	24
37	9·402005	807	9·985712	55	9·416293	862	10·583707	23
38	9·402489	805	9·985679	55	9·416810	860	10·583190	22
39	9·402972	805	9·985646	55	9·417326	860	10·582674	21
40	9·403455	805	9·985613	55	9·417842	860	10·582158	20
41	9·403938	803	9·985580	55	9·418358	858	10·581642	19
42	9·404420	802	9·985547	55	9·418873	857	10·581127	18
43	9·404901	802	9·985514	57	9·419387	857	10·580613	17
44	9·405382	800	9·985480	55	9·419901	857	10·580099	16
45	9·405862	798	9·985447	55	9·420415	853	10·579585	15
46	9·406341	798	9·985414	55	9·420927	855	10·579073	14
47	9·406820	798	9·985381	57	9·421440	853	10·578560	13
48	9·407299	797	9·985347	55	9·421952	852	10·578048	12
49	9·407777	795	9·985314	57	9·422463	852	10·577537	11
50	9·408254	795	9·985280	55	9·422974	850	10·577026	10
51	9·408731	793	9·985247	57	9·423484	848	10·576516	9
52	9·409207	792	9·985213	55	9·423993	850	10·576007	8
53	9·409682	792	9·985180	57	9·424503	847	10·575497	7
54	9·410157	792	9·985146	55	9·425011	847	10·574989	6
55	9·410632	790	9·985113	57	9·425519	847	10·574481	5
56	9·411106	788	9·985079	57	9·426027	845	10·573973	4
57	9·411579	788	9·985045	57	9·426534	845	10·573466	3
58	9·412052	787	9·985011	55	9·427041	843	10·572959	2
59	9·412524	787	9·984978	57	9·427547	842	10·572453	1
60	9·412996		9·984944		9·428052		10·571948	0
′	Cosine.		Sine.		Cotang.		Tangent.	′

0 min. to 30 min.

75 degrees.

15 degrees.

0 min. to 30 min.

′	Sine.	Diff. for 100″	Cosine.	Diff. for 100″	Tangent.	Diff. for 100″	Cotang.	′
0	9·412996	785	9·984944	57	9·428052	843	10·571948	60
1	9·413467	785	9·984910	57	9·428558	840	10·571442	59
2	9·413938	783	9·984876	57	9·429062	840	10·570938	58
3	9·414408	783	9·984842	57	9·429566	840	10·570434	57
4	9·414878	782	9·984808	57	9·430070	838	10·569930	56
5	9·415347	780	9·984774	57	9·430573	837	10·569427	55
6	9·415815	780	9·984740	57	9·431075	837	10·568925	54
7	9·416283	780	9·984706	57	9·431577	837	10·568423	53
8	9·416751	777	9·984672	57	9·432079	835	10·567921	52
9	9·417217	778	9·984638	58	9·432580	833	10·567420	51
10	9·417684	777	9·984603	57	9·433080	833	10·566920	50
11	9·418150	775	9·984569	57	9·433580	833	10·566420	49
12	9·418615	773	9·984535	58	9·434080	832	10·565920	48
13	9·419079	775	9·984500	57	9·434579	832	10·565421	47
14	9·419544	772	9·984466	57	9·435078	830	10·564922	46
15	9·420007	772	9·984432	58	9·435576	828	10·564424	45
16	9·420470	772	9·984397	57	9·436073	828	10·563927	44
17	9·420933	770	9·984363	58	9·436570	828	10·563430	43
18	9·421395	770	9·984328	57	9·437067	827	10·562933	42
19	9·421857	768	9·984294	58	9·437563	827	10·562437	41
20	9·422318	767	9·984259	58	9·438059	825	10·561941	40
21	9·422778	767	9·984224	57	9·438554	823	10·561446	39
22	9·423238	765	9·984190	58	9·439048	825	10·560952	38
23	9·423697	765	9·984155	58	9·439543	822	10·560457	37
24	9·424156	765	9·984120	58	9·440036	822	10·559964	36
25	9·424615	763	9·984085	58	9·440529	822	10·559471	35
26	9·425073	762	9·984050	58	9·441022	820	10·558978	34
27	9·425530	762	9·984015	57	9·441514	820	10·558486	33
28	9·425987	760	9·983981	58	9·442006	818	10·557994	32
29	9·426443	760	9·983946	58	9·442497	818	10·557503	31
30	9·426899		9·983911		9·442988		10·557012	30
′	Cosine.		Sine.		Cotang.		Tangent.	′

30 min. to 60 min.

74 degrees.

15 degrees.

30 min. to 60 min.

′	Sine.	Diff. for 100″	Cosine.	Diff. for 100″	Tangent.	Diff. for 100″	Cotang.	′
30	9·426899	758	9·983911	60	9·442988	818	10·557012	30
31	9·427354	758	9·983875	58	9·443479	815	10·556521	29
32	9·427809	757	9·983840	58	9·443968	817	10·556032	28
33	9·428263	757	9·983805	58	9·444458	815	10·555542	27
34	9·428717	755	9·983770	58	9·444947	813	10·555053	26
35	9·429170	755	9·983735	58	9·445435	813	10·554565	25
36	9·429623	753	9·983700	60	9·445923	813	10·554077	24
37	9·430075	753	9·983664	58	9·446411	812	10·553589	23
38	9·430527	752	9·983629	58	9·446898	810	10·553102	22
39	9·430978	752	9·983594	60	9·447384	810	10·552616	21
40	9·431429	750	9·983558	58	9·447870	810	10·552130	20
41	9·431879	750	9·983523	60	9·448356	808	10·551644	19
42	9·432329	748	9·983487	58	9·448841	808	10·551159	18
43	9·432778	747	9·983452	60	9·449326	807	10·550674	17
44	9·433226	748	9·983416	58	9·449810	807	10·550190	16
45	9·433675	745	9·983381	60	9·450294	805	10·549706	15
46	9·434122	745	9·983345	60	9·450777	805	10·549223	14
47	9·434569	745	9·983309	60	9·451260	805	10·548740	13
48	9·435016	743	9·983273	58	9·451743	803	10·548257	12
49	9·435462	743	9·983238	60	9·452225	802	10·547775	11
50	9·435908	742	9·983202	60	9·452706	802	10·547294	10
51	9·436353	742	9·983166	60	9·453187	802	10·546813	9
52	9·436798	740	9·983130	60	9·453668	800	10·546332	8
53	9·437242	740	9·983094	60	9·454148	800	10·545852	7
54	9·437686	738	9·983058	60	9·454628	798	10·545372	6
55	9·438129	738	9·983022	60	9·455107	798	10·544893	5
56	9·438572	737	9·982986	60	9·455586	797	10·544414	4
57	9·439014	737	9·982950	60	9·456064	797	10·543936	3
58	9·439456	735	9·982914	60	9·456542	795	10·543458	2
59	9·439897	735	9·982878	60	9·457019	795	10·542981	1
60	9·440338		9·982842		9·457496		10·542504	0
′	Cosine.		Sine.		Cotang.		Tangent.	′

0 min. to 30 min.

74 degrees.

16 degrees.

0 min. to 30 min.

′	Sine.	Diff. for 100″	Cosine.	Diff. for 100″	Tangent.	Diff. for 100″	Cotang.	′
0	9·440338		9·982842		9·457496		10·542504	60
1	9·440778	733	9·982805	62	9·457973	795	10·542027	59
2	9·441218	733	9·982769	60	9·458449	793	10·541551	58
3	9·441658	733	9·982733	60	9·458925	793	10·541075	57
4	9·442096	730	9·982696	62	9·459400	792	10·540600	56
5	9·442535	732	9·982660	60	9·459875	792	10·540125	55
6	9·442973	730	9·982624	60	9·460349	790	10·539651	54
7	9·443410	728	9·982587	62	9·460823	790	10·539177	53
8	9·443847	728	9·982551	60	9·461297	790	10·538703	52
9	9·444284	728	9·982514	62	9·461770	788	10·538230	51
10	9·444720	727	9·982477	62	9·462242	787	10·537758	50
11	9·445155	725	9·982441	60	9·462715	788	10·537285	49
12	9·445590	725	9·982404	62	9·463186	785	10·536814	48
13	9·446025	725	9·982367	62	9·463658	787	10·536342	47
14	9·446459	723	9·982331	60	9·464128	783	10·535872	46
15	9·446893	723	9·982294	62	9·464599	785	10·535401	45
16	9·447326	722	9·982257	62	9·465069	783	10·534931	44
17	9·447759	722	9·982220	62	9·465539	783	10·534461	43
18	9·448191	720	9·982183	62	9·466008	782	10·533992	42
19	9·448623	720	9·982146	62	9·466477	782	10·533523	41
20	9·449054	718	9·982109	62	9·466945	780	10·533055	40
21	9·449485	718	9·982072	62	9·467413	780	10·532587	39
22	9·449915	717	9·982035	62	9·467880	778	10·532120	38
23	9·450345	717	9·981998	62	9·468347	778	10·531653	37
24	9·450775	717	9·981961	62	9·468814	778	10·531186	36
25	9·451204	715	9·981924	62	9·469280	777	10·530720	35
26	9·451632	713	9·981886	63	9·469746	777	10·530254	34
27	9·452060	713	9·981849	62	9·470211	775	10·529789	33
28	9·452488	713	9·981812	62	9·470676	775	10·529324	32
29	9·452915	712	9·981774	63	9·471141	775	10·528859	31
30	9·453342	712	9·981737	62	9·471605	773	10·528395	30
′	Cosine.		Sine.		Cotang.		Tangent.	′

30 min. to 60 min.

73 degrees.

16 degrees.

30 min. to 60 min.

′	Sine.	Diff. for 100″	Cosine.	Diff. for 100″	Tangent.	Diff. for 100″	Cotang.	′
30	9·453342	710	9·981737	62	9·471605	773	10·528395	30
31	9·453768	710	9·981700	63	9·472069	772	10·527931	29
32	9·454194	708	9·981662	62	9·472532	772	10·527468	28
33	9·454619	708	9·981625	63	9·472995	770	10·527005	27
34	9·455044	708	9·981587	63	9·473457	770	10·526543	26
35	9·455469	707	9·981549	62	9·473919	770	10·526081	25
36	9·455893	705	9·981512	63	9·474381	768	10·525619	24
37	9·456316	705	9·981474	63	9·474842	768	10·525158	23
38	9·456739	705	9·981436	62	9·475303	767	10·524697	22
39	9·457162	703	9·981399	63	9·475763	767	10·524237	21
40	9·457584	703	9·981361	63	9·476223	767	10·523777	20
41	9·458006	702	9·981323	63	9·476683	765	10·523317	19
42	9·458427	702	9·981285	63	9·477142	765	10·522858	18
43	9·458848	700	9·981247	63	9·477601	763	10·522399	17
44	9·459268	700	9·981209	63	9·478059	763	10·521941	16
45	9·459688	700	9·981171	63	9·478517	763	10·521483	15
46	9·460108	698	9·981133	63	9·478975	762	10·521025	14
47	9·460527	698	9·981095	63	9·479432	762	10·520568	13
48	9·460946	697	9·981057	63	9·479889	760	10·520111	12
49	9·461364	697	9·981019	63	9·480345	760	10·519655	11
50	9·461782	695	9·980981	65	9·480801	760	10·519199	10
51	9·462199	695	9·980942	63	9·481257	758	10·518743	9
52	9·462616	693	9·980904	63	9·481712	758	10·518288	8
53	9·463032	693	9·980866	65	9·482167	757	10·517833	7
54	9·463448	693	9·980827	63	9·482621	757	10·517379	6
55	9·463864	692	9·980789	65	9·483075	757	10·516925	5
56	9·464279	692	9·980750	63	9·483529	755	10·516471	4
57	9·464694	690	9·980712	65	9·483982	755	10·516018	3
58	9·465108	690	9·980673	63	9·484435	753	10·515565	2
59	9·465522	688	9·980635	65	9·484887	753	10·515113	1
60	9·465935		9·980596		9·485339		10·514661	0
′	Cosine.		Sine.		Cotang.		Tangent.	′

0 min. to 30 min.

73 degrees.

17 degrees.

0 min. to 30 min.

′	Sine.	Diff. for 100′	Cosine.	Diff. for 100′	Tangent.	Diff. for 100′	Cotang.	′
0	9·465935		9·980596		9·485339		10·514661	60
1	9·466348	688	9·980558	63	9·485791	753	10·514209	59
2	9·466761	688	9·980519	65	9·486242	752	10·513758	58
3	9·467173	687	9·980480	65	9·486693	752	10·513307	57
4	9·467585	687	9·980442	63	9·487143	750	10·512857	56
5	9·467996	685	9·980403	65	9·487593	750	10·512407	55
6	9·468407	685	9·980364	65	9·488043	750	10·511957	54
7	9·468817	683	9·980325	65	9·488492	748	10·511508	53
8	9·469227	683	9·980286	65	9·488941	748	10·511059	52
9	9·469637	683	9·980247	65	9·489390	748	10·510610	51
10	9·470046	682	9·980208	65	9·489838	747	10·510162	50
11	9·470455	682	9·980169	65	9·490286	747	10·509714	49
12	9·470863	680	9·980130	65	9·490733	745	10·509267	48
13	9·471271	680	9·980091	65	9·491180	745	10·508820	47
14	9·471679	680	9·980052	65	9·491627	745	10·508373	46
15	9·472086	678	9·980012	67	9·492073	743	10·507927	45
16	9·472492	677	9·979973	65	9·492519	743	10·507481	44
17	9·472898	677	9·979934	65	9·492965	743	10·507035	43
18	9·473304	677	9·979895	65	9·493410	742	10·506590	42
19	9·473710	677	9·979855	67	9·493854	740	10·506146	41
20	9·474115	675	9·979816	65	9·494299	742	10·505701	40
21	9·474519	673	9·979776	67	9·494743	740	10·505257	39
22	9·474923	673	9·979737	65	9·495186	738	10·504814	38
23	9·475327	673	9·979697	67	9·495630	740	10·504370	37
24	9·475730	672	9·979658	65	9·496073	738	10·503927	36
25	9·476133	672	9·979618	67	9·496515	737	10·503485	35
26	9·476536	672	9·979579	65	9·496957	737	10·503043	34
27	9·476938	670	9·979539	67	9·497399	737	10·502601	33
28	9·477340	670	9·979499	67	9·497841	737	10·502159	32
29	9·477741	668	9·979459	67	9·498282	735	10·501718	31
30	9·478142	668	9·979420	65	9·498722	733	10·501278	30
′	Cosine.		Sine.		Cotang.		Tangent.	′

30 min. to 60 min.

72 degrees.

17 degrees.

30 min. to 60 min.

′	Sine.	Diff. for 100″	Cosine.	Diff. for 100″	Tangent.	Diff. for 100″	Cotang.	′
30	9·478142	667	9·979420	67	9·498722	735	10·501278	30
31	9·478542	667	9·979380	67	9·499163	733	10·500837	29
32	9·478942	667	9·979340	67	9·499603	732	10·500397	28
33	9·479342	665	9·979300	67	9·500042	732	10·499958	27
34	9·479741	665	9·979260	67	9·500481	732	10·499519	26
35	9·480140	665	9·979220	67	9·500920	732	10·499080	25
36	9·480539	663	9·979180	67	9·501359	730	10·498641	24
37	9·480937	662	9·979140	67	9·501797	730	10·498203	23
38	9·481334	662	9·979100	68	9·502235	728	10·497765	22
39	9·481731	662	9·979059	67	9·502672	728	10·497328	21
40	9·482128	662	9·979019	67	9·503109	728	10·496891	20
41	9·482525	660	9·978979	67	9·503546	727	10·496454	19
42	9·482921	658	9·978939	68	9·503982	727	10·496018	18
43	9·483316	660	9·978898	67	9·504418	727	10·495582	17
44	9·483712	658	9·978858	68	9·504854	725	10·495146	16
45	9·484107	657	9·978817	67	9·505289	725	10·494711	15
46	9·484501	657	9·978777	67	9·505724	725	10·494276	14
47	9·484895	657	9·978737	68	9·506159	723	10·493841	13
48	9·485289	655	9·978696	68	9·506593	723	10·493407	12
49	9·485682	655	9·978655	67	9·507027	722	10·492973	11
50	9·486075	653	9·978615	68	9·507460	722	10·492540	10
51	9·486467	655	9·978574	68	9·507893	722	10·492107	9
52	9·486860	652	9·978533	67	9·508326	722	10·491674	8
53	9·487251	653	9·978493	68	9·508759	720	10·491241	7
54	9·487643	652	9·978452	68	9·509191	718	10·490809	6
55	9·488034	650	9·978411	68	9·509622	720	10·490378	5
56	9·488424	650	9·978370	68	9·510054	718	10·489946	4
57	9·488814	650	9·978329	68	9·510485	718	10·489515	3
58	9·489204	648	9·978288	68	9·510916	717	10·489084	2
59	9·489593	648	9·978247	68	9·511346	717	10·488654	1
60	9·489982		9·978206		9·511776		10·488224	0
′	Cosine.		Sine.		Cotang.		Tangent.	′

0 min. to 30 min.

72 degrees.

18 degrees.

0 min. to 30 min.

′	Sine.	Diff. for 100″	Cosine.	Diff. for 100″	Tangent.	Diff. for 100″	Cotang.	′
0	9·489982	648	9·978206	68	9·511776	717	10·488224	60
1	9·490371	647	9·978165	68	9·512206	715	10·487794	59
2	9·490759	647	9·978124	68	9·512635	715	10·487365	58
3	9·491147	647	9·978083	68	9·513064	715	10·486936	57
4	9·491535	645	9·978042	68	9·513493	713	10·486507	56
5	9·491922	643	9·978001	70	9·513921	713	10·486079	55
6	9·492308	645	9·977959	68	9·514349	713	10·485651	54
7	9·492695	643	9·977918	68	9·514777	712	10·485223	53
8	9·493081	642	9·977877	70	9·515204	712	10·484796	52
9	9·493466	642	9·977835	68	9·515631	710	10·484369	51
10	9·493851	642	9·977794	70	9·516057	712	10·483943	50
11	9·494236	642	9·977752	68	9·516484	710	10·483516	49
12	9·494621	640	9·977711	70	9·516910	708	10·483090	48
13	9·495005	638	9·977669	68	9·517335	710	10·482665	47
14	9·495388	640	9·977628	70	9·517761	708	10·482239	46
15	9·495772	637	9·977586	70	9·518186	707	10·481814	45
16	9·496154	638	9·977544	68	9·518610	707	10·481390	44
17	9·496537	637	9·977503	70	9·519034	707	10·480966	43
18	9·496919	637	9·977461	70	9·519458	707	10·480542	42
19	9·497301	635	9·977419	70	9·519882	705	10·480118	41
20	9·497682	637	9·977377	70	9·520305	705	10·479695	40
21	9·498064	633	9·977335	70	9·520728	705	10·479272	39
22	9·498444	635	9·977293	70	9·521151	703	10·478849	38
23	9·498825	632	9·977251	70	9·521573	703	10·478427	37
24	9·499204	633	9·977209	70	9·521995	703	10·478005	36
25	9·499584	632	9·977167	70	9·522417	702	10·477583	35
26	9·499963	632	9·977125	70	9·522838	702	10·477162	34
27	9·500342	632	9·977083	70	9·523259	702	10·476741	33
28	9·500721	630	9·977041	70	9·523680	700	10·476320	32
29	9·501099	628	9·976999	70	9·524100	700	10·475900	31
30	9·501476		9·976957		9·524520		10·475480	30
′	Cosine.		Sine.		Cotang.		Tangent.	′

30 min. to 60 min.

71 degrees.

18 degrees.

30 min. to 60 min.

′	Sine.	Diff. for 100″	Cosine.	Diff. for 100″	Tangent.	Diff. for 100″	Cotang.	′
30	9·501476	630	9·976957	72	9·524520	700	10·475480	30
31	9·501854	628	9·976914	70	9·524940	698	10·475060	29
32	9·502231	627	9·976872	70	9·525359	698	10·474641	28
33	9·502607	628	9·976830	72	9·525778	698	10·474222	27
34	9·502984	627	9·976787	70	9·526197	697	10·473803	26
35	9·503360	625	9·976745	72	9·526615	697	10·473385	25
36	9·503735	625	9·976702	70	9·527033	697	10·472967	24
37	9·504110	625	9·976660	72	9·527451	695	10·472549	23
38	9·504485	625	9·976617	72	9·527868	695	10·472132	22
39	9·504860	623	9·976574	70	9·528285	695	10·471715	21
40	9·505234	623	9·976532	72	9·528702	695	10·471298	20
41	9·505608	622	9·976489	72	9·529119	693	10·470881	19
42	9·505981	622	9·976446	70	9·529535	693	10·470465	18
43	9·506354	622	9·976404	72	9·529951	692	10·470049	17
44	9·506727	620	9·976361	72	9·530366	692	10·469634	16
45	9·507099	620	9·976318	72	9·530781	692	10·469219	15
46	9·507471	620	9·976275	72	9·531196	692	10·468804	14
47	9·507843	618	9·976232	72	9·531611	690	10·468389	13
48	9·508214	618	9·976189	72	9·532025	690	10·467975	12
49	9·508585	618	9·976146	72	9·532439	690	10·467561	11
50	9·508956	617	9·976103	72	9·532853	688	10·467147	10
51	9·509326	617	9·976060	72	9·533266	688	10·466734	9
52	9·509696	615	9·976017	72	9·533679	688	10·466321	8
53	9·510065	615	9·975974	73	9·534092	687	10·465908	7
54	9·510434	615	9·975930	72	9·534504	687	10·465496	6
55	9·510803	615	9·975887	72	9·534916	687	10·465084	5
56	9·511172	613	9·975844	73	9·535328	685	10·464672	4
57	9·511540	612	9·975800	72	9·535739	685	10·464261	3
58	9·511907	613	9·975757	72	9·536150	685	10·463850	2
59	9·512275	612	9·975714	73	9·536561	685	10·463439	1
60	9·512642		9·975670		9·536972		10·463028	0
′	Cosine.		Sine.		Cotang.		Tangent.	′

0 min. to 30 min.

71 degrees.

19 degrees.

0 min. to 30 min.

'	Sine.	Dif. for	Cosine.	Dif. for	Tangent.	Dif. for	Cotang.	'
0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	683	10·463028	60
1	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	683	10·462618	59
2	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	683	10·462208	58
3	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	682	10·461798	57
4	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	682	10·461389	56
5	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	682	10·460980	55
6	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	680	10·460571	54
7	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	680	10·460163	53
8	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	680	10·459755	52
9	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	10·459347	51
10	[illegible]	[illegible]	[illegible]	[illegible]	9·541061	678	10·458939	50
11	[illegible]	[illegible]	[illegible]	[illegible]	9·541468	678	10·458532	49
12	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	10·458125	48
13	[illegible]	[illegible]	[illegible]	[illegible]	9·542281	678	10·457719	47
14	[illegible]	[illegible]	[illegible]	[illegible]	9·542688	[illegible]	10·457312	46
15	9·518107	602	9·975013	73	9·543094	675	10·456906	45
16	9·518468	602	9·974969	73	9·543499	[illegible]	10·456501	44
17	9·518829	602	9·974925	75	9·543905	675	10·456095	43
18	9·519190	602	9·974880	73	9·544310	675	10·455690	42
19	9·519551	600	9·974836	73	9·544715	673	10·455285	41
20	9·519911	600	9·974792	73	9·545119	675	10·454881	40
21	9·520271	600	9·974748	75	9·545524	673	10·454476	39
22	9·520631	598	9·974703	73	9·545928	672	10·454072	38
23	9·520990	598	9·974659	75	9·546331	673	10·453669	37
24	9·521349	597	9·974614	73	9·546735	672	10·453265	36
25	9·521707	598	9·974570	75	9·547138	670	10·452862	35
26	9·522066	597	9·974525	73	9·547540	672	10·452460	34
27	9·522424	595	9·974481	75	9·547943	670	10·452057	33
28	9·522781	595	9·974436	75	9·548345	670	10·451655	32
29	9·523138	595	9·974391	73	9·548747	670	10·451253	31
30	9·523495		9·974347		9·549149		10·450851	30
'	Cosine.		Sine.		Cotang.		Tangent.	'

30 min. to 60 min.

70 degrees.

19 degrees.

30 min. to 60 min.

′	Sine.	Diff. for 100″	Cosine.	Diff. for 100″	Tangent.	Diff. for 100″	Cotang.	′
30	9·523495	595	9·974347	75	9·549149	668	10·450851	30
31	9·523852	593	9·974302	75	9·549550	668	10·450450	29
32	9·524208	593	9·974257	75	9·549951	668	10·450049	28
33	9·524564	593	9·974212	75	9·550352	667	10·449648	27
34	9·524920	592	9·974167	75	9·550752	668	10·449248	26
35	9·525275	592	9·974122	75	9·551153	665	10·448847	25
36	9·525630	590	9·974077	75	9·551552	667	10·448448	24
37	9·525984	592	9·974032	75	9·551952	665	10·448048	23
38	9·526339	590	9·973987	75	9·552351	665	10·447649	22
39	9·526693	588	9·973942	75	9·552750	665	10·447250	21
40	9·527046	590	9·973897	75	9·553149	665	10·446851	20
41	9·527400	588	9·973852	75	9·553548	663	10·446452	19
42	9·527753	587	9·973807	77	9·553946	663	10·446054	18
43	9·528105	588	9·973761	75	9·554344	662	10·445656	17
44	9·528458	587	9·973716	75	9·554741	663	10·445259	16
45	9·528810	585	9·973671	77	9·555139	662	10·444861	15
46	9·529161	587	9·973625	75	9·555536	662	10·444464	14
47	9·529513	585	9·973580	75	9·555933	660	10·444067	13
48	9·529864	585	9·973535	77	9·556329	660	10·443671	12
49	9·530215	583	9·973489	75	9·556725	660	10·443275	11
50	9·530565	583	9·973444	77	9·557121	660	10·442879	10
51	9·530915	583	9·973398	77	9·557517	660	10·442483	9
52	9·531265	582	9·973352	75	9·557913	658	10·442087	8
53	9·531614	582	9·973307	77	9·558308	658	10·441692	7
54	9·531963	582	9·973261	77	9·558703	657	10·441297	6
55	9·532312	582	9·973215	77	9·559097	657	10·440903	5
56	9·532661	580	9·973169	75	9·559491	657	10·440509	4
57	9·533009	580	9·973124	77	9·559885	657	10·440115	3
58	9·533357	578	9·973078	77	9·560279	657	10·439721	2
59	9·533704	580	9·973032	77	9·560673	655	10·439327	1
60	9·534052		9·972986		9·561066		10·438934	0
′	Cosine.		Sine.		Cotang.		Tangent.	′

0 min. to 30 min.

70 degrees.

20 degrees.

0 min. to 30 min.

′	Sine.	Diff. for 100″	Cosine.	Diff. for 100″	Tangent.	Diff. for 100″	Cotang.	′
0	9·534052	578	9·972986	77	9·561066	655	10·438934	60
1	9·534399	577	9·972940	77	9·561459	653	10·438541	59
2	9·534745	578	9·972894	77	9·561851	655	10·438149	58
3	9·535092	577	9·972848	77	9·562244	653	10·437756	57
4	9·535438	575	9·972802	78	9·562636	653	10·437364	56
5	9·535783	577	9·972755	77	9·563028	652	10·436972	55
6	9·536129	575	9·972709	77	9·563419	653	10·436581	54
7	9·536474	573	9·972663	77	9·563811	652	10·436189	53
8	9·536818	575	9·972617	78	9·564202	652	10·435798	52
9	9·537163	573	9·972570	77	9·564593	650	10·435407	51
10	9·537507	573	9·972524	77	9·564983	650	10·435017	50
11	9·537851	572	9·972478	78	9·565373	650	10·434627	49
12	9·538194	573	9·972431	77	9·565763	650	10·434237	48
13	9·538538	570	9·972385	78	9·566153	648	10·433847	47
14	9·538880	572	9·972338	78	9·566542	650	10·433458	46
15	9·539223	570	9·972291	77	9·566932	647	10·433068	45
16	9·539565	570	9·972245	78	9·567320	648	10·432680	44
17	9·539907	570	9·972198	78	9·567709	648	10·432291	43
18	9·540249	568	9·972151	77	9·568098	647	10·431902	42
19	9·540590	568	9·972105	78	9·568486	645	10·431514	41
20	9·540931	568	9·972058	78	9·568873	647	10·431127	40
21	9·541272	568	9·972011	78	9·569261	645	10·430739	39
22	9·541613	567	9·971964	78	9·569648	645	10·430352	38
23	9·541953	567	9·971917	78	9·570035	645	10·429965	37
24	9·542293	565	9·971870	78	9·570422	645	10·429578	36
25	9·542632	565	9·971823	78	9·570809	643	10·429191	35
26	9·542971	565	9·971776	78	9·571195	643	10·428805	34
27	9·543310	565	9·971729	78	9·571581	643	10·428419	33
28	9·543649	563	9·971682	78	9·571967	642	10·428033	32
29	9·543987	563	9·971635	78	9·572352	643	10·427648	31
30	9·544325		9·971588		9·572738		10·427262	30
′	Cosine.		Sine.		Cotang.		Tangent.	′

30 min. to 60 min.

69 degrees.

20 degrees.

30 min. to 60 min.

′	Sine.	Diff. for 100″	Cosine.	Diff. for 100″	Tangent.	Diff. for 100″	Cotang.	′
30	9·544325	563	9·971588	80	9·572738	642	10·427262	30
31	9·544663	562	9·971540	78	9·573123	640	10·426877	29
32	9·545000	563	9·971493	78	9·573507	642	10·426493	28
33	9·545338	560	9·971446	80	9·573892	640	10·426108	27
34	9·545674	562	9·971398	78	9·574276	640	10·425724	26
35	9·546011	560	9·971351	80	9·574660	640	10·425340	25
36	9·546347	560	9·971303	78	9·575044	638	10·424956	24
37	9·546683	560	9·971256	80	9·575427	638	10·424573	23
38	9·547019	558	9·971208	78	9·575810	638	10·424190	22
39	9·547354	558	9·971161	80	9·576193	638	10·423807	21
40	9·547689	558	9·971113	78	9·576576	638	10·423424	20
41	9·548024	558	9·971066	80	9·576959	637	10·423041	19
42	9·548359	557	9·971018	80	9·577341	637	10·422659	18
43	9·548693	557	9·970970	80	9·577723	635	10·422277	17
44	9·549027	555	9·970922	80	9·578104	637	10·421896	16
45	9·549360	555	9·970874	78	9·578486	635	10·421514	15
46	9·549693	555	9·970827	80	9·578867	635	10·421133	14
47	9·550026	555	9·970779	80	9·579248	635	10·420752	13
48	9·550359	555	9·970731	80	9·579629	633	10·420371	12
49	9·550692	553	9·970683	80	9·580009	633	10·419991	11
50	9·551024	553	9·970635	82	9·580389	633	10·419611	10
51	9·551356	552	9·970586	80	9·580769	633	10·419231	9
52	9·551687	552	9·970538	80	9·581149	632	10·418851	8
53	9·552018	552	9·970490	80	9·581528	632	10·418472	7
54	9·552349	552	9·970442	80	9·581907	632	10·418093	6
55	9·552680	550	9·970394	82	9·582286	632	10·417714	5
56	9·553010	552	9·970345	80	9·582665	632	10·417335	4
57	9·553341	548	9·970297	80	9·583044	630	10·416956	3
58	9·553670	550	9·970249	82	9·583422	630	10·416578	2
59	9·554000	548	9·970200	80	9·583800	628	10·416200	1
60	9·554329		9·970152		9·584177		10·415823	0
′	Cosine.		Sine.		Cotang.		Tangent.	′

0 min. to 30 min.

69 degrees.

21 degrees.

0 min. to 30 min.

′	Sine.	Diff. for 100″	Cosine.	Diff. for 100″	Tangent.	Diff. for 100″	Cotang.	′
0	9·554329	548	9·970152	82	9·584177	630	10·415823	60
1	9·554658	548	9·970103	80	9·584555	628	10·415445	59
2	9·554987	547	9·970055	82	9·584932	628	10·415068	58
3	9·555315	547	9·970006	82	9·585309	628	10·414691	57
4	9·555643	547	9·969957	80	9·585686	627	10·414314	56
5	9·555971	547	9·969909	82	9·586062	628	10·413938	55
6	9·556299	545	9·969860	82	9·586439	627	10·413561	54
7	9·556626	545	9·969811	82	9·586815	625	10·413185	53
8	9·556953	545	9·969762	80	9·587190	627	10·412810	52
9	9·557280	543	9·969714	82	9·587566	625	10·412434	51
10	9·557606	543	9·969665	82	9·587941	625	10·412059	50
11	9·557932	543	9·969616	82	9·588316	625	10·411684	49
12	9·558258	542	9·969567	82	9·588691	625	10·411309	48
13	9·558583	543	9·969518	82	9·589066	623	10·410934	47
14	9·558909	542	9·969469	82	9·589440	623	10·410560	46
15	9·559234	540	9·969420	83	9·589814	623	10·410186	45
16	9·559558	542	9·969370	82	9·590188	623	10·409812	44
17	9·559883	540	9·969321	82	9·590562	622	10·409438	43
18	9·560207	540	9·969272	82	9·590935	622	10·409065	42
19	9·560531	540	9·969223	83	9·591308	622	10·408692	41
20	9·560855	538	9·969173	82	9·591681	622	10·408319	40
21	9·561178	538	9·969124	82	9·592054	620	10·407946	39
22	9·561501	538	9·969075	83	9·592426	622	10·407574	38
23	9·561824	537	9·969025	82	9·592799	620	10·407201	37
24	9·562146	537	9·968976	83	9·593171	618	10·406829	36
25	9·562468	537	9·968926	82	9·593542	620	10·406458	35
26	9·562790	537	9·968877	83	9·593914	618	10·406086	34
27	9·563112	535	9·968827	83	9·594285	618	10·405715	33
28	9·563433	537	9·968777	82	9·594656	618	10·405344	32
29	9·563755	533	9·968728	83	9·595027	618	10·404973	31
30	9·564075		9·968678		9·595398		10·404602	30
′	Cosine.		Sine.		Cotang.		Tangent.	′

30 min. to 60 min.

68 degrees.

21 degrees.

30 min. to 60 min.

′	Sine.	Diff. for 100″	Cosine.	Diff. for 100″	Tangent.	Diff. for 100″	Cotang.	′
30	9·564075	535	9·968678	83	9·595398	617	10·404602	30
31	9·564396	533	9·968628	83	9·595768	617	10·404232	29
32	9·564716	533	9·968578	83	9·596138	617	10·403862	28
33	9·565036	533	9·968528	82	9·596508	617	10·403492	27
34	9·565356	533	9·968479	83	9·596878	615	10·403122	26
35	9·565676	532	9·968429	83	9·597247	615	10·402753	25
36	9·565995	532	9·968379	83	9·597616	615	10·402384	24
37	9·566314	530	9·968329	85	9·597985	615	10·402015	23
38	9·566632	532	9·968278	83	9·598354	613	10·401646	22
39	9·566951	530	9·968228	83	9·598722	615	10·401278	21
40	9·567269	530	9·968178	83	9·599091	613	10·400909	20
41	9·567587	528	9·968128	83	9·599459	613	10·400541	19
42	9·567904	530	9·968078	85	9·599827	612	10·400173	18
43	9·568222	528	9·968027	83	9·600194	613	10·399806	17
44	9·568539	528	9·967977	83	9·600562	612	10·399438	16
45	9·568856	527	9·967927	85	9·600929	612	10·399071	15
46	9·569172	527	9·967876	83	9·601296	612	10·398704	14
47	9·569488	527	9·967826	85	9·601663	610	10·398337	13
48	9·569804	527	9·967775	83	9·602029	610	10·397971	12
49	9·570120	525	9·967725	85	9·602395	610	10·397605	11
50	9·570435	527	9·967674	83	9·602761	610	10·397239	10
51	9·570751	525	9·967624	85	9·603127	610	10·396873	9
52	9·571066	523	9·967573	85	9·603493	608	10·396507	8
53	9·571380	525	9·967522	85	9·603858	608	10·396142	7
54	9·571695	523	9·967471	83	9·604223	608	10·395777	6
55	9·572009	523	9·967421	85	9·604588	608	10·395412	5
56	9·572323	522	9·967370	85	9·604953	607	10·395047	4
57	9·572636	523	9·967319	85	9·605317	608	10·394683	3
58	9·572950	522	9·967268	85	9·605682	607	10·394318	2
59	9·573263	520	9·967217	85	9·606046	607	10·393954	1
60	9·573575		9·967166		9·606410		10·393590	0
′	Cosine.		Sine.		Cotang.		Tangent.	′

0 min. to 30 min.

68 degrees.

22 degrees.

0 min. to 30 min.

′	Sine.	Diff. for 100″	Cosine.	Diff. for 100″	Tangent.	Diff. for 100″	Cotang.	′
0	9·573575	522	9·967166	85	9·606410	605	10·393590	60
1	9·573888	520	9·967115	85	9·606773	607	10·393227	59
2	9·574200	520	9·967064	85	9·607137	605	10·392863	58
3	9·574512	520	9·967013	87	9·607500	605	10·392500	57
4	9·574824	520	9·966961	85	9·607863	603	10·392137	56
5	9·575136	518	9·966910	85	9·608225	605	10·391775	55
6	9·575447	518	9·966859	85	9·608588	603	10·391412	54
7	9·575758	518	9·966808	87	9·608950	603	10·391050	53
8	9·576069	517	9·966756	85	9·609312	603	10·390688	52
9	9·576379	517	9·966705	87	9·609674	603	10·390326	51
10	9·576689	517	9·966653	85	9·610036	602	10·389964	50
11	9·576999	517	9·966602	87	9·610397	603	10·389603	49
12	9·577309	515	9·966550	85	9·610759	602	10·389241	48
13	9·577618	515	9·966499	87	9·611120	600	10·388880	47
14	9·577927	515	9·966447	87	9·611480	602	10·388520	46
15	9·578236	515	9·966395	85	9·611841	600	10·388159	45
16	9·578545	513	9·966344	87	9·612201	600	10·387799	44
17	9·578853	515	9·966292	87	9·612561	600	10·387439	43
18	9·579162	513	9·966240	87	9·612921	600	10·387079	42
19	9·579470	512	9·966188	87	9·613281	600	10·386719	41
20	9·579777	513	9·966136	85	9·613641	598	10·386359	40
21	9·580085	512	9·966085	87	9·614000	598	10·386000	39
22	9·580392	512	9·966033	87	9·614359	598	10·385641	38
23	9·580699	510	9·965981	87	9·614718	598	10·385282	37
24	9·581005	512	9·965929	88	9·615077	597	10·384923	36
25	9·581312	510	9·965876	87	9·615435	597	10·384565	35
26	9·581618	510	9·965824	87	9·615793	597	10·384207	34
27	9·581924	508	9·965772	87	9·616151	597	10·383849	33
28	9·582229	510	9·965720	87	9·616509	597	10·383491	32
29	9·582535	508	9·965668	88	9·616867	595	10·383133	31
30	9·582840		9·965615		9·617224		10·382776	30
′	Cosine.		Sine.		Cotang.		Tangent.	′

30 min. to 60 min.

67 degrees.

22 degrees.

30 min. to 60 min.

′	Sine.	Diff. for 100″	Cosine.	Diff. for 100″	Tangent.	Diff. for 100″	Cotang.	′
30	9·582840	508	9·965615	87	9·617224	597	10·382776	30
31	9·583145	507	9·965563	87	9·617582	595	10·382418	29
32	9·583449	508	9·965511	88	9·617939	593	10·382061	28
33	9·583754	507	9·965458	87	9·618295	595	10·381705	27
34	9·584058	505	9·965406	88	9·618652	593	10·381348	26
35	9·584361	507	9·965353	87	9·619008	593	10·380992	25
36	9·584665	505	9·965301	88	9·619364	593	10·380636	24
37	9·584968	507	9·965248	88	9·619720	593	10·380280	23
38	9·585272	503	9·965195	87	9·620076	593	10·379924	22
39	9·585574	505	9·965143	88	9·620432	592	10·379568	21
40	9·585877	503	9·965090	88	9·620787	592	10·379213	20
41	9·586179	505	9·965037	88	9·621142	592	10·378858	19
42	9·586482	502	9·964984	88	9·621497	592	10·378503	18
43	9·586783	503	9·964931	87	9·621852	592	10·378148	17
44	9·587085	502	9·964879	88	9·622207	590	10·377793	16
45	9·587386	503	9·964826	88	9·622561	590	10·377439	15
46	9·587688	502	9·964773	88	9·622915	590	10·377085	14
47	9·587989	500	9·964720	90	9·623269	590	10·376731	13
48	9·588289	502	9·964666	88	9·623623	588	10·376377	12
49	9·588590	500	9·964613	88	9·623976	590	10·376024	11
50	9·588890	500	9·964560	88	9·624330	588	10·375670	10
51	9·589190	498	9·964507	88	9·624683	588	10·375317	9
52	9·589489	500	9·964454	90	9·625036	587	10·374964	8
53	9·589789	498	9·964400	88	9·625388	588	10·374612	7
54	9·590088	498	9·964347	88	9·625741	587	10·374259	6
55	9·590387	498	9·964294	90	9·626093	587	10·373907	5
56	9·590686	497	9·964240	88	9·626445	587	10·373555	4
57	9·590984	497	9·964187	90	9·626797	587	10·373203	3
58	9·591282	497	9·964133	88	9·627149	587	10·372851	2
59	9·591580	497	9·964080	90	9·627501	585	10·372499	1
60	9·591878		9·964026		9·627852		10·372148	0
′	Cosine.		Sine.		Cotang.		Tangent.	′

0 min. to 30 min.

67 degrees.

23 degrees.

0 min. to 30 min.

	Sine.	Diff. for 100″	Cosine.	Diff. for 100″	Tangent.	Diff. for 100″	Cotang.	′
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	9·627852	585	10·372148	60
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	9·628203	585	10·371797	59
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	9·628554	585	10·371446	58
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	9·628905	583	10·371095	57
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	9·629255	585	10·370745	56
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	9·629606	583	10·370394	55
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	9·629956	583	10·370044	54
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	9·630306	583	10·369694	53
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	9·630656	582	10·369344	52
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	9·631005	583	10·368995	51
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	9·631355	582	10·368645	50
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	9·631704	582	10·368296	49
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	9·632053	582	10·367947	48
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	9·632402	580	10·367598	47
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	9·632750	582	10·367250	46
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	9·633099	580	10·366901	45
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	9·633447	580	10·366553	44
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	9·633795	580	10·366205	43
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	9·634143	578	10·365857	42
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	9·634490	580	10·365510	41
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	9·634838	578	10·365162	40
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	9·635185	578	10·364815	39
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	9·635532	578	10·364468	38
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	9·635879	578	10·364121	37
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	9·636226	577	10·363774	36
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	9·636572	578	10·363428	35
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	9·636919	577	10·363081	34
[illegible]	[illegible]	[illegible]	9·962562	[illegible]	9·637265	577	10·362735	33
[illegible]	[illegible]	[illegible]	9·962508	[illegible]	9·637611	575	10·362389	32
[illegible]	[illegible]	[illegible]	9·962453	[illegible]	9·637956	577	10·362044	31
[illegible]	[illegible]		9·962398		9·638302		10·361698	30
	Cosine.		Sine.		Cotang.		Tangent.	′

30 min. to 60 min.

66 degrees.

23 degrees.

30 min. to 60 min.

	Sine.	Diff. for 100″	Cosine.	Diff for 100″	Tangent.	Diff. for 100″	Cotang.	′
30	9·600700	483	9·962398	92	9·638302	575	10·361698	30
31	9·600990	483	9·962343	92	9·638647	575	10·361353	29
32	9·601280	483	9·962288	92	9·638992	575	10·361008	28
33	9·601570	483	9·962233	92	9·639337	575	10·360663	27
34	9·601860	483	9·962178	92	9·639682	575	10·360318	26
35	9·602150	482	9·962123	93	9·640027	573	10·359973	25
36	9·602439	482	9·962067	92	9·640371	575	10·359629	24
37	9·602728	482	9·962012	92	9·640716	573	10·359284	23
38	9·603017	480	9·961957	92	9·641060	573	10·358940	22
39	9·603305	482	9·961902	93	9·641404	572	10·358596	21
40	9·603594	480	9·961846	92	9·641747	573	10·358253	20
41	9·603882	480	9·961791	93	9·642091	572	10·357909	19
42	9·604170	478	9·961735	92	9·642434	572	10·357566	18
43	9·604457	480	9·961680	93	9·642777	572	10·357223	17
44	9·604745	478	9·961624	92	9·643120	572	10·356880	16
45	9·605032	478	9·961569	93	9·643463	572	10·356537	15
46	9·605319	478	9·961513	92	9·643806	570	10·356194	14
47	9·605606	477	9·961458	93	9·644148	570	10·355852	13
48	9·605892	478	9·961402	93	9·644490	570	10·355510	12
49	9·606179	477	9·961346	93	9·644832	570	10·355168	11
50	9·606465	477	9·961290	92	9·645174	570	10·354826	10
51	9·606751	475	9·961235	93	9·645516	568	10·354484	9
52	9·607036	477	9·961179	93	9·645857	570	10·354143	8
53	9·607322	475	9·961123	93	9·646199	568	10·353801	7
54	9·607607	475	9·961067	93	9·646540	568	10·353460	6
55	9·607892	475	9·961011	93	9·646881	568	10·353119	5
56	9·608177	473	9·960955	93	9·647222	567	10·352778	4
57	9·608461	473	9·960899	93	9·647562	568	10·352438	3
58	9·608745	473	9·960843	95	9·647903	567	10·352097	2
59	9·609029	473	9·960786	93	9·648243	567	10·351757	1
60	9·609313		9·960730		9·648583		10·351417	0
′	Cosine.		Sine.		Cotang.		Tangent.	′

0 min. to 30 min.

66 degrees.

24 degrees.

0 min. to 30 min.

′	Sine.	Dif. for 100″	Cosine.	Dif. for 100″	Tangent.	Dif. for 100″	Cotang.	′
0	[illegible]		9·960730		9·648583		10·351417	60
1	[illegible]	473	9·960674	93	9·648923	567	10·351077	59
2	[illegible]	472	9·960618	93	9·649263	567	10·350737	58
3	[illegible]	473	9·960561	95	9·649602	565	10·350398	57
4	[illegible]	472	9·960505	93	9·649942	567	10·350058	56
5	[illegible]	472	9·960448	95	9·650281	565	10·349719	55
6	[illegible]	472	9·960392	93	9·650620	565	10·349380	54
7	[illegible]	472	9·960335	95	9·650959	565	10·349041	53
8	[illegible]	472	9·960279	93	9·651297	563	10·348703	52
9	[illegible]	472	9·960222	95	9·651636	565	10·348364	51
10	[illegible]	472	9·960165	95	9·651974	563	10·348026	50
11	[illegible]	468	9·960109	93	9·652312	563	10·347688	49
12	[illegible]	468	9·960053	95	9·652650	563	10·347350	48
13	[illegible]	468	9·959996	95	9·652988	563	10·347012	47
14	[illegible]	468	9·959938	95	9·653326	563	10·346674	46
15	[illegible]	468	9·959882	93	9·653663	562	10·346337	45
16	[illegible]	467	9·959825	95	9·654000	562	10·346000	44
17	[illegible]	467	9·959768	95	9·654337	562	10·345663	43
18	[illegible]	467	9·959711	95	9·654674	562	10·345326	42
19	[illegible]	467	9·959654	95	9·655011	562	10·344989	41
20	9·614944	465	9·959596	97	9·655348	562	10·344652	40
21	9·615223	465	9·959539	95	9·655684	560	10·344316	39
22	9·615502	465	9·959482	95	9·656020	560	10·343980	38
23	9·615781	465	9·959425	95	9·656356	560	10·343644	37
24	9·616060	465	9·959368	95	9·656692	560	10·343308	36
25	9·616338	463	9·959310	97	9·657028	560	10·342972	35
26	9·616616	463	9·959253	95	9·657364	560	10·342636	34
27	9·616894	463	9·959195	97	9·657699	558	10·342301	33
28	9·617172	463	9·959138	95	9·658034	558	10·341966	32
29	9·617450	463	9·959080	97	9·658369	558	10·341631	31
30	9·617727	462	9·959023	95	9·658704	558	10·341296	30
′	Cosine.		Sine.		Cotang.		Tangent.	′

30 min. to 60 min.

65 degrees.

24 degrees.

30 min. to 60 min.

′	Sine.	Diff. for 100″	Cosine.	Diff. for 100″	Tangent.	Diff. for 100″	Cotang.	′
30	9·617727	462	9·959023	97	9·658704	558	10·341296	30
31	9·618004	462	9·958965	95	9·659039	557	10·340961	29
32	9·618281	462	9·958908	97	9·659373	558	10·340627	28
33	9·618558	460	9·958850	97	9·659708	557	10·340292	27
34	9·618834	460	9·958792	97	9·660042	557	10·339958	26
35	9·619110	460	9·958734	95	9·660376	557	10·339624	25
36	9·619386	460	9·958677	97	9·660710	555	10·339290	24
37	9·619662	460	9·958619	97	9·661043	557	10·338957	23
38	9·619938	458	9·958561	97	9·661377	555	10·338623	22
39	9·620213	458	9·958503	97	9·661710	555	10·338290	21
40	9·620488	458	9·958445	97	9·662043	555	10·337957	20
41	9·620763	458	9·958387	97	9·662376	555	10·337624	19
42	9·621038	458	9·958329	97	9·662709	555	10·337291	18
43	9·621313	457	9·958271	97	9·663042	555	10·336958	17
44	9·621587	457	9·958213	98	9·663375	553	10·336625	16
45	9·621861	457	9·958154	97	9·663707	553	10·336293	15
46	9·622135	457	9·958096	97	9·664039	553	10·335961	14
47	9·622409	455	9·958038	98	9·664371	553	10·335629	13
48	9·622682	457	9·957979	97	9·664703	553	10·335297	12
49	9·622956	455	9·957921	97	9·665035	552	10·334965	11
50	9·623229	455	9·957863	98	9·665366	553	10·334634	10
51	9·623502	453	9·957804	97	9·665698	552	10·334302	9
52	9·623774	455	9·957746	98	9·666029	552	10·333971	8
53	9·624047	453	9·957687	98	9·666360	552	10·333640	7
54	9·624319	453	9·957628	97	9·666691	550	10·333309	6
55	9·624591	453	9·957570	98	9·667021	552	10·332979	5
56	9·624863	453	9·957511	98	9·667352	550	10·332648	4
57	9·625135	452	9·957452	98	9·667682	552	10·332318	3
58	9·625406	452	9·957393	97	9·668013	550	10·331987	2
59	9·625677	452	9·957335	98	9·668343	550	10·331657	1
60	9·625948		9·957276		9·668673		10·331327	0
′	Cosine.		Sine.		Cotang.		Tangent.	′

0 min. to 30 min.

65 degrees.

25 degrees.

0 min. to 30 min.

′	Sine.	Diff. for 100″	Cosine.	Diff. for 100″	Tangent.	Diff. for 100″	Cotang.	′
0	9·625948		9·957276		9·668673		10·331327	60
1	9·626219	452	9·957217	98	9·669002	548	10·330998	59
2	9·626490	452	9·957158	98	9·669332	550	10·330668	58
3	9·626760	450	9·957099	98	9·669661	548	10·330339	57
4	9·627030	450	9·957040	98	9·669991	550	10·330009	56
5	9·627300	450	9·956981	98	9·670320	548	10·329680	55
6	9·627570	450	9·956921	100	9·670649	548	10·329351	54
7	9·627840	450	9·956862	98	9·670977	547	10·329023	53
8	9·628109	448	9·956803	98	9·671306	548	10·328694	52
9	9·628378	448	9·956744	98	9·671635	548	10·328365	51
10	9·628647	448	9·956684	100	9·671963	547	10·328037	50
11	9·628916	448	9·956625	98	9·672291	547	10·327709	49
12	9·629185	448	9·956566	98	9·672619	547	10·327381	48
13	9·629453	447	9·956506	100	9·672947	547	10·327053	47
14	9·629721	447	9·956447	98	9·673274	545	10·326726	46
15	9·629989	447	9·956387	100	9·673602	547	10·326398	45
16	9·630257	447	9·956327	100	9·673929	545	10·326071	44
17	9·630524	445	9·956268	98	9·674257	547	10·325743	43
18	9·630792	447	9·956208	100	9·674584	545	10·325416	42
19	9·631059	445	9·956148	100	9·674911	545	10·325089	41
20	9·631326	445	9·956089	98	9·675237	543	10·324763	40
21	9·631593	445	9·956029	100	9·675564	545	10·324436	39
22	9·631859	443	9·955969	100	9·675890	543	10·324110	38
23	9·632125	443	9·955909	100	9·676217	545	10·323783	37
24	9·632392	445	9·955849	100	9·676543	543	10·323457	36
25	9·632658	443	9·955789	100	9·676869	543	10·323131	35
26	9·632923	442	9·955729	100	9·677194	542	10·322806	34
27	9·633189	443	9·955669	100	9·677520	543	10·322480	33
28	9·633454	442	9·955609	100	9·677846	543	10·322154	32
29	9·633719	442	9·955548	102	9·678171	542	10·321829	31
30	9·633984	442	9·955488	100	9·678496	542	10·321504	30
′	Cosine.		Sine.		Cotang.		Tangent.	′

30 min. to 60 min.

64 degrees.

25 degrees.

30 min. to 60 min.

′	Sine.	Diff. for 100″	Cosine.	Diff. for 100″	Tangent.	Diff. for 100″	Cotang.	′
30	9·633984	442	9·955488	100	9·678496	542	10·321504	30
31	9·634249	442	9·955428	100	9·678821	542	10·321179	29
32	9·634514	440	9·955368	102	9·679146	542	10·320854	28
33	9·634778	440	9·955307	100	9·679471	540	10·320529	27
34	9·635042	440	9·955247	102	9·679795	542	10·320205	26
35	9·635306	440	9·955186	100	9·680120	540	10·319880	25
36	9·635570	440	9·955126	102	9·680444	540	10·319556	24
37	9·635834	438	9·955065	100	9·680768	540	10·319232	23
38	9·636097	438	9·955005	102	9·681092	540	10·318908	22
39	9·636360	438	9·954944	102	9·681416	540	10·318584	21
40	9·636623	438	9·954883	100	9·681740	538	10·318260	20
41	9·636886	437	9·954823	102	9·682063	540	10·317937	19
42	9·637148	438	9·954762	102	9·682387	538	10·317613	18
43	9·637411	437	9·954701	102	9·682710	538	10·317290	17
44	9·637673	437	9·954640	102	9·683033	538	10·316967	16
45	9·637935	437	9·954579	102	9·683356	538	10·316644	15
46	9·638197	435	9·954518	102	9·683679	537	10·316321	14
47	9·638458	437	9·954457	102	9·684001	538	10·315999	13
48	9·638720	435	9·954396	102	9·684324	537	10·315676	12
49	9·638981	435	9·954335	102	9·684646	537	10·315354	11
50	9·639242	435	9·954274	102	9·684968	537	10·315032	10
51	9·639503	435	9·954213	102	9·685290	537	10·314710	9
52	9·639764	433	9·954152	103	9·685612	537	10·314388	8
53	9·640024	433	9·954090	102	9·685934	535	10·314066	7
54	9·640284	433	9·954029	102	9·686255	537	10·313745	6
55	9·640544	433	9·953968	103	9·686577	535	10·313423	5
56	9·640804	433	9·953906	102	9·686898	535	10·313102	4
57	9·641064	433	9·953845	103	9·687219	535	10·312781	3
58	9·641324	432	9·953783	102	9·687540	535	10·312460	2
59	9·641583	432	9·953722	103	9·687861	535	10·312139	1
60	9·641842		9·953660		9·688182		10·311818	0
′	Cosine.		Sine.		Cotang.		Tangent.	′

0 min. to 30 min.

64 degrees.

26 degrees.

0 min. to 30 min.

′	Sine.	Diff for 100″	Cosine.	Diff for 100″	Tangent.	Diff for 100″	Cotang.	′
0	9·641842		9·953660		9·688182		10·311818	60
1	9·642101	432	9·953599	102	9·688502	533	10·311498	59
2	9·642360	432	9·953537	103	9·688823	533	10·311177	58
3	9·642618	430	9·953475	103	9·689143	533	10·310857	57
4	9·642877	432	9·953413	103	9·689463	533	10·310537	56
5	9·643135	430	9·953352	102	9·689783	533	10·310217	55
6	9·643393	430	9·953290	103	9·690103	533	10·309897	54
7	9·643650	428	9·953228	103	9·690423	533	10·309577	53
8	9·643908	430	9·953166	103	9·690742	532	10·309258	52
9	9·644165	428	9·953104	103	9·691062	533	10·308938	51
10	9·644423	430	9·953042	103	9·691381	532	10·308619	50
11	9·644680	428	9·952980	103	9·691700	532	10·308300	49
12	9·644936	427	9·952918	103	9·692019	532	10·307981	48
13	9·645193	428	9·952855	105	9·692338	532	10·307662	47
14	9·645450	428	9·952793	103	9·692656	530	10·307344	46
15	9·645706	427	9·952731	103	9·692975	532	10·307025	45
16	9·645962	427	9·952669	103	9·693293	530	10·306707	44
17	9·646218	427	9·952606	105	9·693612	532	10·306388	43
18	9·646474	427	9·952544	103	9·693930	530	10·306070	42
19	9·646729	425	9·952481	105	9·694248	530	10·305752	41
20	9·646984	425	9·952419	103	9·694566	530	10·305434	40
21	9·647240	427	9·952356	105	9·694883	528	10·305117	39
22	9·647494	423	9·952294	103	9·695201	530	10·304799	38
23	9·647749	425	9·952231	105	9·695518	528	10·304482	37
24	9·648004	425	9·952168	105	9·695836	530	10·304164	36
25	9·648258	423	9·952106	103	9·696153	528	10·303847	35
26	9·648512	423	9·952043	105	9·696470	528	10·303530	34
27	9·648766	423	9·951980	105	9·696787	528	10·303213	33
28	9·649020	423	9·951917	105	9·697103	527	10·302897	32
29	9·649274	423	9·951854	105	9·697420	528	10·302580	31
30	9·649527	422	9·951791	105	9·697736	527	10·302264	30
′	Cosine.		Sine.		Cotang.		Tangent.	

30 min. to 60 min.

63 degrees.

26 degrees.

30 min. to 60 min.

′	Sine.	Diff. for 100″	Cosine.	Diff. for 100″	Tangent.	Diff. for 100″	Cotang.	′
30	9·649527	423	9·951791	105	9·697736	528	10·302264	30
31	9·649781	422	9·951728	105	9·698053	527	10·301947	29
32	9·650034	422	9·951665	105	9·698369	527	10·301631	28
33	9·650287	420	9·951602	105	9·698685	527	10·301315	27
34	9·650539	422	9·951539	105	9·699001	525	10·300999	26
35	9·650792	420	9·951476	107	9·699316	527	10·300684	25
36	9·651044	422	9·951412	105	9·699632	525	10·300368	24
37	9·651297	420	9·951349	105	9·699947	527	10·300053	23
38	9·651549	418	9·951286	107	9·700263	525	10·299737	22
39	9·651800	420	9·951222	105	9·700578	525	10·299422	21
40	9·652052	420	9·951159	105	9·700893	525	10·299107	20
41	9·652304	418	9·951096	107	9·701208	525	10·298792	19
42	9·652555	418	9·951032	107	9·701523	523	10·298477	18
43	9·652806	418	9·950968	105	9·701837	525	10·298163	17
44	9·653057	418	9·950905	107	9·702152	523	10·297848	16
45	9·653308	417	9·950841	105	9·702466	525	10·297534	15
46	9·653558	417	9·950778	107	9·702781	523	10·297219	14
47	9·653808	418	9·950714	107	9·703095	523	10·296905	13
48	9·654059	417	9·950650	107	9·703409	522	10·296591	12
49	9·654309	415	9·950586	107	9·703722	523	10·296278	11
50	9·654558	417	9·950522	107	9·704036	523	10·295964	10
51	9·654808	417	9·950458	107	9·704350	522	10·295650	9
52	9·655058	415	9·950394	107	9·704663	522	10·295337	8
53	9·655307	415	9·950330	107	9·704976	523	10·295024	7
54	9·655556	415	9·950266	107	9·705290	522	10·294710	6
55	9·655805	415	9·950202	107	9·705603	522	10·294397	5
56	9·656054	413	9·950138	107	9·705916	520	10·294084	4
57	9·656302	415	9·950074	107	9·706228	522	10·293772	3
58	9·656551	413	9·950010	108	9·706541	522	10·293459	2
59	9·656799	413	9·949945	107	9·706854	520	10·293146	1
60	9·657047		9·949881		9·707166		10·292834	0
′	Cosine.		Sine.		Cotang.		Tangent.	′

0 min. to 30 min.

63 degrees.

27 degrees.

0 min. to 30 min.

′	Sine.	Diff. for 100″	Cosine.	Diff. for 100″	Tangent.	Diff. for 100″	Cotang.	′
0	9·657047	413	9·949881	108	9·707166	520	10·292834	60
1	9·657295	412	9·949816	107	9·707478	520	10·292522	59
2	9·657542	413	9·949752	107	9·707790	520	10·292210	58
3	9·657790	412	9·949688	108	9·708102	520	10·291898	57
4	9·658037	412	9·949623	108	9·708414	520	10·291586	56
5	9·658284	412	9·949558	107	9·708726	518	10·291274	55
6	9·658531	412	9·949494	108	9·709037	520	10·290963	54
7	9·658778	412	9·949429	108	9·709349	518	10·290651	53
8	9·659025	410	9·949364	107	9·709660	518	10·290340	52
9	9·659271	410	9·949300	108	9·709971	518	10·290029	51
10	9·659517	410	9·949235	108	9·710282	518	10·289718	50
11	9·659763	410	9·949170	108	9·710593	518	10·289407	49
12	9·660009	410	9·949105	108	9·710904	518	10·289096	48
13	9·660255	410	9·949040	108	9·711215	517	10·288785	47
14	9·660501	408	9·948975	108	9·711525	518	10·288475	46
15	9·660746	408	9·948910	108	9·711836	517	10·288164	45
16	9·660991	408	9·948845	108	9·712146	517	10·287854	44
17	9·661236	408	9·948780	108	9·712456	517	10·287544	43
18	9·661481	408	9·948715	108	9·712766	517	10·287234	42
19	9·661726	407	9·948650	110	9·713076	517	10·286924	41
20	9·661970	407	9·948584	108	9·713386	517	10·286614	40
21	9·662214	408	9·948519	108	9·713696	515	10·286304	39
22	9·662459	407	9·948454	110	9·714005	515	10·285995	38
23	9·662703	405	9·948388	108	9·714314	517	10·285686	37
24	9·662946	407	9·948323	110	9·714624	515	10·285376	36
25	9·663190	405	9·948257	108	9·714933	515	10·285067	35
26	9·663433	407	9·948192	110	9·715242	515	10·284758	34
27	9·663677	405	9·948126	110	9·715551	515	10·284449	33
28	9·663920	405	9·948060	108	9·715860	513	10·284140	32
29	9·664163	405	9·947995	110	9·716168	515	10·283832	31
30	9·664406		9·947929		9·716477		10·283523	30
′	Cosine.		Sine.		Cotang.		Tangent.	′

30 min. to 60 min.

62 degrees.

27 degrees.

30 min. to 60 min.

′	Sine.	Diff. for 100″	Cosine.	Diff. for 100″	Tangent.	Diff. for 100″	Cotang.	′
30	9·664406	403	9·947929	110	9·716477	513	10·283523	30
31	9·664648	405	9·947863	110	9·716785	513	10·283215	29
32	9·664891	403	9·947797	110	9·717093	513	10·282907	28
33	9·665133	403	9·947731	110	9·717401	513	10·282599	27
34	9·665375	403	9·947665	108	9·717709	513	10·282291	26
35	9·665617	403	9·947600	112	9·718017	513	10·281983	25
36	9·665859	402	9·947533	110	9·718325	513	10·281675	24
37	9·666100	403	9·947467	110	9·718633	512	10·281367	23
38	9·666342	402	9·947401	110	9·718940	513	10·281060	22
39	9·666583	402	9·947335	110	9·719248	512	10·280752	21
40	9·666824	402	9·947269	110	9·719555	512	10·280445	20
41	9·667065	400	9·947203	112	9·719862	512	10·280138	19
42	9·667305	402	9·947136	110	9·720169	512	10·279831	18
43	9·667546	400	9·947070	110	9·720476	512	10·279524	17
44	9·667786	402	9·947004	112	9·720783	510	10·279217	16
45	9·668027	400	9·946937	110	9·721089	512	10·278911	15
46	9·668267	398	9·946871	112	9·721396	510	10·278604	14
47	9·668506	400	9·946804	110	9·721702	512	10·278298	13
48	9·668746	400	9·946738	112	9·722009	510	10·277991	12
49	9·668986	398	9·946671	112	9·722315	510	10·277685	11
50	9·669225	398	9·946604	110	9·722621	510	10·277379	10
51	9·669464	398	9·946538	112	9·722927	508	10·277073	9
52	9·669703	398	9·946471	112	9·723232	510	10·276768	8
53	9·669942	398	9·946404	112	9·723538	510	10·276462	7
54	9·670181	397	9·946337	112	9·723844	508	10·276156	6
55	9·670419	398	9·946270	112	9·724149	508	10·275851	5
56	9·670658	397	9·946203	112	9·724454	510	10·275546	4
57	9·670896	397	9·946136	112	9·724760	508	10·275240	3
58	9·671134	397	9·946069	112	9·725065	508	10·274935	2
59	9·671372	395	9·946002	112	9·725370	507	10·274630	1
60	9·671609		9·945935		9·725674		10·274326	0
′	Cosine.		Sine.		Cotang.		Tangent.	′

0 min. to 30 min.

62 degrees.

28 degrees.

0 min. to 30 min.

′	Sine.	Diff. for 100″	Cosine.	Diff. for 100″	Tangent.	Diff. for 100″	Cotang.	′
0	9·671609	397	9·945935	112	9·725674	508	10·274326	60
1	9·671847	395	9·945868	113	9·725979	508	10·274021	59
2	9·672084	395	9·945800	112	9·726284	507	10·273716	58
3	9·672321	395	9·945733	112	9·726588	507	10·273412	57
4	9·672558	395	9·945666	113	9·726892	508	10·273108	56
5	9·672795	395	9·945598	112	9·727197	507	10·272803	55
6	9·673032	393	9·945531	112	9·727501	507	10·272499	54
7	9·673268	395	9·945464	113	9·727805	507	10·272195	53
8	9·673505	393	9·945396	113	9·728109	505	10·271891	52
9	9·673741	393	9·945328	112	9·728412	507	10·271588	51
10	9·673977	393	9·945261	113	9·728716	507	10·271284	50
11	9·674213	392	9·945193	113	9·729020	505	10·270980	49
12	9·674448	393	9·945125	112	9·729323	505	10·270677	48
13	9·674684	392	9·945058	113	9·729626	505	10·270374	47
14	9·674919	393	9·944990	113	9·729929	507	10·270071	46
15	9·675155	392	9·944922	113	9·730233	503	10·269767	45
16	9·675390	390	9·944854	113	9·730535	505	10·269465	44
17	9·675624	392	9·944786	113	9·730838	505	10·269162	43
18	9·675859	392	9·944718	113	9·731141	505	10·268859	42
19	9·676094	390	9·944650	113	9·731444	503	10·268556	41
20	9·676328	390	9·944582	113	9·731746	503	10·268254	40
21	9·676562	390	9·944514	113	9·732048	505	10·267952	39
22	9·676796	390	9·944446	115	9·732351	503	10·267649	38
23	9·677030	390	9·944377	113	9·732653	503	10·267347	37
24	9·677264	390	9·944309	113	9·732955	503	10·267045	36
25	9·677498	388	9·944241	115	9·733257	502	10·266743	35
26	9·677731	388	9·944172	113	9·733558	503	10·266442	34
27	9·677964	388	9·944104	113	9·733860	503	10·266140	33
28	9·678197	388	9·944036	115	9·734162	502	10·265838	32
29	9·678430	388	9·943967	113	9·734463	502	10·265537	31
30	9·678663		9·943899		9·734764		10·265236	30
′	Cosine.		Sine.		Cotang.		Tangent.	′

30 min. to 60 min.

61 degrees.

28 degrees.

30 min. to 60 min.

′	Sine.	Diff. for 100″	Cosine.	Diff. for 100″	Tangent.	Diff. for 100″	Cotang.	′
30	9·678663	387	9·943899	115	9·734764	503	10·265236	30
31	9·678895	388	9·943830	115	9·735066	502	10·264934	29
32	9·679128	387	9·943761	113	9·735367	502	10·264633	28
33	9·679360	387	9·943693	115	9·735668	502	10·264332	27
34	9·679592	387	9·943624	115	9·735969	500	10·264031	26
35	9·679824	387	9·943555	115	9·736269	502	10·263731	25
36	9·680056	387	9·943486	115	9·736570	500	10·263430	24
37	9·680288	385	9·943417	115	9·736870	502	10·263130	23
38	9·680519	385	9·943348	115	9·737171	500	10·262829	22
39	9·680750	387	9·943279	115	9·737471	500	10·262529	21
40	9·680982	385	9·943210	115	9·737771	500	10·262229	20
41	9·681213	383	9·943141	115	9·738071	500	10·261929	19
42	9·681443	385	9·943072	115	9·738371	500	10·261629	18
43	9·681674	385	9·943003	115	9·738671	500	10·261329	17
44	9·681905	383	9·942934	117	9·738971	500	10·261029	16
45	9·682135	383	9·942864	115	9·739271	498	10·260729	15
46	9·682365	383	9·942795	115	9·739570	500	10·260430	14
47	9·682595	383	9·942726	117	9·739870	498	10·260130	13
48	9·682825	383	9·942656	115	9·740169	498	10·259831	12
49	9·683055	382	9·942587	117	9·740468	498	10·259532	11
50	9·683284	383	9·942517	115	9·740767	498	10·259233	10
51	9·683514	382	9·942448	117	9·741066	498	10·258934	9
52	9·683743	382	9·942378	117	9·741365	498	10·258635	8
53	9·683972	382	9·942308	115	9·741664	497	10·258336	7
54	9·684201	382	9·942239	117	9·741962	498	10·258038	6
55	9·684430	380	9·942169	117	9·742261	497	10·257739	5
56	9·684658	382	9·942099	117	9·742559	498	10·257441	4
57	9·684887	380	9·942029	117	9·742858	497	10·257142	3
58	9·685115	380	9·941959	117	9·743156	497	10·256844	2
59	9·685343	380	9·941889	117	9·743454	497	10·256546	1
60	9·685571		9·941819		9·743752		10·256248	0
′	Cosine.		Sine.		Cotang.		Tangent.	′

0 min. to 30 min.

61 degrees.

29 degrees.

0 min. to 30 min.

′	Sine.	Diff. for 100″	Cosine.	Diff. for 100″	Tangent.	Diff. for 100″	Cotang.	′
0	9·685571	380	9·941819	117	9·743752	497	10·256248	60
1	9·685799	380	9·941749	117	9·744050	497	10·255950	59
2	9·686027	378	9·941679	117	9·744348	495	10·255652	58
3	9·686254	380	9·941609	117	9·744645	497	10·255355	57
4	9·686482	378	9·941539	117	9·744943	495	10·255057	56
5	9·686709	378	9·941469	118	9·745240	497	10·254760	55
6	9·686936	378	9·941398	117	9·745538	495	10·254462	54
7	9·687163	377	9·941328	117	9·745835	495	10·254165	53
8	9·687389	378	9·941258	118	9·746132	495	10·253868	52
9	9·687616	378	9·941187	117	9·746429	495	10·253571	51
10	9·687843	377	9·941117	118	9·746726	495	10·253274	50
11	9·688069	377	9·941046	118	9·747023	493	10·252977	49
12	9·688295	377	9·940975	117	9·747319	495	10·252681	48
13	9·688521	377	9·940905	118	9·747616	495	10·252384	47
14	9·688747	375	9·940834	118	9·747913	493	10·252087	46
15	9·688972	377	9·940763	117	9·748209	493	10·251791	45
16	9·689198	375	9·940693	118	9·748505	493	10·251495	44
17	9·689423	375	9·940622	118	9·748801	493	10·251199	43
18	9·689648	375	9·940551	118	9·749097	493	10·250903	42
19	9·689873	375	9·940480	118	9·749393	493	10·250607	41
20	9·690098	375	9·940409	118	9·749689	493	10·250311	40
21	9·690323	375	9·940338	118	9·749985	493	10·250015	39
22	9·690548	373	9·940267	118	9·750281	492	10·249719	38
23	9·690772	373	9·940196	118	9·750576	493	10·249424	37
24	9·690996	373	9·940125	118	9·750872	492	10·249128	36
25	9·691220	373	9·940054	120	9·751167	492	10·248833	35
26	9·691444	373	9·939982	118	9·751462	492	10·248538	34
27	9·691668	373	9·939911	118	9·751757	492	10·248243	33
28	9·691892	372	9·939840	120	9·752052	492	10·247948	32
29	9·692115	373	9·939768	118	9·752347	492	10·247653	31
30	9·692339		9·939697		9·752642		10·247358	30
′	Cosine.		Sine.		Cotang.		Tangent.	′

30 min. to 60 min.

60 degrees.

29 degrees.

30 min. to 60 min.

′	Sine.	Diff. for 100″	Cosine.	Diff. for 100″	Tangent.	Diff. for 100″	Cotang.	′
30	9·692339	372	9·939697	120	9·752642	492	10·247358	30
31	9·692562	372	9·939625	118	9·752937	490	10·247063	29
32	9·692785	372	9·939554	120	9·753231	492	10·246769	28
33	9·693008	372	9·939482	120	9·753526	490	10·246474	27
34	9·693231	370	9·939410	118	9·753820	492	10·246180	26
35	9·693453	372	9·939339	120	9·754115	490	10·245885	25
36	9·693676	370	9·939267	120	9·754409	490	10·245591	24
37	9·693898	370	9·939195	120	9·754703	490	10·245297	23
38	9·694120	370	9·939123	118	9·754997	490	10·245003	22
39	9·694342	370	9·939052	120	9·755291	490	10·244709	21
40	9·694564	370	9·938980	120	9·755585	488	10·244415	20
41	9·694786	368	9·938908	120	9·755878	490	10·244122	19
42	9·695007	370	9·938836	122	9·756172	488	10·243828	18
43	9·695229	368	9·938763	120	9·756465	490	10·243535	17
44	9·695450	368	9·938691	120	9·756759	488	10·243241	16
45	9·695671	368	9·938619	120	9·757052	488	10·242948	15
46	9·695892	368	9·938547	120	9·757345	488	10·242655	14
47	9·696113	368	9·938475	122	9·757638	488	10·242362	13
48	9·696334	367	9·938402	120	9·757931	488	10·242069	12
49	9·696554	368	9·938330	120	9·758224	488	10·241776	11
50	9·696775	367	9·938258	122	9·758517	488	10·241483	10
51	9·696995	367	9·938185	120	9·758810	487	10·241190	9
52	9·697215	367	9·938113	122	9·759102	488	10·240898	8
53	9·697435	365	9·938040	122	9·759395	487	10·240605	7
54	9·697654	367	9·937967	120	9·759687	487	10·240313	6
55	9·697874	367	9·937895	122	9·759979	488	10·240021	5
56	9·698094	365	9·937822	122	9·760272	487	10·239728	4
57	9·698313	365	9·937749	122	9·760564	487	10·239436	3
58	9·698532	365	9·937676	120	9·760856	487	10·239144	2
59	9·698751	365	9·937604	122	9·761148	485	10·238852	1
60	9·698970		9·937531		9·761439		10·238561	0
′	Cosine.		Sine.		Cotang.		Tangent.	′

0 min. to 30 min.

60 degrees.

30 degrees.

0 min. to 30 min.

′	Sine.	Diff. for 100″	Cosine.	Diff. for 100″	Tangent.	Diff. for 100″	Cotang.	′
0	9·698970	365	9·937531	122	9·761439	487	10·238561	60
1	9·699189	363	9·937458	122	9·761731	487	10·238269	59
2	9·699407	365	9·937385	122	9·762023	485	10·237977	58
3	9·699626	363	9·937312	123	9·762314	487	10·237686	57
4	9·699844	363	9·937238	122	9·762606	485	10·237394	56
5	9·700062	363	9·937165	122	9·762897	485	10·237103	55
6	9·700280	363	9·937092	122	9·763188	485	10·236812	54
7	9·700498	363	9·937019	122	9·763479	485	10·236521	53
8	9·700716	362	9·936946	123	9·763770	485	10·236230	52
9	9·700933	363	9·936872	122	9·764061	485	10·235939	51
10	9·701151	362	9·936799	123	9·764352	485	10·235648	50
11	9·701368	362	9·936725	122	9·764643	483	10·235357	49
12	9·701585	362	9·936652	123	9·764933	485	10·235067	48
13	9·701802	362	9·936578	122	9·765224	483	10·234776	47
14	9·702019	362	9·936505	123	9·765514	485	10·234486	46
15	9·702236	360	9·936431	123	9·765805	483	10·234195	45
16	9·702452	362	9·936357	122	9·766095	483	10·233905	44
17	9·702669	360	9·936284	123	9·766385	483	10·233615	43
18	9·702885	360	9·936210	123	9·766675	483	10·233325	42
19	9·703101	360	9·936136	123	9·766965	483	10·233035	41
20	9·703317	360	9·936062	123	9·767255	483	10·232745	40
21	9·703533	360	9·935988	123	9·767545	482	10·232455	39
22	9·703749	358	9·935914	123	9·767834	483	10·232166	38
23	9·703964	358	9·935840	123	9·768124	483	10·231876	37
24	9·704179	360	9·935766	123	9·768414	482	10·231586	36
25	9·704395	358	9·935692	123	9·768703	482	10·231297	35
26	9·704610	358	9·935618	125	9·768992	482	10·231008	34
27	9·704825	358	9·935543	123	9·769281	483	10·230719	33
28	9·705040	357	9·935469	123	9·769571	482	10·230429	32
29	9·705254	358	9·935395	125	9·769860	480	10·230140	31
30	9·705469		9·935320		9·770148		10·229852	30
′	Cosine.		Sine.		Cotang.		Tangent.	′

30 min. to 60 min.

59 degrees.

30 degrees.

30 min. to 60 min.

′	Sine.	Diff. for 100″	Cosine.	Diff. for 100″	Tangent.	Diff. for 100″	Cotang.	′
30	9·705469	357	9·935320	123	9·770148	482	10·229852	30
31	9·705683	358	9·935246	125	9·770437	482	10·229563	29
32	9·705898	357	9·935171	123	9·770726	482	10·229274	28
33	9·706112	357	9·935097	125	9·771015	480	10·228985	27
34	9·706326	355	9·935022	123	9·771303	482	10·228697	26
35	9·706539	357	9·934948	125	9·771592	480	10·228408	25
36	9·706753	357	9·934873	125	9·771880	480	10·228120	24
37	9·706967	355	9·934798	125	9·772168	482	10·227832	23
38	9·707180	355	9·934723	123	9·772457	480	10·227543	22
39	9·707393	355	9·934649	125	9·772745	480	10·227255	21
40	9·707606	355	9·934574	125	9·773033	480	10·226967	20
41	9·707819	355	9·934499	125	9·773321	478	10·226679	19
42	9·708032	355	9·934424	125	9·773608	480	10·226392	18
43	9·708245	355	9·934349	125	9·773896	480	10·226104	17
44	9·708458	353	9·934274	125	9·774184	478	10·225816	16
45	9·708670	353	9·934199	127	9·774471	480	10·225529	15
46	9·708882	353	9·934123	125	9·774759	478	10·225241	14
47	9·709094	353	9·934048	125	9·775046	478	10·224954	13
48	9·709306	353	9·933973	125	9·775333	480	10·224667	12
49	9·709518	353	9·933898	127	9·775621	478	10·224379	11
50	9·709730	352	9·933822	125	9·775908	478	10·224092	10
51	9·709941	353	9·933747	127	9·776195	478	10·223805	9
52	9·710153	352	9·933671	125	9·776482	477	10·223518	8
53	9·710364	352	9·933596	127	9·776768	478	10·223232	7
54	9·710575	352	9·933520	125	9·777055	478	10·222945	6
55	9·710786	352	9·933445	127	9·777342	477	10·222658	5
56	9·710997	352	9·933369	127	9·777628	478	10·222372	4
57	9·711208	352	9·933293	127	9·777915	477	10·222085	3
58	9·711419	350	9·933217	127	9·778201	478	10·221799	2
59	9·711629	350	9·933141	125	9·778488	477	10·221512	1
60	9·711839		9·933066		9·778774		10·221226	0
′	Cosine.		Sine.		Cotang.		Tangent.	′

0 min. to 30 min.

59 degrees.

31 degrees.

0 min. to 30 min.

	Sine.	Diff. for 100″	Cosine.	Diff. for 100″	Tangent.	Diff. for 100″	Cotang.	′
0	9·711839		9·933066		9·778774		10·221226	60
1	9·712050	352	9·932990	127	9·779060	477	10·220940	59
2	9·712260	350	9·932914	127	9·779346	477	10·220654	58
3	9·712469	348	9·932838	127	9·779632	477	10·220368	57
4	9·712679	350	9·932762	127	9·779918	477	10·220082	56
5	9·712889	350	9·932685	128	9·780203	475	10·219797	55
6	9·713098	348	9·932609	127	9·780489	477	10·219511	54
7	9·713308	350	9·932533	127	9·780775	477	10·219225	53
8	9·713517	348	9·932457	127	9·781060	475	10·218940	52
9	9·713726	348	9·932380	128	9·781346	477	10·218654	51
10	9·713935	348	9·932304	127	9·781631	475	10·218369	50
11	9·714144	348	9·932228	127	9·781916	475	10·218084	49
12	9·714352	347	9·932151	128	9·782201	475	10·217799	48
13	9·714561	348	9·932075	127	9·782486	475	10·217514	47
14	9·714769	347	9·931998	128	9·782771	475	10·217229	46
15	9·714978	348	9·931921	128	9·783056	475	10·216944	45
16	9·715186	347	9·931845	127	9·783341	475	10·216659	44
17	9·715394	347	9·931768	128	9·783626	475	10·216374	43
18	9·715602	347	9·931691	128	9·783910	473	10·216090	42
19	9·715809	345	9·931614	128	9·784195	475	10·215805	41
20	9·716017	347	9·931537	128	9·784479	473	10·215521	40
21	9·716224	345	9·931460	128	9·784764	475	10·215236	39
22	9·716432	347	9·931383	128	9·785048	473	10·214952	38
23	9·716639	345	9·931306	128	9·785332	473	10·214668	37
24	9·716846	345	9·931229	128	9·785616	473	10·214384	36
25	9·717053	345	9·931152	128	9·785900	473	10·214100	35
26	9·717259	343	9·931075	128	9·786184	473	10·213816	34
27	9·717466	345	9·930998	128	9·786468	473	10·213532	33
28	9·717673	345	9·930921	128	9·786752	473	10·213248	32
29	9·717879	343	9·930843	130	9·787036	473	10·212964	31
30	9·718085	343	9·930766	128	9·787319	472	10·212681	30
′	Cosine.		Sine.		Cotang.		Tangent.	′

30 min. to 60 min.

58 degrees.

31 degrees.

30 min. to 60 min.

′	Sine.	Diff. for 100″	Cosine.	Diff. for 100″	Tangent.	Diff. for 100″	Cotang	′
30	9·718085	343	9·930766	130	9·787319	473	10·212681	30
31	9·718291	343	9·930688	128	9·787603	472	10·212397	29
32	9·718497	343	9·930611	130	9·787886	473	10·212114	28
33	9·718703	343	9·930533	128	9·788170	472	10·211830	27
34	9·718909	342	9·930456	130	9·788453	472	10·211547	26
35	9·719114	343	9·930378	130	9·788736	472	10·211264	25
36	9·719320	342	9·930300	128	9·789019	472	10·210981	24
37	9·719525	342	9·930223	130	9·789302	472	10·210698	23
38	9·719730	342	9·930145	130	9·789585	472	10·210415	22
39	9·719935	342	9·930067	130	9·789868	472	10·210132	21
40	9·720140	342	9·929989	130	9·790151	472	10·209849	20
41	9·720345	340	9·929911	130	9·790434	470	10·209566	19
42	9·720549	342	9·929833	130	9·790716	472	10·209284	18
43	9·720754	340	9·929755	130	9·790999	470	10·209001	17
44	9·720958	340	9·929677	130	9·791281	470	10·208719	16
45	9·721162	340	9·929599	130	9·791563	472	10·208437	15
46	9·721366	340	9·929521	132	9·791846	470	10·208154	14
47	9·721570	340	9·929442	130	9·792128	470	10·207872	13
48	9·721774	340	9·929364	130	9·792410	470	10·207590	12
49	9·721978	338	9·929286	132	9·792692	470	10·207308	11
50	9·722181	340	9·929207	130	9·792974	470	10·207026	10
51	9·722385	338	9·929129	132	9·793256	470	10·206744	9
52	9·722588	338	9·929050	130	9·793538	468	10·206462	8
53	9·722791	338	9·928972	132	9·793819	470	10·206181	7
54	9·722994	338	9·928893	130	9·794101	470	10·205899	6
55	9·723197	338	9·928815	132	9·794383	468	10·205617	5
56	9·723400	338	9·928736	132	9·794664	470	10·205336	4
57	9·723603	337	9·928657	132	9·794946	468	10·205054	3
58	9·723805	337	9·928578	132	9·795227	468	10·204773	2
59	9·724007	338	9·928499	132	9·795508	468	10·204492	1
60	9·724210		9·928420		9·795789		10·204211	0
′	Cosine.		Sine.		Cotang.		Tangent.	′

0 min. to 30 min.

58 degrees.

32 degrees.

0 min. to 30 min.

′	Sine.	Diff. for 100″	Cosine.	Diff. for 100″	Tangent.	Diff. for 100″	Cotang.	′
0	9·724210	337	9·928420	130	9·795789	468	10·204211	60
1	9·724412	337	9·928342	132	9·796070	468	10·203930	59
2	9·724614	337	9·928263	133	9·796351	468	10·203649	58
3	9·724816	335	9·928183	132	9·796632	468	10·203368	57
4	9·725017	337	9·928104	132	9·796913	468	10·203087	56
5	9·725219	335	9·928025	132	9·797194	467	10·202806	55
6	9·725420	337	9·927946	132	9·797474	468	10·202526	54
7	9·725622	335	9·927867	133	9·797755	468	10·202245	53
8	9·725823	335	9·927787	132	9·798036	467	10·201964	52
9	9·726024	335	9·927708	132	9·798316	467	10·201684	51
10	9·726225	335	9·927629	133	9·798596	468	10·201404	50
11	9·726426	333	9·927549	132	9·798877	467	10·201123	49
12	9·726626	335	9·927470	133	9·799157	467	10·200843	48
13	9·726827	333	9·927390	133	9·799437	467	10·200563	47
14	9·727027	335	9·927310	132	9·799717	467	10·200283	46
15	9·727228	333	9·927231	133	9·799997	467	10·200003	45
16	9·727428	333	9·927151	133	9·800277	467	10·199723	44
17	9·727628	333	9·927071	133	9·800557	465	10·199443	43
18	9·727828	332	9·926991	133	9·800836	467	10·199164	42
19	9·728027	333	9·926911	133	9·801116	467	10·198884	41
20	9·728227	333	9·926831	133	9·801396	465	10·198604	40
21	9·728427	332	9·926751	133	9·801675	467	10·198325	39
22	9·728626	332	9·926671	133	9·801955	465	10·198045	38
23	9·728825	332	9·926591	133	9·802234	465	10·197766	37
24	9·729024	332	9·926511	133	9·802513	465	10·197487	36
25	9·729223	332	9·926431	133	9·802792	467	10·197208	35
26	9·729422	332	9·926351	135	9·803072	465	10·196928	34
27	9·729621	332	9·926270	133	9·803351	465	10·196649	33
28	9·729820	330	9·926190	133	9·803630	465	10·196370	32
29	9·730018	332	9·926110	135	9·803909	463	10·196091	31
30	9·730217		9·926029		9·804187		10·195813	30
′	Cosine.		Sine.		Cotang.		Tangent.	′

30 min. to 60 min.

57 degrees.

32 degrees.

30 min. to 60 min.

′	Sine.	Diff. for 100″	Cosine.	Diff. for 100″	Tangent.	Diff. for 100″	Cotang.	′
30	9·730217	330	9·926029	133	9·804187	465	10·195813	30
31	9·730415	330	9·925949	135	9·804466	465	10·195534	29
32	9·730613	330	9·925868	133	9·804745	463	10·195255	28
33	9·730811	330	9·925788	135	9·805023	465	10·194977	27
34	9·731009	328	9·925707	135	9·805302	463	10·194698	26
35	9·731206	330	9·925626	135	9·805580	465	10·194420	25
36	9·731404	330	9·925545	133	9·805859	463	10·194141	24
37	9·731602	328	9·925465	135	9·806137	463	10·193863	23
38	9·731799	328	9·925384	135	9·806415	463	10·193585	22
39	9·731996	328	9·925303	135	9·806693	463	10·193307	21
40	9·732193	328	9·925222	135	9·806971	463	10·193029	20
41	9·732390	328	9·925141	135	9·807249	463	10·192751	19
42	9·732587	328	9·925060	135	9·807527	463	10·192473	18
43	9·732784	327	9·924979	137	9·807805	463	10·192195	17
44	9·732980	328	9·924897	135	9·808083	463	10·191917	16
45	9·733177	327	9·924816	135	9·808361	462	10·191639	15
46	9·733373	327	9·924735	135	9·808638	463	10·191362	14
47	9·733569	327	9·924654	137	9·808916	462	10·191084	13
48	9·733765	327	9·924572	135	9·809193	463	10·190807	12
49	9·733961	327	9·924491	137	9·809471	462	10·190529	11
50	9·734157	327	9·924409	135	9·809748	462	10·190252	10
51	9·734353	327	9·924328	137	9·810025	462	10·189975	9
52	9·734549	325	9·924246	137	9·810302	463	10·189698	8
53	9·734744	325	9·924164	135	9·810580	462	10·189420	7
54	9·734939	327	9·924083	137	9·810857	462	10·189143	6
55	9·735135	325	9·924001	137	9·811134	460	10·188866	5
56	9·735330	325	9·923919	137	9·811410	462	10·188590	4
57	9·735525	323	9·923837	137	9·811687	462	10·188313	3
58	9·735719	325	9·923755	137	9·811964	462	10·188036	2
59	9·735914	325	9·923673	137	9·812241	460	10·187759	1
60	9·736109		9·923591		9·812517		10·187483	0
′	Cosine.		Sine.		Cotang.		Tangent.	′

0 min. to 30 min.

57 degrees.

33 degrees.

0 min. to 30 min.

′	Sine.	Diff. for 100″	Cosine.	Diff. for 100″	Tangent.	Diff. for 100″	Cotang.	′
0	9·736109		9·923591		9·812517		10·187483	60
1	9·736303	323	9·923509	137	9·812794	462	10·187206	59
2	9·736498	325	9·923427	137	9·813070	460	10·186930	58
3	9·736692	323	9·923345	137	9·813347	462	10·186653	57
4	9·736886	323	9·923263	137	9·813623	460	10·186377	56
5	9·737080	323	9·923181	137	9·813899	460	10·186101	55
6	9·737274	323	9·923098	138	9·814176	462	10·185824	54
7	9·737467	322	9·923016	137	9·814452	460	10·185548	53
8	9·737661	323	9·922933	138	9·814728	460	10·185272	52
9	9·737855	323	9·922851	137	9·815004	460	10·184996	51
10	9·738048	322	9·922768	138	9·815280	460	10·184720	50
11	9·738241	322	9·922686	137	9·815555	458	10·184445	49
12	9·738434	322	9·922603	138	9·815831	460	10·184169	48
13	9·738627	322	9·922520	138	9·816107	460	10·183893	47
14	9·738820	322	9·922438	137	9·816382	458	10·183618	46
15	9·739013	322	9·922355	138	9·816658	460	10·183342	45
16	9·739206	322	9·922272	138	9·816933	458	10·183067	44
17	9·739398	320	9·922189	138	9·817209	460	10·182791	43
18	9·739590	320	9·922106	138	9·817484	458	10·182516	42
19	9·739783	322	9·922023	138	9·817759	458	10·182241	41
20	9·739975	320	9·921940	138	9·818035	460	10·181965	40
21	9·740167	320	9·921857	138	9·818310	458	10·181690	39
22	9·740359	320	9·921774	138	9·818585	458	10·181415	38
23	9·740550	318	9·921691	138	9·818860	458	10·181140	37
24	9·740742	320	9·921607	140	9·819135	458	10·180865	36
25	9·740934	320	9·921524	138	9·819410	458	10·180590	35
26	9·741125	318	9·921441	138	9·819684	457	10·180316	34
27	9·741316	318	9·921357	140	9·819959	458	10·180041	33
28	9·741508	320	9·921274	138	9·820234	458	10·179766	32
29	9·741699	318	9·921190	140	9·820508	457	10·179492	31
30	9·741889	317	9·921107	138	9·820783	458	10·179217	30
′	Cosine.		Sine.		Cotang.		Tangent.	′

30 min. to 60 min.

56 degrees.

33 degrees.

30 min. to 60 min.

′	Sine.	Diff. for 100″	Cosine.	Diff. for 100″	Tangent.	Diff. for 100″	Cotang.	′
30	9·741889	318	9·921107	140	9·820783	457	10·179217	30
31	9·742080	318	9·921023	140	9·821057	458	10·178943	29
32	9·742271	318	9·920939	138	9·821332	457	10·178668	28
33	9·742462	317	9·920856	140	9·821606	457	10·178394	27
34	9·742652	317	9·920772	140	9·821880	457	10·178120	26
35	9·742842	318	9·920688	140	9·822154	458	10·177846	25
36	9·743033	317	9·920604	140	9·822429	457	10·177571	24
37	9·743223	317	9·920520	140	9·822703	457	10·177297	23
38	9·743413	315	9·920436	140	9·822977	457	10·177023	22
39	9·743602	317	9·920352	140	9·823251	455	10·176749	21
40	9·743792	317	9·920268	140	9·823524	457	10·176476	20
41	9·743982	315	9·920184	142	9·823798	457	10·176202	19
42	9·744171	317	9·920099	140	9·824072	455	10·175928	18
43	9·744361	315	9·920015	140	9·824345	457	10·175655	17
44	9·744550	315	9·919931	142	9·824619	457	10·175381	16
45	9·744739	315	9·919846	140	9·824893	455	10·175107	15
46	9·744928	315	9·919762	142	9·825166	455	10·174834	14
47	9·745117	315	9·919677	140	9·825439	457	10·174561	13
48	9·745306	313	9·919593	142	9·825713	455	10·174287	12
49	9·745494	315	9·919508	140	9·825986	455	10·174014	11
50	9·745683	313	9·919424	142	9·826259	455	10·173741	10
51	9·745871	315	9·919339	142	9·826532	455	10·173468	9
52	9·746060	313	9·919254	142	9·826805	455	10·173195	8
53	9·746248	313	9·919169	140	9·827078	455	10·172922	7
54	9·746436	313	9·919085	142	9·827351	455	10·172649	6
55	9·746624	313	9·919000	142	9·827624	455	10·172376	5
56	9·746812	312	9·918915	142	9·827897	455	10·172103	4
57	9·746999	313	9·918830	142	9·828170	453	10·171830	3
58	9·747187	312	9·918745	143	9·828442	455	10·171558	2
59	9·747374	313	9·918659	142	9·828715	453	10·171285	1
60	9·747562		9·918574		9·828987		10·171013	0
′	Cosine.		Sine.		Cotang.		Tangent.	′

0 min. to 30 min.

56 degrees.

34 degrees.

0 min. to 30 min.

′	Sine.	Diff. for 100″	Cosine.	Diff. for 100″	Tangent.	Diff. for 100″	Cotang.	′
0	9·747562	312	9·918574	142	9·828987	455	10·171013	60
1	9·747749	312	9·918489	142	9·829260	453	10·170740	59
2	9·747936	312	9·918404	143	9·829532	455	10·170468	58
3	9·748123	312	9·918318	142	9·829805	453	10·170195	57
4	9·748310	312	9·918233	143	9·830077	453	10·169923	56
5	9·748497	310	9·918147	142	9·830349	453	10·169651	55
6	9·748683	312	9·918062	143	9·830621	453	10·169379	54
7	9·748870	310	9·917976	142	9·830893	453	10·169107	53
8	9·749056	312	9·917891	143	9·831165	453	10·168835	52
9	9·749243	310	9·917805	143	9·831437	453	10·168563	51
10	9·749429	310	9·917719	142	9·831709	453	10·168291	50
11	9·749615	310	9·917634	143	9·831981	453	10·168019	49
12	9·749801	310	9·917548	143	9·832253	453	10·167747	48
13	9·749987	308	9·917462	143	9·832525	452	10·167475	47
14	9·750172	310	9·917376	143	9·832796	453	10·167204	46
15	9·750358	308	9·917290	143	9·833068	452	10·166932	45
16	9·750543	310	9·917204	143	9·833339	453	10·166661	44
17	9·750729	308	9·917118	143	9·833611	452	10·166389	43
18	9·750914	308	9·917032	143	9·833882	453	10·166118	42
19	9·751099	308	9·916946	145	9·834154	452	10·165846	41
20	9·751284	308	9·916859	143	9·834425	452	10·165575	40
21	9·751469	308	9·916773	143	9·834696	452	10·165304	39
22	9·751654	308	9·916687	145	9·834967	452	10·165033	38
23	9·751839	307	9·916600	143	9·835238	452	10·164762	37
24	9·752023	308	9·916514	145	9·835509	452	10·164491	36
25	9·752208	307	9·916427	143	9·835780	452	10·164220	35
26	9·752392	307	9·916341	145	9·836051	452	10·163949	34
27	9·752576	307	9·916254	145	9·836322	452	10·163678	33
28	9·752760	307	9·916167	143	9·836593	452	10·163407	32
29	9·752944	307	9·916081	145	9·836864	450	10·163136	31
30	9·753128		9·915994		9·837134		10·162866	30
′	Cosine.		Sine.		Cotang.		Tangent.	′

80 min. to 60 min.

55 degrees.

34 degrees.

30 min. to 60 min.

′	Sine.	Diff. for 100″	Cosine.	Diff. for 100″	Tangent.	Diff. for 100″	Cotang.	′
30	9·753128	307	9·915994	145	9·837134	452	10·162866	30
31	9·753312	305	9·915907	145	9·837405	450	10·162595	29
32	9·753495	307	9·915820	145	9·837675	452	10·162325	28
33	9·753679	305	9·915733	145	9·837946	450	10·162054	27
34	9·753862	307	9·915646	145	9·838216	452	10·161784	26
35	9·754046	305	9·915559	145	9·838487	450	10·161513	25
36	9·754229	305	9·915472	145	9·838757	450	10·161243	24
37	9·754412	305	9·915385	147	9·839027	450	10·160973	23
38	9·754595	305	9·915297	145	9·839297	451	10·160703	22
39	9·754778	303	9·915210	145	9·839568	450	10·160432	21
40	9·754960	305	9·915123	147	9·839838	450	10·160162	20
41	9·755143	305	9·915035	145	9·840108	450	10·159892	19
42	9·755326	303	9·914948	147	9·840378	450	10·159622	18
43	9·755508	303	9·914860	145	9·840648	448	10·159352	17
44	9·755690	303	9·914773	147	9·840917	450	10·159083	16
45	9·755872	303	9·914685	145	9·841187	450	10·158813	15
46	9·756054	303	9·914598	147	9·841457	450	10·158543	14
47	9·756236	303	9·914510	147	9·841727	448	10·158273	13
48	9·756418	303	9·914422	147	9·841996	450	10·158004	12
49	9·756600	303	9·914334	147	9·842266	448	10·157734	11
50	9·756782	302	9·914246	147	9·842535	450	10·157465	10
51	9·756963	302	9·914158	147	9·842805	448	10·157195	9
52	9·757144	303	9·914070	147	9·843074	448	10·156926	8
53	9·757326	302	9·913982	147	9·843343	448	10·156657	7
54	9·757507	302	9·913894	147	9·843612	450	10·156388	6
55	9·757688	302	9·913806	147	9·843882	448	10·156118	5
56	9·757869	302	9·913718	147	9·844151	448	10·155849	4
57	9·758050	300	9·913630	148	9·844420	448	10·155580	3
58	9·758230	302	9·913541	147	9·844689	448	10·155311	2
59	9·758411	300	9·913453	147	9·844958	448	10·155042	1
60	9·758591		9·913365		9·845227		10·154773	0
′	Cosine.		Sine.		Cotang.		Tangent.	′

0 min. to 30 min.

55 degrees.

35 degrees.

0 min. to 30 min.

′	Sine.	Diff. for 100″	Cosine.	Diff. for 100″	Tangent.	Diff. for 100″	Cotang.	′
0	9·758591	302	9·913365	148	9·845227	448	10·154773	60
1	9·758772	300	9·913276	148	9·845496	447	10·154504	59
2	9·758952	300	9·913187	147	9·845764	448	10·154236	58
3	9·759132	300	9·913099	148	9·846033	448	10·153967	57
4	9·759312	300	9·913010	147	9·846302	447	10·153698	56
5	9·759492	300	9·912922	148	9·846570	448	10·153430	55
6	9·759672	300	9·912833	148	9·846839	448	10·153161	54
7	9·759852	298	9·912744	148	9·847108	447	10·152892	53
8	9·760031	300	9·912655	148	9·847376	447	10·152624	52
9	9·760211	298	9·912566	148	9·847644	448	10·152356	51
10	9·760390	298	9·912477	148	9·847913	447	10·152087	50
11	9·760569	298	9·912388	148	9·848181	447	10·151819	49
12	9·760748	298	9·912299	148	9·848449	447	10·151551	48
13	9·760927	298	9·912210	148	9·848717	448	10·151283	47
14	9·761106	298	9·912121	150	9·848986	447	10·151014	46
15	9·761285	298	9·912031	148	9·849254	447	10·150746	45
16	9·761464	297	9·911942	148	9·849522	447	10·150478	44
17	9·761642	298	9·911853	150	9·849790	445	10·150210	43
18	9·761821	297	9·911763	148	9·850057	447	10·149943	42
19	9·761999	297	9·911674	150	9·850325	447	10·149675	41
20	9·762177	298	9·911584	148	9·850593	447	10·149407	40
21	9·762356	297	9·911495	150	9·850861	447	10·149139	39
22	9·762534	297	9·911405	150	9·851129	445	10·148871	38
23	9·762712	295	9·911315	148	9·851396	447	10·148604	37
24	9·762889	297	9·911226	150	9·851664	445	10·148336	36
25	9·763067	297	9·911136	150	9·851931	447	10·148069	35
26	9·763245	295	9·911046	150	9·852199	445	10·147801	34
27	9·763422	297	9·910956	150	9·852466	445	10·147534	33
28	9·763600	295	9·910866	150	9·852733	447	10·147267	32
29	9·763777	295	9·910776	150	9·853001	445	10·146999	31
30	9·763954		9·910686		9·853268		10·146732	30
′	Cosine.		Sine.		Cotang.		Tangent.	′

30 min. to 60 min.

54 degrees.

35 degrees.

30 min. to 60 min.

′	Sine.	Diff. for 100″	Cosine.	Diff. for 100″	Tangent.	Diff. for 100″	Cotang.	′
30	9·763954	295	9·910686	150	9·853268	445	10·146732	30
31	9·764131	295	9·910596	150	9·853535	445	10·146465	29
32	9·764308	295	9·910506	152	9·853802	445	10·146198	28
33	9·764485	295	9·910415	150	9·854069	445	10·145931	27
34	9·764662	293	9·910325	150	9·854336	445	10·145664	26
35	9·764838	295	9·910235	152	9·854603	445	10·145397	25
36	9·765015	293	9·910144	150	9·854870	445	10·145130	24
37	9·765191	293	9·910054	152	9·855137	445	10·144863	23
38	9·765367	295	9·909963	150	9·855404	445	10·144596	22
39	9·765544	293	9·909873	152	9·855671	445	10·144329	21
40	9·765720	293	9·909782	152	9·855938	443	10·144062	20
41	9·765896	293	9·909691	150	9·856204	445	10·143796	19
42	9·766072	292	9·909601	152	9·856471	443	10·143529	18
43	9·766247	293	9·909510	152	9·856737	445	10·143263	17
44	9·766423	292	9·909419	152	9·857004	443	10·142996	16
45	9·766598	293	9·909328	152	9·857270	445	10·142730	15
46	9·766774	292	9·909237	152	9·857537	443	10·142463	14
47	9·766949	292	9·909146	152	9·857803	443	10·142197	13
48	9·767124	293	9·909055	152	9·858069	445	10·141931	12
49	9·767300	292	9·908964	152	9·858336	443	10·141664	11
50	9·767475	290	9·908873	153	9·858602	443	10·141398	10
51	9·767649	292	9·908781	152	9·858868	443	10·141132	9
52	9·767824	292	9·908690	152	9·859134	443	10·140866	8
53	9·767999	290	9·908599	153	9·859400	443	10·140600	7
54	9·768173	292	9·908507	152	9·859666	443	10·140334	6
55	9·768348	290	9·908416	153	9·859932	443	10·140068	5
56	9·768522	292	9·908324	152	9·860198	443	10·139802	4
57	9·768697	290	9·908233	153	9·860464	443	10·139536	3
58	9·768871	290	9·908141	153	9·860730	442	10·139270	2
59	9·769045	290	9·908049	152	9·860995	443	10·139005	1
60	9·769219		9·907958		9·861261		10·138739	0
′	Cosine.		Sine.		Cotang.		Tangent.	′

0 min. to 30 min.

54 degrees.

36 degrees.

0 min. to 30 min.

′	Sine.	Diff. for 100″	Cosine.	Diff. for 100″	Tangent.	Diff. for 100″	Cotang.	′
0	9·769219	290	9·907958	153	9·861261	443	10·138739	60
1	9·769393	288	9·907866	153	9·861527	442	10·138473	59
2	9·769566	290	9·907774	153	9·861792	443	10·138208	58
3	9·769740	288	9·907682	153	9·862058	442	10·137942	57
4	9·769913	290	9·907590	153	9·862323	443	10·137677	56
5	9·770087	288	9·907498	153	9·862589	442	10·137411	55
6	9·770260	288	9·907406	153	9·862854	442	10·137146	54
7	9·770433	288	9·907314	153	9·863119	443	10·136881	53
8	9·770606	288	9·907222	155	9·863385	442	10·136615	52
9	9·770779	288	9·907129	153	9·863650	442	10·136350	51
10	9·770952	288	9·907037	153	9·863915	442	10·136085	50
11	9·771125	288	9·906945	155	9·864180	442	10·135820	49
12	9·771298	287	9·906852	153	9·864445	442	10·135555	48
13	9·771470	288	9·906760	155	9·864710	442	10·135290	47
14	9·771643	287	9·906667	153	9·864975	442	10·135025	46
15	9·771815	287	9·906575	155	9·865240	442	10·134760	45
16	9·771987	287	9·906482	155	9·865505	442	10·134495	44
17	9·772159	287	9·906389	155	9·865770	442	10·134230	43
18	9·772331	287	9·906296	153	9·866035	442	10·133965	42
19	9·772503	287	9·906204	155	9·866300	440	10·133700	41
20	9·772675	287	9·906111	155	9·866564	442	10·133436	40
21	9·772847	285	9·906018	155	9·866829	442	10·133171	39
22	9·773018	287	9·905925	155	9·867094	440	10·132906	38
23	9·773190	285	9·905832	155	9·867358	442	10·132642	37
24	9·773361	287	9·905739	157	9·867623	440	10·132377	36
25	9·773533	285	9·905645	155	9·867887	442	10·132113	35
26	9·773704	285	9·905552	155	9·868152	440	10·131848	34
27	9·773875	285	9·905459	155	9·868416	440	10·131584	33
28	9·774046	285	9·905366	157	9·868680	442	10·131320	32
29	9·774217	285	9·905272	155	9·868945	440	10·131055	31
30	9·774388		9·905179		9·869209		10·130791	30
′	Cosine.		Sine.		Cotang.		Tangent.	′

30 min. to 60 min.

53 degrees.

36 degrees.

30 min. to 60 min.

′	Sine.	Diff. for 100′	Cosine.	Diff. for 100′	Tangent.	Diff. for 100′	Cotang.	′
30	9·774388	283	9·905179	157	9·869209	440	10·130791	30
31	9·774558	285	9·905085	155	9·869473	440	10·130527	29
32	9·774729	283	9·904992	157	9·869737	440	10·130263	28
33	9·774899	285	9·904898	157	9·870001	440	10·129999	27
34	9·775070	283	9·904804	155	9·870265	440	10·129735	26
35	9·775240	283	9·904711	157	9·870529	440	10·129471	25
36	9·775410	283	9·904617	157	9·870793	440	10·129207	24
37	9·775580	283	9·904523	157	9·871057	440	10·128943	23
38	9·775750	283	9·904429	157	9·871321	440	10·128679	22
39	9·775920	283	9·904335	157	9·871585	440	10·128415	21
40	9·776090	282	9·904241	157	9·871849	438	10·128151	20
41	9·776259	283	9·904147	157	9·872112	440	10·127888	19
42	9·776429	282	9·904053	157	9·872376	440	10·127624	18
43	9·776598	283	9·903959	158	9·872640	438	10·127360	17
44	9·776768	282	9·903864	157	9·872903	440	10·127097	16
45	9·776937	282	9·903770	157	9·873167	438	10·126833	15
46	9·777106	282	9·903676	158	9·873430	440	10·126570	14
47	9·777275	282	9·903581	157	9·873694	438	10·126306	13
48	9·777444	282	9·903487	158	9·873957	438	10·126043	12
49	9·777613	280	9·903392	157	9·874220	440	10·125780	11
50	9·777781	282	9·903298	158	9·874484	438	10·125516	10
51	9·777950	282	9·903203	158	9·874747	438	10·125253	9
52	9·778119	280	9·903108	157	9·875010	438	10·124990	8
53	9·778287	280	9·903014	158	9·875273	440	10·124727	7
54	9·778455	282	9·902919	158	9·875537	438	10·124463	6
55	9·778624	280	9·902824	158	9·875800	438	10·124200	5
56	9·778792	280	9·902729	158	9·876063	438	10·123937	4
57	9·778960	280	9·902634	158	9·876326	438	10·123674	3
58	9·779128	278	9·902539	158	9·876589	438	10·123411	2
59	9·779295	280	9·902444	158	9·876852	437	10·123148	1
60	9·779463		9·902349		9·877114		10·122886	
′	Cosine.		Sine.		Cotang.		Tangent.	′

0 min. to 30 min.

53 degrees.

37 degrees.

0 min. to 30 min.

′	Sine.	Diff. for 100″	Cosine.	Diff. for 100″	Tangent.	Diff. for 100″	Cotang.	′
0	9·779463	280	9·902349	160	9·877114	438	10·122886	60
1	9·779631	278	9·902253	158	9·877377	438	10·122623	59
2	9·779798	280	9·902158	158	9·877640	438	10·122360	58
3	9·779966	278	9·902063	160	9·877903	437	10·122097	57
4	9·780133	278	9·901967	158	9·878165	438	10·121835	56
5	9·780300	278	9·901872	160	9·878428	438	10·121572	55
6	9·780467	278	9·901776	158	9·878691	437	10·121309	54
7	9·780634	278	9·901681	160	9·878953	438	10·121047	53
8	9·780801	278	9·901585	158	9·879216	437	10·120784	52
9	9·780968	277	9·901490	160	9·879478	438	10·120522	51
10	9·781134	278	9·901394	160	9·879741	437	10·120259	50
11	9·781301	278	9·901298	160	9·880003	437	10·119997	49
12	9·781468	277	9·901202	160	9·880265	438	10·119735	48
13	9·781634	277	9·901106	160	9·880528	437	10·119472	47
14	9·781800	277	9·901010	160	9·880790	437	10·119210	46
15	9·781966	277	9·900914	160	9·881052	437	10·118948	45
16	9·782132	277	9·900818	160	9·881314	438	10·118686	44
17	9·782298	277	9·900722	160	9·881577	437	10·118423	43
18	9·782464	277	9·900626	162	9·881839	437	10·118161	42
19	9·782630	277	9·900529	160	9·882101	437	10·117899	41
20	9·782796	275	9·900433	160	9·882363	437	10·117637	40
21	9·782961	277	9·900337	162	9·882625	437	10·117375	39
22	9·783127	275	9·900240	160	9·882887	435	10·117113	38
23	9·783292	277	9·900144	162	9·883148	437	10·116852	37
24	9·783458	275	9·900047	160	9·883410	437	10·116590	36
25	9·783623	275	9·899951	162	9·883672	437	10·116328	35
26	9·783788	275	9·899854	162	9·883934	437	10·116066	34
27	9·783953	275	9·899757	162	9·884196	435	10·115804	33
28	9·784118	273	9·899660	160	9·884457	437	10·115543	32
29	9·784282	275	9·899564	162	9·884719	435	10·115281	31
30	9·784447		9·899467		9·884980		10·115020	30
′	Cosine.		Sine.		Cotang.		Tangent.	′

30 min. to 60 min.

52 degrees.

37 degrees.

30 min. to 60 min.

′	Sine.	Diff. for 100″	Cosine.	Diff. for 100″	Tangent.	Diff. for 100″	Cotang.	′
30	9·784447	275	9·899467	162	9·884980	437	10·115020	30
31	9·784612	273	9·899370	162	9·885242	437	10·114758	29
32	9·784776	275	9·899273	162	9·885504	435	10·114496	28
33	9·784941	273	9·899176	163	9·885765	435	10·114235	27
34	9·785105	273	9·899078	162	9·886026	437	10·113974	26
35	9·785269	273	9·898981	162	9·886288	435	10·113712	25
36	9·785433	273	9·898884	162	9·886549	437	10·113451	24
37	9·785597	273	9·898787	163	9·886811	435	10·113189	23
38	9·785761	273	9·898689	162	9·887072	435	10·112928	22
39	9·785925	273	9·898592	163	9·887333	435	10·112667	21
40	9·786089	272	9·898494	162	9·887594	435	10·112406	20
41	9·786252	273	9·898397	163	9·887855	435	10·112145	19
42	9·786416	272	9·898299	162	9·888116	437	10·111884	18
43	9·786579	272	9·898202	163	9·888378	435	10·111622	17
44	9·786742	273	9·898104	163	9·888639	435	10·111361	16
45	9·786906	272	9·898006	163	9·888900	435	10·111100	15
46	9·787069	272	9·897908	163	9·889161	433	10·110839	14
47	9·787232	272	9·897810	163	9·889421	435	10·110579	13
48	9·787395	270	9·897712	163	9·889682	435	10·110318	12
49	9·787557	272	9·897614	163	9·889943	435	10·110057	11
50	9·787720	272	9·897516	163	9·890204	435	10·109796	10
51	9·787883	270	9·897418	163	9·890465	433	10·109535	9
52	9·788045	272	9·897320	163	9·890725	435	10·109275	8
53	9·788208	270	9·897222	165	9·890986	435	10·109014	7
54	9·788370	270	9·897123	163	9·891247	433	10·108753	6
55	9·788532	270	9·897025	165	9·891507	435	10·108493	5
56	9·788694	270	9·896926	163	9·891768	433	10·108232	4
57	9·788856	270	9·896828	165	9·892028	435	10·107972	3
58	9·789018	270	9·896729	163	9·892289	433	10·107711	2
59	9·789180	270	9·896631	165	9·892549	435	10·107451	1
60	9·789342		9·896532		9·892810		10·107190	0
′	Cosine.		Sine.		Cotang.		Tangent.	′

0 min. to 30 min.

52 degrees.

38 degrees.

0 min. to 30 min.

′	Sine.	Diff. for 100′	Cosine.	Diff. for 100′	Tangent.	Diff. for 100′	Cotang.	
0	9·789342		9·896532		9·892810		10·107190	60
1	9·789504	270	9·896433	165	9·893070	433	10·106930	59
2	9·789665	268	9·896335	163	9·893331	435	10·106669	58
3	9·789827	270	9·896236	165	9·893591	433	10·106409	57
4	9·789988	268	9·896137	165	9·893851	433	10·106149	56
5	9·790149	268	9·896038	165	9·894111	433	10·105889	55
6	9·790310	268	9·895939	165	9·894372	435	10·105628	54
7	9·790471	268	9·895840	165	9·894632	433	10·105368	53
8	9·790632	268	9·895741	165	9·894892	433	10·105108	52
9	9·790793	268	9·895641	167	9·895152	433	10·104848	51
10	9·790954	268	9·895542	165	9·895412	433	10·104588	50
11	9·791115	268	9·895443	165	9·895672	433	10·104328	49
12	9·791275	267	9·895343	167	9·895932	433	10·104068	48
13	9·791436	268	9·895244	165	9·896192	433	10·103808	47
14	9·791596	267	9·895145	165	9·896452	433	10·103548	46
15	9·791757	268	9·895045	167	9·896712	433	10·103288	45
16	9·791917	267	9·894945	167	9·896971	432	10·103029	44
17	9·792077	267	9·894846	165	9·897231	433	10·102769	43
18	9·792237	267	9·894746	167	9·897491	433	10·102509	42
19	9·792397	267	9·894646	167	9·897751	433	10·102249	41
20	9·792557	267	9·894546	167	9·898010	432	10·101990	40
21	9·792716	265	9·894446	167	9·898270	433	10·101730	39
22	9·792876	267	9·894346	167	9·898530	433	10·101470	38
23	9·793035	265	9·894246	167	9·898789	432	10·101211	37
24	9·793195	267	9·894146	167	9·899049	433	10·100951	36
25	9·793354	265	9·894046	167	9·899308	432	10·100692	35
26	9·793514	267	9·893946	167	9·899568	433	10·100432	34
27	9·793673	265	9·893846	167	9·899827	432	10·100173	33
28	9·793832	265	9·893745	168	9·900087	433	10·099913	32
29	9·793991	265	9·893645	167	9·900346	433	10·099654	31
30	9·794150	265	9·893544	168	9·900605	432	10·099395	30
′	Cosine.		Sine.		Cotang.		Tangent.	′

30 min. to 60 min.

51 degrees.

38 degrees.

30 min. to 60 min.

′	Sine.	Diff. for 100″	Cosine.	Diff. for 100″	Tangent.	Diff. for 100″	Cotang.	′
30	9·794150	263	9·893544	167	9·900605	432	10·099395	30
31	9·794308	265	9·893444	168	9·900864	433	10·099136	29
32	9·794467	265	9·893343	167	9·901124	432	10·098876	28
33	9·794626	263	9·893243	168	9·901383	432	10·098617	27
34	9·794784	263	9·893142	168	9·901642	432	10·098358	26
35	9·794942	265	9·893041	168	9·901901	432	10·098099	25
36	9·795101	263	9·892940	168	9·902160	433	10·097840	24
37	9·795259	263	9·892839	167	9·902420	432	10·097580	23
38	9·795417	263	9·892739	168	9·902679	432	10·097321	22
39	9·795575	263	9·892638	170	9·902938	432	10·097062	21
40	9·795733	263	9·892536	168	9·903197	432	10·096803	20
41	9·795891	263	9·892435	168	9·903456	430	10·096544	19
42	9·796049	262	9·892334	168	9·903714	432	10·096286	18
43	9·796206	263	9·892233	168	9·903973	432	10·096027	17
44	9·796364	262	9·892132	170	9·904232	432	10·095768	16
45	9·796521	263	9·892030	168	9·904491	432	10·095509	15
46	9·796679	262	9·891929	170	9·904750	430	10·095250	14
47	9·796836	262	9·891827	168	9·905008	432	10·094992	13
48	9·796993	262	9·891726	170	9·905267	432	10·094733	12
49	9·797150	262	9·891624	168	9·905526	432	10·094474	11
50	9·797307	262	9·891523	170	9·905785	430	10·094215	10
51	9·797464	262	9·891421	170	9·906043	432	10·093957	9
52	9·797621	260	9·891319	170	9·906302	430	10·093698	8
53	9·797777	262	9·891217	170	9·906560	432	10·093440	7
54	9·797934	262	9·891115	170	9·906819	430	10·093181	6
55	9·798091	260	9·891013	170	9·907077	432	10·092923	5
56	9·798247	260	9·890911	170	9·907336	430	10·092664	4
57	9·798403	262	9·890809	170	9·907594	432	10·092406	3
58	9·798560	260	9·890707	170	9·907853	430	10·092147	2
59	9·798716	260	9·890605	170	9·908111	430	10·091889	1
60	9·798872		9·890503		9·908369		10·091631	0
′	Cosine.		Sine.		Cotang.		Tangent.	′

0 min. to 30 min.

51 degrees.

40 degrees.

0 min. to 30 min.

′	Sine.	Diff. for 100″	Cosine.	Diff. for 100″	Tangent.	Diff. for 100″	Cotang.	′
0	9 808067	252	9·884254	177	9·923814	427	10·076186	60
1	9·808218	250	9·884148	177	9·924070	428	10·075930	59
2	9·808368	252	9·884042	177	9·924327	427	10·075673	58
3	9·808519	250	9·883936	178	9·924583	428	10·075417	57
4	9·808669	250	9·883829	177	9·924840	427	10·075160	56
5	9·808819	250	9·883723	177	9·925096	427	10·074904	55
6	9·808969	250	9·883617	178	9·925352	428	10·074648	54
7	9·809119	250	9·883510	177	9·925609	427	10·074391	53
8	9·809269	250	9·883404	178	9·925865	428	10·074135	52
9	9·809419	250	9·883297	177	9·926122	427	10·073878	51
10	9·809569	248	9·883191	178	9·926378	427	10·073622	50
11	9·809718	250	9·883084	178	9·926634	427	10·073366	49
12	9·809868	248	9·882977	177	9·926890	428	10·073110	48
13	9·810017	250	9·882871	178	9·927147	427	10·072853	47
14	9·810167	248	9·882764	178	9·927403	427	10·072597	46
15	9·810316	248	9·882657	178	9·927659	427	10·072341	45
16	9·810465	248	9·882550	178	9·927915	427	10·072085	44
17	9·810614	248	9·882443	178	9·928171	427	10·071829	43
18	9·810763	248	9·882336	178	9·928427	428	10·071573	42
19	9·810912	248	9·882229	180	9·928684	427	10·071316	41
20	9·811061	248	9·882121	178	9·928940	427	10·071060	40
21	9·811210	247	9·882014	178	9·929196	427	10·070804	39
22	9·811358	248	9·881907	180	9·929452	427	10·070548	38
23	9·811507	247	9·881799	178	9·929708	427	10·070292	37
24	9·811655	248	9·881692	180	9·929964	427	10·070036	36
25	9·811804	247.	9·881584	178	9·930220	425	10·069780	35
26	9·811952	247	9·881477	180	9·930475	427	10·069525	34
27	9·812100	247	9·881369	180	9·930731	427	10·069269	33
28	9·812248	247	9·881261	180	9·930987	427	10·069013	32
29	9·812396	247	9·881153	178	9·931243	427	10·068757	31
30	9·812544		9·881046		9·931499		10·068501	30
′	Cosine.		Sine.		Cotang.		Tangent.	′

30 min. to 60 min.

49 degrees.

40 degrees.

30 min. to 60 min.

′	Sine.	Diff. for 100″	Cosine.	Diff. for 100″	Tangent.	Diff. for 100″	Cotang.	′
30	9·812544	247	9·881046	180	9·931499	427	10·068501	30
31	9·812692	247	9·880938	180	9·931755	425	10·068245	29
32	9·812840	247	9·880830	180	9·932010	427	10·067990	28
33	9·812988	245	9·880722	182	9·932266	427	10·067734	27
34	9·813135	247	9·880613	180	9·932522	427	10·067478	26
35	9·813283	245	9·880505	180	9·932778	425	10·067222	25
36	9·813430	247	9·880397	180	9·933033	427	10·066967	24
37	9·813578	245	9·880289	182	9·933289	427	10·066711	23
38	9·813725	245	9·880180	180	9·933545	425	10·066455	22
39	9·813872	245	9·880072	182	9·933800	427	10·066200	21
40	9·814019	245	9·879963	180	9·934056	425	10·065944	20
41	9·814166	245	9·879855	182	9·934311	427	10·065689	19
42	9·814313	245	9·879746	182	9·934567	425	10·065433	18
43	9·814460	245	9·879637	180	9·934822	427	10·065178	17
44	9·814607	243	9·879529	182	9·935078	425	10·064922	16
45	9·814753	245	9·879420	182	9·935333	427	10·064667	15
46	9·814900	243	9·879311	182	9·935589	425	10·064411	14
47	9·815046	245	9·879202	182	9·935844	427	10·064156	13
48	9·815193	243	9·879093	182	9·936100	425	10·063900	12
49	9·815339	243	9·878984	182	9·936355	427	10·063645	11
50	9·815485	245	9·878875	182	9·936611	425	10·063389	10
51	9·815632	243	9·878766	183	9·936866	425	10·063134	9
52	9·815778	243	9·878656	182	9·937121	427	10·062879	8
53	9·815924	242	9·878547	182	9·937377	425	10·062623	7
54	9·816069	243	9·878438	183	9·937632	425	10·062368	6
55	9·816215	243	9·878328	182	9·937887	425	10·062113	5
56	9·816361	243	9·878219	183	9·938142	427	10·061858	4
57	9·816507	242	9·878109	183	9·938398	425	10·061602	3
58	9·816652	243	9·877999	182	9·938653	425	10·061347	2
59	9·816798	242	9·877890	183	9·938908	425	10·061092	1
60	9·816943		9·877780		9·939163		10·060837	0
′	Cosine.		Sine.		Cotang.		Tangent.	′

0 min. to 30 min.

49 degrees.

41 degrees.

0 min. to 30 min.

′	Sine.	Diff. for 100″	Cosine.	Diff. for 100″	Tangent.	Diff. for 100″	Cotang.	′
0	9·816943	242	9·877780	183	9·939163	425	10·060837	60
1	9·817088	242	9·877670	183	9·939418	425	10·060582	59
2	9·817233	243	9·877560	183	9·939673	425	10·060327	58
3	9·817379	242	9·877450	183	9·939928	425	10·060072	57
4	9·817524	240	9·877340	183	9·940183	427	10·059817	56
5	9·817668	242	9·877230	183	9·940439	425	10·059561	55
6	9·817813	242	9·877120	183	9·940694	425	10·059306	54
7	9·817958	242	9·877010	185	9·940949	425	10·059051	53
8	9·818103	240	9·876899	183	9·941204	425	10·058796	52
9	9·818247	242	9·876789	185	9·941459	423	10·058541	51
10	9·818392	240	9·876678	183	9·941713	425	10·058287	50
11	9·818536	242	9·876568	185	9·941968	425	10·058032	49
12	9·818681	240	9·876457	183	9·942223	425	10·057777	48
13	9·818825	240	9·876347	185	9·942478	425	10·057522	47
14	9·818969	240	9·876236	185	9·942733	425	10·057267	46
15	9·819113	240	9·876125	185	9·942988	425	10·057012	45
16	9·819257	240	9·876014	183	9·943243	425	10·056757	44
17	9·819401	240	9·875904	185	9·943498	423	10·056502	43
18	9·819545	240	9·875793	185	9·943752	425	10·056248	42
19	9·819689	238	9·875682	185	9·944007	425	10·055993	41
20	9·819832	240	9·875571	187	9·944262	425	10·055738	40
21	9·819976	240	9·875459	185	9·944517	423	10·055483	39
22	9·820120	238	9·875348	185	9·944771	425	10·055229	38
23	9·820263	238	9·875237	185	9·945026	425	10·054974	37
24	9·820406	240	9·875126	187	9·945281	423	10·054719	36
25	9·820550	238	9·875014	185	9·945535	425	10·054465	35
26	9·820693	238	9·874903	187	9·945790	425	10·054210	34
27	9·820836	238	9·874791	185	9·946045	423	10·053955	33
28	9·820979	238	9·874680	187	9·946299	425	10·053701	32
29	9·821122	238	9·874568	187	9·946554	423	10·053446	31
30	9·821265		9·874456		9·946808		10·053192	30
′	Cosine.		Sine.		Cotang.		Tangent.	′

30 min. to 60 min.

48 degrees.

41 degrees.

30 min. to 60 min.

′	Sine.	Diff. for 100″	Cosine.	Diff. for 100″	Tangent.	Diff. for 100″	Cotang.	′
30	9·821265	237	9·874456	187	9·946808	425	10·053192	30
31	9·821407	238	9·874344	187	9·947063	425	10·052937	29
32	9·821550	238	9·874232	185	9·947318	423	10·052682	28
33	9·821693	237	9·874121	187	9·947572	425	10·052428	27
34	9·821835	237	9·874009	188	9·947827	423	10·052173	26
35	9·821977	238	9·873896	187	9·948081	423	10·051919	25
36	9·822120	237	9·873784	187	9·948335	425	10·051665	24
37	9·822262	237	9·873672	187	9·948590	423	10·051410	23
38	9·822404	237	9·873560	187	9·948844	425	10·051156	22
39	9·822546	237	9·873448	188	9·949099	423	10·050901	21
40	9·822688	237	9·873335	187	9·949353	425	10·050647	20
41	9·822830	237	9·873223	188	9·949608	423	10·050392	19
42	9·822972	237	9·873110	187	9·949862	423	10·050138	18
43	9·823114	236	9·872998	188	9·950116	425	10·049884	17
44	9·823255	237	9·872885	188	9·950371	423	10·049629	16
45	9·823397	237	9·872772	188	9·950625	423	10·049375	15
46	9·823539	235	9·872659	187	9·950879	423	10·049121	14
47	9·823680	235	9·872547	188	9·951133	425	10·048867	13
48	9·823821	237	9·872434	188	9·951388	423	10·048612	12
49	9·823963	235	9·872321	188	9·951642	423	10·048358	11
50	9·824104	235	9·872208	188	9·951896	423	10·048104	10
51	9·824245	235	9·872095	190	9·952150	425	10·047850	9
52	9·824386	235	9·871981	188	9·952405	423	10·047595	8
53	9·824527	235	9·871868	188	9·952659	423	10·047341	7
54	9·824668	233	9·871755	190	9·952913	423	10·047087	6
55	9·824808	235	9·871641	188	9·953167	423	10·046833	5
56	9·824949	235	9·871528	190	9·953421	423	10·046579	4
57	9·825090	233	9·871414	188	9·953675	423	10·046325	3
58	9·825230	235	9·871301	190	9·953929	423	10·046071	2
59	9·825371	233	9·871187	190	9·954183	423	10·045817	1
60	9·825511		9·871073		9·954437		10·045563	0
′	Cosine.		Sine.		Cotang.		Tangent.	′

0 min. to 30 min.

48 degrees.

42 degrees.

0 min. to 30 min.

′	Sine.	Diff. for 100″	Cosine.	Diff. for 100″	Tangent.	Diff. for 100″	Cotang.	′
0	9·825511		9·871073		9·954437		10·045563	60
1	9·825651	233	9·870960	188	9·954691	423	10·045309	59
2	9·825791	233	9·870846	190	9·954946	425	10·045054	58
3	9·825931	233	9·870732	190	9·955200	423	10·044800	57
4	9·826071	233	9·870618	190	9·955454	423	10·044546	56
5	9·826211	233	9·870504	190	9·955708	423	10·044292	55
6	9·826351	233	9·870390	190	9·955961	422	10·044039	54
7	9·826491	233	9·870276	190	9·956215	423	10·043785	53
8	9·826631	233	9·870161	192	9·956469	423	10·043531	52
9	9·826770	232	9·870047	190	9·956723	423	10·043277	51
10	9·826910	233	9·869933	190	9·956977	423	10·043023	50
11	9·827049	232	9·869818	192	9·957231	423	10·042769	49
12	9·827189	233	9·869704	190	9·957485	423	10·042515	48
13	9·827328	232	9·869589	192	9·957739	423	10·042261	47
14	9·827467	232	9·869474	192	9·957993	423	10·042007	46
15	9·827606	232	9·869360	190	9·958247	423	10·041753	45
16	9·827745	232	9·869245	192	9·958500	422	10·041500	44
17	9·827884	232	9·869130	192	9·958754	423	10·041246	43
18	9·828023	232	9·869015	192	9·959008	423	10·040992	42
19	9·828162	232	9·868900	192	9·959262	423	10·040738	41
20	9·828301	232	9·868785	192	9·959516	423	10·040484	40
21	9·828439	230	9·868670	192	9·959769	422	10·040231	39
22	9·828578	232	9·868555	192	9·960023	423	10·039977	38
23	9·828716	230	9·868440	192	9·960277	423	10·039723	37
24	9·828855	232	9·868324	193	9·960530	422	10·039470	36
25	9·828993	230	9·868209	192	9·960784	423	10·039216	35
26	9·829131	230	9·868093	193	9·961038	423	10·038962	34
27	9·829269	230	9·867978	192	9·961292	423	10·038708	33
28	9·829407	230	9·867862	193	9·961545	422	10·038455	32
29	9·829545	230	9·867747	192	9·961799	423	10·038201	31
30	9·829683	230	9·867631	193	9·962052	422	10·037948	30
′	Cosine.		Sine.		Cotang.		Tangent.	′

30 min. to 60 min.

47 degrees.

42 degrees.

30 min. to 60 min.

′	Sine.	Diff. for 100″	Cosine.	Diff. for 100″	Tangent.	Diff. for 100″	Cotang.	′
30	9·829683		9·867631		9·962052		10·037948	30
31	9·829821	230	9·867515	193	9·962306	423	10·037694	29
32	9·829959	230	9·867399	193	9·962560	423	10·037440	28
33	9·830097	230	9·867283	193	9·962813	422	10·037187	27
34	9·830234	228	9·867167	193	9·963067	423	10·036933	26
35	9·830372	230	9·867051	193	9·963320	422	10·036680	25
36	9·830509	228	9·866935	193	9·963574	423	10·036426	24
37	9·830646	228	9·866819	193	9·963828	423	10·036172	23
38	9·830784	230	9·866703	193	9·964081	422	10·035919	22
39	9·830921	228	9·866586	195	9·964335	423	10·035665	21
40	9·831058	228	9·866470	193	9·964588	422	10·035412	20
41	9·831195	228	9·866353	195	9·964842	423	10·035158	19
42	9·831332	228	9·866237	193	9·965095	422	10·034905	18
43	9·831469	228	9·866120	195	9·965349	423	10·034651	17
44	9·831606	228	9·866004	193	9·965602	422	10·034398	16
45	9·831742	227	9·865887	195	9·965855	422	10·034145	15
46	9·831879	228	9·865770	195	9·966109	423	10·033891	14
47	9·832015	227	9·865653	195	9·966362	422	10·033638	13
48	9·832152	228	9·865536	195	9·966616	423	10·033384	12
49	9·832288	227	9·865419	195	9·966869	422	10·033131	11
50	9·832425	228	9·865302	195	9·967123	423	10·032877	10
51	9·832561	227	9·865185	195	9·967376	422	10·032624	9
52	9·832697	227	9·865068	195	9·967629	422	10·032371	8
53	9·832833	227	9·864950	197	9·967883	423	10·032117	7
54	9·832969	227	9·864833	195	9·968136	422	10·031864	6
55	9·833105	227	9·864716	195	9·968389	422	10·031611	5
56	9·833241	227	9·864598	197	9·968643	423	10·031357	4
57	9·833377	227	9·864481	195	9·968896	422	10·031104	3
58	9·833512	225	9·864363	197	9·969149	422	10·030851	2
59	9·833648	227	9·864245	197	9·969403	423	10·030597	1
60	9·833783	225	9·864127	197	9·969656	422	10·030344	0
′	Cosine.		Sine.		Cotang.		Tangent.	′

0 min. to 30 min.

47 degrees.

43 degrees.

0 min. to 30 min.

′	Sine.	Diff. for 100″	Cosine.	Diff. for 100″	Tangent.	Diff. for 100″	Cotang.	′
0	9·833783		9·864127		9·969656		10·030344	60
1	9·833919	227	9·864010	195	9·969909	422	10·030091	59
2	9·834054	225	9·863892	197	9·970162	422	10·029838	58
3	9·834189	225	9·863774	197	9·970416	423	10·029584	57
4	9·834325	227	9·863656	197	9·970669	422	10·029331	56
5	9·834460	225	9·863538	197	9·970922	422	10·029078	55
6	9·834595	225	9·863419	198	9·971175	422	10·028825	54
7	9·834730	225	9·863301	197	9·971429	423	10·028571	53
8	9·834865	225	9·863183	197	9·971682	422	10·028318	52
9	9·834999	223	9·863064	198	9·971935	422	10·028065	51
10	9·835134	225	9·862946	197	9·972188	422	10·027812	50
11	9·835269	225	9·862827	198	9·972441	422	10·027559	49
12	9·835403	223	9·862709	197	9·972695	423	10·027305	48
13	9·835538	225	9·862590	198	9·972948	422	10·027052	47
14	9·835672	223	9·862471	198	9·973201	422	10·026799	46
15	9·835807	225	9·862353	197	9·973454	422	10·026546	45
16	9·835941	223	9·862234	198	9·973707	422	10·026293	44
17	9·836075	223	9·862115	198	9·973960	422	10·026040	43
18	9·836209	223	9·861996	198	9·974213	422	10·025787	42
19	9·836343	223	9·861877	198	9·974466	422	10·025534	41
20	9·836477	223	9·861758	198	9·974720	423	10·025280	40
21	9·836611	223	9·861638	200	9·974973	422	10·025027	39
22	9·836745	223	9·861519	198	9·975226	422	10·024774	38
23	9·836878	222	9·861400	198	9·975479	422	10·024521	37
24	9·837012	223	9·861280	200	9·975732	422	10·024268	36
25	9·837146	223	9·861161	198	9·975985	422	10·024015	35
26	9·837279	222	9·861041	200	9·976238	422	10·023762	34
27	9·837412	222	9·860922	198	9·976491	422	10·023509	33
28	9·837546	223	9·860802	200	9·976744	422	10·023256	32
29	9·837679	222	9·860682	200	9·976997	422	10·023003	31
30	9·837812	222	9·860562	200	9·977250	422	10·022750	30
′	Cosine.		Sine.		Cotang.		Tangent.	′

30 min. to 60 min.

46 degrees.

43 degrees.

30 min. to 60 min.

′	Sine.	Diff. for 100′	Cosine.	Diff. for 100′	Tangent.	Diff. for 100′	Cotang.	′
30	9·837812	222	9·860562	200	9·977250	422	10·022750	30
31	9·837945	222	9·860442	200	9·977503	422	10·022497	29
32	9·838078	222	9·860322	200	9·977756	422	10·022244	28
33	9·838211	222	9·860202	200	9·978009	422	10·021991	27
34	9·838344	222	9·860082	200	9·978262	422	10·021738	26
35	9·838477	222	9·859962	200	9·978515	422	10·021485	25
36	9·838610	220	9·859842	202	9·978768	422	10·021232	24
37	9·838742	222	9·859721	200	9·979021	422	10·020979	23
38	9·838875	220	9·859601	202	9·979274	422	10·020726	22
39	9·839007	222	9·859480	200	9·979527	422	10·020473	21
40	9·839140	220	9·859360	202	9·979780	422	10·020220	20
41	9·839272	220	9·859239	200	9·980033	422	10·019967	19
42	9·839404	220	9·859119	202	9·980286	420	10·019714	18
43	9·839536	220	9·858998	202	9·980538	422	10·019462	17
44	9·839668	220	9·858877	202	9·980791	422	10·019209	16
45	9·839800	220	9·858756	202	9·981044	422	10·018956	15
46	9·839932	220	9·858635	202	9·981297	422	10·018703	14
47	9·840064	220	9·858514	202	9·981550	422	10·018450	13
48	9·840196	220	9·858393	202	9·981803	422	10·018197	12
49	9·840328	218	9·858272	202	9·982056	422	10·017944	11
50	9·840459	220	9·858151	203	9·982309	422	10·017691	10
51	9·840591	218	9·858029	202	9·982562	420	10·017438	9
52	9·840722	220	9·857908	203	9·982814	422	10·017186	8
53	9·840854	218	9·857786	202	9·983067	422	10·016933	7
54	9·840985	218	9·857665	203	9·983320	422	10·016680	6
55	9·841116	218	9·857543	202	9·983573	422	10·016427	5
56	9·841247	218	9·857422	203	9·983826	422	10·016174	4
57	9·841378	218	9·857300	203	9·984079	422	10·015921	3
58	9·841509	218	9·857178	203	9·984332	420	10·015668	2
59	9·841640	218	9·857056	203	9·984584	422	10·015416	1
60	9·841771		9·856934		9·984837		10·015163	0
′	Cosine.		Sine.		Cotang.		Tangent.	′

0 min. to 30 min.

46 degrees.

44 degrees.

0 min. to 30 min.

′	Sine.	Diff. for 100″	Cosine.	Diff. for 100″	Tangent.	Diff. for 100″	Cotang.	′
0	9·841771	218	9·856934	203	9·984837	422	10·015163	60
1	9·841902	218	9·856812	203	9·985090	422	10·014910	59
2	9·842033	217	9·856690	203	9·985343	422	10·014657	58
3	9·842163	218	9·856568	203	9·985596	420	10·014404	57
4	9·842294	217	9·856446	205	9·985848	422	10·014152	56
5	9·842424	218	9·856323	203	9·986101	422	10·013899	55
6	9·842555	217	9·856201	205	9·986354	422	10·013646	54
7	9·842685	217	9·856078	203	9·986607	422	10·013393	53
8	9·842815	218	9·855956	205	9·986860	420	10·013140	52
9	9·842946	217	9·855833	203	9·987112	422	10·012888	51
10	9·843076	217	9·855711	205	9·987365	422	10·012635	50
11	9·843206	217	9·855588	205	9·987618	422	10·012382	49
12	9·843336	217	9·855465	205	9·987871	420	10·012129	48
13	9·843466	215	9·855342	205	9·988123	422	10·011877	47
14	9·843595	217	9·855219	205	9·988376	422	10·011624	46
15	9·843725	217	9·855096	205	9·988629	422	10·011371	45
16	9·843855	215	9·854973	205	9·988882	420	10·011118	44
17	9·843984	217	9·854850	205	9·989134	422	10·010866	43
18	9·844114	215	9·854727	207	9·989387	422	10·010613	42
19	9·844243	215	9·854603	205	9·989640	422	10·010360	41
20	9·844372	217	9·854480	207	9·989893	420	10·010107	40
21	9·844502	215	9·854356	205	9·990145	422	10·009855	39
22	9·844631	215	9·854233	207	9·990398	422	10·009602	38
23	9·844760	215	9·854109	205	9·990651	420	10·009349	37
24	9·844889	215	9·853986	207	9·990903	422	10·009097	36
25	9·845018	215	9·853862	207	9·991156	422	10·008844	35
26	9·845147	215	9·853738	207	9·991409	422	10·008591	34
27	9·845276	215	9·853614	207	9·991662	420	10·008338	33
28	9·845405	215	9·853490	207	9·991914	422	10·008086	32
29	9·845533	215	9·853366	207	9·992167	422	10·007833	31
30	9·845662		9·853242		9·992420		10·007580	30
′	Cosine.		Sine.		Cotang.		Tangent.	′

30 min. to 60 min.

45 degrees.

44 degrees.

30 min. to 60 min.

'	Sine.	Diff. for 100"	Cosine.	Diff. for 100"	Tangent.	Diff. for 100"	Cotang.	'
30	9·845662	213	9·853242	207	9·992420	420	10·007580	30
31	9·845790	215	9·853118	207	9·992672	422	10·007328	29
32	9·845919	213	9·852994	208	9·992925	422	10·007075	28
33	9·846047	213	9·852869	207	9·993178	422	10·006822	27
34	9·846175	215	9·852745	208	9·993431	420	10·006569	26
35	9·846304	213	9·852620	207	9·993683	422	10·006317	25
36	9·846432	213	9·852496	208	9·993936	422	10·006064	24
37	9·846560	213	9·852371	207	9·994189	420	10·005811	23
38	9·846688	213	9·852247	208	9·994441	422	10·005559	22
39	9·846816	213	9·852122	208	9·994694	422	10·005306	21
40	9·846944	212	9·851997	208	9·994947	420	10·005053	20
41	9·847071	213	9·851872	208	9·995199	422	10·004801	19
42	9·847199	213	9·851747	208	9·995452	422	10·004548	18
43	9·847327	212	9·851622	208	9·995705	420	10·004295	17
44	9·847454	213	9·851497	208	9·995957	422	10·004043	16
45	9·847582	212	9·851372	210	9·996210	422	10·003790	15
46	9·847709	212	9·851246	208	9·996463	420	10·003537	14
47	9·847836	213	9·851121	208	9·996715	422	10·003285	13
48	9·847964	212	9·850996	210	9·996968	422	10·003032	12
49	9·848091	212	9·850870	208	9·997221	420	10·002779	11
50	9·848218	212	9·850745	210	9·997473	422	10·002527	10
51	9·848345	212	9·850619	210	9·997726	422	10·002274	9
52	9·848472	212	9·850493	208	9·997979	420	10·002021	8
53	9·848599	212	9·850368	210	9·998231	422	10·001769	7
54	9·848726	210	9·850242	210	9·998484	422	10·001516	6
55	9·848852	212	9·850116	210	9·998737	420	10·001263	5
56	9·848979	212	9·849990	210	9·998989	422	10·001011	4
57	9·849106	210	9·849864	210	9·999242	422	10·000758	3
58	9·849232	212	9·849738	212	9·999495	420	10·000505	2
59	9·849359	210	9·849611	210	9·999747	422	10·000253	1
60	9·849485		9·849485		10·000000		10·000000	0
'	Cosine.		Sine.		Cotang.		Tangent.	'

0 min. to 30 min.

45 degrees.

0 degrees.

Sine.

′	0″	6″	12″	18″	24″	′
0	— ∞	5·463726	5·764756	5·940847	6·065786	59
1	6·463726	6·505119	6·542907	6·577669	6·609854	58
2	6·764756	6·785945	6·806149	6·825454	6·843937	57
3	6·940847	6·955088	6·968876	6·982240	6·995205	56
4	7·065786	7·076510	7·086975	7·097194	7·107179	55
5	7·162696	7·171296	7·179729	7·188002	7·196120	54
6	7·241877	7·249056	7·256118	7·263066	7·269906	53
7	7·308824	7·314984	7·321058	7·327049	7·332958	52
8	7·366816	7·372211	7·377540	7·382804	7·388005	51
9	7·417968	7·422767	7·427513	7·432209	7·436853	50
10	7·463726	7·468047	7·472326	7·476563	7·480759	49
11	7·505118	7·509048	7·512943	7·516804	7·520630	48
12	7·542906	7·546511	7·550085	7·553630	7·557147	47
13	7·577668	7·580996	7·584299	7·587577	7·590830	46
14	7·609853	7·612944	7·616013	7·619061	7·622087	45
15	7·639816	7·642702	7·645568	7·648416	7·651245	44
16	7·667845	7·670550	7·673240	7·675912	7·678568	43
17	7·694173	7·696720	7·699253	7·701770	7·704274	42
18	7·718997	7·721403	7·723795	7·726175	7·728542	41
19	7·742478	7·744757	7·747025	7·749281	7·751526	40
20	7·764754	7·766920	7·769075	7·771220	7·773354	39
21	7·785943	7·788006	7·790059	7·792103	7·794137	38
22	7·806146	7·808115	7·810076	7·812028	7·813971	37
23	7·825451	7·827335	7·829211	7·831079	7·832939	36
24	7·843934	7·845740	7·847538	7·849329	7·851112	35
25	7·861662	7·863396	7·865123	7·866843	7·868556	34
26	7·878695	7·880362	7·882023	7·883678	7·885326	33
27	7·895085	7·896691	7·898290	7·899884	7·901472	32
28	7·910879	7·912428	7·913970	7·915508	7·917040	31
29	7·926119	7·927614	7·929104	7·930588	7·932068	30
′	60″	54″	48″	42″	36″	′

Cosine.

89 degrees.

0 degrees.

Sine.

′	30″	36″	42″	48″	54″	60″	′
0	6·162696	6·241877	6·308824	6·366816	6·417969	6·463726	59
1	6·639817	6·667846	6·694175	6·718999	6·742480	6·764756	58
2	6·861666	6·878699	6·895090	6·910884	6·926124	6·940847	57
3	7·007794	7·020029	7·031928	7·043510	7·054791	7·065786	56
4	7·116939	7·126484	7·135824	7·144967	7·153922	7·162696	55
5	7·204089	7·211914	7·219601	7·227154	7·234578	7·241877	54
6	7·276639	7·283270	7·289801	7·296235	7·302575	7·308824	53
7	7·338787	7·344539	7·350216	7·355820	7·361353	7·366816	52
8	7·393145	7·398224	7·403245	7·408208	7·413116	7·417968	51
9	7·441449	7·445997	7·450497	7·454952	7·459361	7·463726	50
10	7·484915	7·489031	7·493109	7·497149	7·501152	7·505118	49
11	7·524423	7·528183	7·531911	7·535607	7·539272	7·542906	48
12	7·560635	7·564096	7·567529	7·570935	7·574315	7·577668	47
13	7·594059	7·597264	7·600446	7·603604	7·606740	7·609853	46
14	7·625093	7·628078	7·631042	7·633986	7·636911	7·639816	45
15	7·654056	7·656849	7·659624	7·662382	7·665122	7·667845	44
16	7·681208	7·683833	7·686441	7·689034	7·691611	7·694173	43
17	7·706762	7·709237	7·711697	7·714144	7·716577	7·718997	42
18	7·730896	7·733237	7·735566	7·737882	7·740186	7·742478	41
19	7·753758	7·755980	7·758190	7·760389	7·762577	7·764754	40
20	7·775477	7·777591	7·779694	7·781787	7·783870	7·785943	39
21	7·796162	7·798177	7·800183	7·802180	7·804167	7·806146	38
22	7·815906	7·817831	7·819749	7·821658	7·823558	7·825451	37
23	7·834791	7·836635	7·838471	7·840300	7·842121	7·843934	36
24	7·852889	7·854658	7·856419	7·858174	7·859922	7·861662	35
25	7·870262	7·871962	7·873655	7·875342	7·877022	7·878695	34
26	7·886968	7·888603	7·890233	7·891857	7·893474	7·895085	33
27	7·903054	7·904631	7·906201	7·907766	7·909326	7·910879	32
28	7·918566	7·920087	7·921603	7·923114	7·924619	7·926119	31
29	7·933543	7·935012	7·936477	7·937937	7·939392	7·940842	30
′	30″	24″	18″	12″	6″	0″	′

Cosine.

89 degrees.

0 degrees.

Sine.

′	0″	6″	12″	18″	24″	′
30	7·940842	7·942287	7·943727	7·945163	7·946594	29
31	7·955082	7·956481	7·957875	7·959264	7·960650	28
32	7·968870	7·970225	7·971576	7·972922	7·974265	27
33	7·982233	7·983547	7·984857	7·986164	7·987466	26
34	7·995198	7·996473	7·997745	7·999013	8·000277	25
35	8·007787	8·009026	8·010261	8·011493	8·012722	24
36	8·020021	8·021225	8·022427	8·023625	8·024819	23
37	8·031919	8·033092	8·034261	8·035426	8·036589	22
38	8·043501	8·044642	8·045781	8·046916	8·048048	21
39	8·054781	8·055894	8·057003	8·058109	8·059213	20
40	8·065776	8·066861	8·067942	8·069021	8·070097	19
41	8·076500	8·077558	8·078613	8·079666	8·080716	18
42	8·086965	8·087997	8·089028	8·090056	8·091081	17
43	8·097183	8·098192	8·099198	8·100203	8·101204	16
44	8·107167	8·108153	8·109136	8·110118	8·111097	15
45	8·116926	8·117890	8·118852	8·119812	8·120769	14
46	8·126471	8·127414	8·128355	8·129294	8·130231	13
47	8·135810	8·136733	8·137654	8·138574	8·139491	12
48	8·144953	8·145857	8·146759	8·147659	8·148557	11
49	8·153907	8·154793	8·155676	8·156558	8·157438	10
50	8·162681	8·163548	8·164414	8·165279	8·166141	9
51	8·171280	8·172131	8·172980	8·173827	8·174673	8
52	8·179713	8·180547	8·181380	8·182211	8·183041	7
53	8·187985	8·188803	8·189620	8·190436	8·191250	6
54	8·196102	8·196905	8·197707	8·198508	8·199307	5
55	8·204070	8·204859	8·205647	8·206433	8·207217	4
56	8·211895	8·212670	8·213443	8·214215	8·214986	3
57	8·219581	8·220342	8·221102	8·221861	8·222618	2
58	8·227134	8·227882	8·228628	8·229374	8·230118	1
59	8·234557	8·235292	8·236026	8·236759	8·237491	0
′	60″	54″	48″	42″	36″	′

Cosine.

89 degrees.

0 degrees.

Sine.

′	30″	36″	42″	48″	54″	60″	′
30	7·948020	7·949442	7·950859	7·952271	7·953679	7·955082	29
31	7·962031	7·963407	7·964779	7·966147	7·967511	7·968870	28
32	7·975603	7·976937	7·978267	7·979593	7·980915	7·982233	27
33	7·988764	7·990058	7·991349	7·992636	7·993919	7·995198	26
34	8·001538	8·002795	8·004048	8·005298	8·006544	8·007787	25
35	8·013947	8·015168	8·016387	8·017601	8·018813	8·020021	24
36	8·026011	8·027199	8·028384	8·029566	8·030744	8·031919	23
37	8·037749	8·038905	8·040059	8·041209	8·042357	8·043501	22
38	8·049178	8·050304	8·051428	8·052549	8·053666	8·054781	21
39	8·060314	8·061412	8·062507	8·063599	8·064689	8·065776	20
40	8·071171	8·072242	8·073310	8·074376	8·075439	8·076500	19
41	8·081764	8·082809	8·083852	8·084892	8·085929	8·086965	18
42	8·092104	8·093125	8·094143	8·095159	8·096172	8·097183	17
43	8·102204	8·103201	8·104196	8·105188	8·106179	8·107167	16
44	8·112074	8·113049	8·114021	8·114992	8·115960	8·116926	15
45	8·121725	8·122678	8·123630	8·124579	8·125526	8·126471	14
46	8·131166	8·132099	8·133030	8·133959	8·134885	8·135810	13
47	8·140406	8·141319	8·142231	8·143140	8·144048	8·144953	12
48	8·149453	8·150348	8·151241	8·152131	8·153020	8·153907	11
49	8·158316	8·159193	8·160067	8·160940	8·161811	8·162681	10
50	8·167002	8·167861	8·168718	8·169574	8·170428	8·171280	9
51	8·175517	8·176360	8·177200	8·178039	8·178877	8·179713	8
52	8·183869	8·184695	8·185520	8·186343	8·187165	8·187985	7
53	8·192062	8·192873	8·193683	8·194491	8·195297	8·196102	6
54	8·200104	8·200901	8·201695	8·202488	8·203280	8·204070	5
55	8·208000	8·208782	8·209562	8·210341	8·211119	8·211895	4
56	8·215755	8·216523	8·217289	8·218055	8·218819	8·219581	3
57	8·223374	8·224128	8·224882	8·225633	8·226384	8·227134	2
58	8·230861	8·231603	8·232343	8·233082	8·233820	8·234557	1
59	8·238221	8·238951	8·239679	8·240405	8·241131	8·241855	0
′	30″	24″	18″	12″	6″	0″	′

Cosine.

89 degrees.

1 degree.

Sine.

′	0″	6″	12″	18″	24″	′
0	8·241855	8·242578	8·243300	8·244021	8·244741	59
1	8·249033	8·249744	8·250455	8·251164	8·251871	58
2	8·256094	8·256794	8·257493	8·258190	8·258887	57
3	8·263042	8·263731	8·264419	8·265105	8·265791	56
4	8·269881	8·270559	8·271236	8·271912	8·272587	55
5	8·276614	8·277281	8·277948	8·278613	8·279278	54
6	8·283243	8·283901	8·284557	8·285213	8·285867	53
7	8·289773	8·290421	8·291068	8·291713	8·292358	52
8	8·296207	8·296845	8·297482	8·298118	8·298754	51
9	8·302546	8·303175	8·303803	8·304430	8·305056	50
10	8·308794	8·309414	8·310033	8·310651	8·311268	49
11	8·314954	8·315565	8·316175	8·316785	8·317393	48
12	8·321027	8·321630	8·322231	8·322832	8·323433	47
13	8·327016	8·327611	8·328204	8·328797	8·329389	46
14	8·332924	8·333511	8·334096	8·334681	8·335265	45
15	8·338753	8·339332	8·339909	8·340486	8·341063	44
16	8·344504	8·345075	8·345646	8·346215	8·346784	43
17	8·350181	8·350744	8·351307	8·351869	8·352430	42
18	8·355783	8·356340	8·356895	8·357450	8·358005	41
19	8·361315	8·361864	8·362413	8·362961	8·363508	40
20	8·366777	8·367319	8·367861	8·368402	8·368943	39
21	8·372171	8·372707	8·373242	8·373776	8·374310	38
22	8·377499	8·378028	8·378557	8·379084	8·379612	37
23	8·382762	8·383285	8·383807	8·384329	8·384850	36
24	8·387962	8·388479	8·388995	8·389510	8·390025	35
25	8·393101	8·393611	8·394121	8·394631	8·395139	34
26	8·398179	8·398684	8·399188	8·399691	8·400194	33
27	8·403199	8·403698	8·404196	8·404694	8·405191	32
28	8·408161	8·408654	8·409147	8·409639	8·410131	31
29	8·413068	8·413555	8·414042	8·414529	8·415015	30
′	60″	54″	48″	42″	36″	′

Cosine.

88 degrees.

1 degree.

Sine.

′	30″	36″	42″	48″	54″	60″	′
0	8·245459	8·246176	8·246892	8·247607	8·248321	8·249033	59
1	8·252578	8·253284	8·253988	8·254691	8·255393	8·256094	58
2	8·259582	8·260276	8·260970	8·261662	8·262353	8·263042	57
3	8·266475	8·267158	8·267841	8·268522	8·269202	8·269881	56
4	8·273260	8·273933	8·274605	8·275275	8·275945	8·276614	55
5	8·279941	8·280604	8·281265	8·281925	8·282585	8·283243	54
6	8·286521	8·287173	8·287825	8·288475	8·289125	8·289773	53
7	8·293002	8·293645	8·294287	8·294928	8·295568	8·296207	52
8	8·299388	8·300021	8·300654	8·301286	8·301916	8·302546	51
9	8·305681	8·306306	8·306929	8·307552	8·308173	8·308794	50
10	8·311885	8·312500	8·313115	8·313729	8·314342	8·314954	49
11	8·318001	8·318608	8·319214	8·319819	8·320423	8·321027	48
12	8·324032	8·324630	8·325228	8·325825	8·326421	8·327016	47
13	8·329980	8·330571	8·331160	8·331749	8·332337	8·332924	46
14	8·335848	8·336431	8·337013	8·337593	8·338174	8·338753	45
15	8·341638	8·342213	8·342787	8·343360	8·343933	8·344504	44
16	8·347352	8·347919	8·348485	8·349051	8·349616	8·350181	43
17	8·352991	8·353551	8·354110	8·354669	8·355226	8·355783	42
18	8·358558	8·359111	8·359663	8·360214	8·360765	8·361315	41
19	8·364055	8·364600	8·365146	8·365690	8·366234	8·366777	40
20	8·369482	8·370021	8·370560	8·371097	8·371635	8·372171	39
21	8·374843	8·375375	8·375907	8·376438	8·376969	8·377499	38
22	8·380138	8·380664	8·381190	8·381714	8·382239	8·382762	37
23	8·385370	8·385890	8·386409	8·386927	8·387445	8·387962	36
24	8·390539	8·391053	8·391566	8·392078	8·392590	8·393101	35
25	8·395647	8·396155	8·396662	8·397168	8·397674	8·398179	34
26	8·400696	8·401198	8·401699	8·402200	8·402700	8·403199	33
27	8·405687	8·406183	8·406679	8·407173	8·407668	8·408161	32
28	8·410621	8·411112	8·411602	8·412091	8·412579	8·413068	31
29	8·415500	8·415985	8·416469	8·416953	8·417436	8·417919	30
′	30″	24″	18″	12″	6″	0″	′

Cosine.

88 degrees.

1 degree.

Sine.

′	0″	6″	12″	18″	24″	′
30	8·417919	8·418401	8·418883	8·419364	8·419844	29
31	8·422717	8·423194	8·423670	8·424146	8·424621	28
32	8·427462	8·427934	8·428405	8·428876	8·429346	27
33	8·432156	8·432623	8·433089	8·433554	8·434020	26
34	8·436800	8·437262	8·437723	8·438183	8·438644	25
35	8·441394	8·441851	8·442308	8·442763	8·443219	24
36	8·445941	8·446393	8·446845	8·447296	8·447746	23
37	8·450440	8·450888	8·451335	8·451781	8·452227	22
38	8·454893	8·455336	8·455779	8·456220	8·456662	21
39	8·459301	8·459740	8·460178	8·460615	8·461052	20
40	8·463665	8·464099	8·464532	8·464965	8·465398	19
41	8·467985	8·468415	8·468844	8·469273	8·469701	18
42	8·472263	8·472688	8·473113	8·473538	8·473962	17
43	8·476498	8·476920	8·477341	8·477761	8·478181	16
44	8·480693	8·481110	8·481527	8·481944	8·482360	15
45	8·484848	8·485261	8·485674	8·486087	8·486499	14
46	8·488963	8·489373	8·489782	8·490190	8·490598	13
47	8·493040	8·493445	8·493851	8·494255	8·494660	12
48	8·497078	8·497480	8·497882	8·498283	8·498683	11
49	8·501080	8·501478	8·501876	8·502273	8·502670	10
50	8·505045	8·505439	8·505833	8·506227	8·506621	9
51	8·508974	8·509365	8·509755	8·510145	8·510535	8
52	8·512867	8·513255	8·513642	8·514029	8·514415	7
53	8·516726	8·517110	8·517494	8·517877	8·518260	6
54	8·520551	8·520932	8·521312	8·521692	8·522072	5
55	8·524343	8·524720	8·525097	8·525474	8·525850	4
56	8·528102	8·528476	8·528850	8·529223	8·529596	3
57	8·531828	8·532199	8·532570	8·532940	8·533310	2
58	8·535523	8·535891	8·536258	8·536625	8·536992	1
59	8·539186	8·539551	8·539915	8·540279	8·540643	0
	60″	54″	48″	42″	36″	′

Cosine.

88 degrees.

1 degree.

Sine.

′	30″	36″	42″	48″	54″	60″	′
30	8·420325	8·420804	8·421283	8·421761	8·422239	8·422717	29
31	8·425096	8·425570	8·426044	8·426517	8·426990	8·427462	28
32	8·429815	8·430285	8·430753	8·431221	8·431689	8·432156	27
33	8·434484	8·434948	8·435412	8·435875	8·436338	8·436800	26
34	8·439103	8·439562	8·440021	8·440479	8·440937	8·441394	25
35	8·443674	8·444128	8·444582	8·445035	8·445488	8·445941	24
36	8·448196	8·448646	8·449095	8·449544	8·449992	8·450440	23
37	8·452673	8·453118	8·453562	8·454006	8·454450	8·454893	22
38	8·457103	8·457543	8·457984	8·458423	8·458863	8·459301	21
39	8·461489	8·461925	8·462360	8·462796	8·463230	8·463665	20
40	8·465830	8·466262	8·466693	8·467124	8·467555	8·467985	19
41	8·470129	8·470557	8·470984	8·471410	8·471837	8·472263	18
42	8·474386	8·474809	8·475232	8·475655	8·476077	8·476498	17
43	8·478601	8·479020	8·479439	8·479857	8·480276	8·480693	16
44	8·482776	8·483191	8·483606	8·484020	8·484434	8·484848	15
45	8·486910	8·487322	8·487733	8·488143	8·488553	8·488963	14
46	8·491006	8·491414	8·491821	8·492228	8·492634	8·493040	13
47	8·495064	8·495467	8·495871	8·496274	8·496676	8·497078	12
48	8·499084	8·499484	8·499883	8·500283	8·500681	8·501080	11
49	8·503067	8·503463	8·503859	8·504255	8·504650	8·505045	10
50	8·507014	8·507406	8·507799	8·508191	8·508582	8·508974	9
51	8·510925	8·511314	8·511703	8·512091	8·512480	8·512867	8
52	8·514801	8·515187	8·515572	8·515957	8·516342	8·516726	7
53	8·518643	8·519025	8·519407	8·519789	8·520170	8·520551	6
54	8·522451	8·522830	8·523209	8·523587	8·523965	8·524343	5
55	8·526226	8·526602	8·526977	8·527353	8·527727	8·528102	4
56	8·529969	8·530341	8·530714	8·531085	8·531457	8·531828	3
57	8·533679	8·534049	8·534418	8·534786	8·535155	8·535523	2
58	8·537358	8·537725	8·538091	8·538456	8·538821	8·539186	1
59	8·541007	8·541370	8·541733	8·542095	8·542457	8·542819	0
′	30″	24″	18″	12″	6″	0″	′

Cosine.

88 degrees.

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